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Optimization models to characterize the broadcast capacity of vehicular ad hoc networks

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A B S T R A C T

Broadcast capacity of the entire network is one of the fundamental properties of vehicular ad hoc networks (VANETs). It measures how efficiently the information can be transmitted in the network and usually it is limited by the interference between the concurrent transmissions in the physical layer of the network. This study defines the broadcast capacity of vehicular ad hoc network as the maximum successful concurrent transmissions. In other words, we measure the maximum number of packets which can be transmitted in VANET simultaneously, which characterizes how fast a new message such as a traffic incident can be transmitted in VANET. Integer programming (IP) models are first developed to explore the maximum number of successful receiving nodes as well as the maximum number of transmitting nodes in VANET. The models embed an traffic flow model in the optimization problem. Since IP model cannot be efficiently solved as the network size increases, this study develops a statistical model to predict the network capacity based on the significant parameters in the transportation and communication networks. MITSIMLab is used to generate the necessary traffic flow data. Response surface method and linear regression technologies are applied to build the statistical models. Thus, this paper brings together an array of tools to solve the broadcast capacity problem in VANET. The proposed methodology provides an efficient approach to estimate the performance of VANET in real-time, which will impact the efficacy of travel decision making.

1. Introduction

Vehicular ad hoc networks (VANETs) are infrastructure light vehicular networks, in which instrumented vehicles communicate with one another across wireless links directly or through possible intermediate vehicles. Different paradigms for information communication have been proposed under the umbrella of vehicle infrastructure integration (VII, 2007) by the USDOT: this includes vehicle to vehicle (V2V) communication systems such as VANET; vehicle to infrastructure communication (V2I) and a hybrid mix of both V2V and V2I. The recent deployment projects have demonstrated the significant benefits of V2V and V2I systems on alleviating congestion, improving safety and in providing other non-traffic applications. Similarly, VANETs have a great potential to be applied to traffic control and traffic safety management so that the transportation system performance can be improved.

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There is a growing belief that the success of VANET-based inter-vehicle communications will facilitate a plethora of potential applications. In general, VANET applications will deliver functions and services in five key areas: (i) *traffic* safety: provides drivers with timely information to enable cooperative vehicle interactions with roadway environment to mitigate collision/injuries/deaths; (ii) *vehicular* mobility and traffic operation: increases road capacity by dynamic cooperative traffic operation and management; (iii) *consumer* convenience: provides information share for people sharing common interests; (iv) *vehicle* probe data acquisitions: aggregates real-time vehicular data for use in mobility and environmental applications; (v) *opportunistic* data sharing: provides opportunistic extension of the wireless mesh networks in improving the capacity and coverage, or information service for emergency situations. The first real 5.9 GHz dedicated short range communication technology (DSRC) was approved by FCC in December 2003 for vehicle infrastructure integration initiative (VII) applications in USA. Similarly, vehicle to vehicle communication for safety applications was tested in the advanced safety vehicle (ASV) program sponsored by the Japanese Ministry of Land, Infrastructure and Transport (MLIT) (ITS, 2008). Other projects in Europe such as NOW (2008) and FleetNet (2008) demonstrate the significant traffic management benefits that can be obtained by a meaningful integration of wireless communication and vehicular networks. VANET technologies, once successfully developed, will have significant impacts to the society in general.

The potential applications of VANET have sparked a significant amount of research in both wireless communication and transportation areas. Although, VANETs have certain similarities to other mobile ad hoc networks, they are characterized by certain fundamental properties which makes their study more challenging (Blum et al., 2004; Moreno et al., 2005; Nekovee, 2007). For instance, VANETs are characterized by dynamic vehicular network in which mobile nodes are varying both spatially and temporally. In addition, driver behavior, mobility constraints due to space and existing infrastructure and high speed/acceleration of vehicles cause unique challenges to solve these problems. Furthermore, the number of vehicles in VANET cannot be scheduled or controlled in real-time because drivers will enter or exit the traffic network at different time instances. Several efforts have been invested on the performance of VANET in the recent years. For example, Artimy et al. (2004, 2005a,b) studied the information propagation in VANET by simulation of the transportation networks with an embedded communication network. Their simulation results show that the traffic factors, such as traffic density and traffic speed, affect the connectivity of VANET significantly. Wu et al. (2004) studied the spatial information propagation in VANET. Their analytical expression calculates the time delay of information transmission. Jin and Recker (2006) employed stochastic models to discuss the reliability of inter-vehicle communication under both uniform and general traffic streams. Assuming the presence of equipped vehicles follows an independent homogeneous Poisson process, Wang (2007) proposed a closed form expression for the expected propagation distance and its variance in the case of no transmission delay. Du et al. (2007) and Ukkusuri and Du (in press) explored the connectivity of VANET at every time instant by integrating traffic flow features.

Recently, there is a great deal of interest in understanding the fundamental transmission capacity of wireless networks in which transmission is permitted only between single source–destination pairs. This is because this allows us to characterize the throughput of the wireless network which determines the efficiency and amount of information flow in the ad hoc wireless network. In a VANET, a related problem of characterizing is very important since the throughput of the information is dependent on the vehicular mobility and the opportunities available for communication. As in a self-organized system, each individual vehicle broadcasts its own messages to the entire network nodes without cooperation. Therefore, one of the primary problems for VANET is data loss due to packet collision among concurrent transmissions (Jamieson and Balakrishnan, 2008). When the network loses data, retransmissions have to be conducted if the application requires a certain level of data fidelity, and doing so delay information propagation and degrade the information propagation efficiency. Therefore, the broadcast capacity acts as a key performance limit for information propagation efficiency in VANET. Our *goal* in this paper is to understand this issue comprehensively using optimization approaches to characterize the efficiency of information flow in VANET.

1.1. Related work

In the context of wireless ad hoc networks, there has been some related work which addresses the issue of network capacity. A seminal work is the research by Gupta and Kumar (2000) which defines per node throughput in a wireless network by a bit-distance product. This work supposes the network transports one bit-meter when one bit has been transported a distance of 1 m. The throughput of the network is measured by the number of bit-meters that are transported per second. Their results show that as a wireless network is formed by n identical randomly located nodes, each node capable of transmitting at W bits/s and with a fixed broadcast radius, the local throughput per node is $\Theta\left(\frac{W}{\sqrt{n\log n}}\right)$ bits/s under a non-interference protocol. It is also shown that if the distribution of nodes, the traffic patterns and the transmission range of each node are optimally chosen, the bit-meter product that can be transported by the network per second is $\Theta(\sqrt{An})$ bit-meters/s. Extending the case of stationary nodes in Gupta and Kumar (2000), Grossglauser and Tse (2002) study a mobile ad hoc network, in which n mobile nodes communicate in random source–destination pairs. Allowing the unbounded delay and using only the one-hop relaying, their results show the mobility of nodes increases per node throughput asymptotically as the number of nodes becomes arbitrary large. Studying a wireless ad hoc network with finite immobile nodes, Jain et al. (2005) propose methods to compute the upper and lower bounds of the network capacity with no power control. Behzad and Rubin (2006) propose that independent of nodal distribution and traffic pattern, the throughput capacity of an ad hoc wireless network is maximized by properly increasing the nodal transmission power. Gastpar and Vetterli (2002) extend the work of Gupta and Kumar (2000) by assuming a network coding model where nodes could cooperate in arbitrary ways.

In addition, there is only a random picked source and destination, and the rest of the nodes act as relays. Their results show that under these conditions the throughput is $O(\log(n))$.

Based on different network features, there is other research, for instance, Yi et al. (2003) and Huang et al. (2007) propose the throughput of the wireless network increases with directional antennas. Li et al. (2001) present simulation-based results on the throughput of the small ad hoc network. Negi and Rajeswaran (2004) study the throughput of the wireless network, in which communication links apply ultra-wide band links.

Only a few papers deal with the transmission limits in wireless ad hoc network, in which transmissions are permitted from a source to a group of destinations. Koskinen and Virtamo (2005) evaluated the probability of successful transmission in a random time slot with the assumptions that all nodes employ the same transmission range and nodes are randomly located. Son et al. (2006) used Mica2 platform to study the relationship between concurrent transmissions and *signal-to-interference-plus-noise ratio* in static wireless networks. Their research emphasizes the effects that the concurrent transmissions have on packet error rate.

In relation to ~~VANET~~, there are two key limitations which should be considered to model the capacity of the network. First, most of the previous research ~~focus~~ on general wireless ad hoc networks, and either assume the network is static or assume nodes move randomly in the study area. Obviously, those assumptions are not applicable to ~~VANET~~. For example, in ~~VANET~~, vehicles move on the fixed road network under a set of traffic control policies. Transportation science studies have shown that under the given traffic demand, road capacity, and speed limit, average traffic density and speed are not fully predictable. Consequently, the movements of vehicles in ~~VANET~~ can be simulated using data about the traffic conditions. Second, even though nodal capacity of wireless network has been extensively studied in terms of network size, transmission power, bandwidth constraints, and antenna directionality, very few studies discuss about the concurrent transmissions which influence the capacity of wireless network. Moreover, it is clear that the node density, node distribution and nodal mobility in road networks will characterize the message transmission limits in ~~VANET~~. Therefore, it is valuable to investigate the interaction between the transmission limits in ~~VANET~~ and the traffic flow features. To the best of our knowledge, there is no previous work which addresses the above issues in ~~VANET~~.

Motivated by the above points, assuming the given VANET employs a broadcast transmission protocol, we consider that the number of successful concurrent transmissions in VANET reflects the information broadcast capacity of the entire network, and also may cause the performance bottlenecks. In addition, we account for the fact that number of successful concurrent transmissions is affected by various factors, such as traffic density, vehicular distribution, traffic pattern, network size, transmission power, bandwidth constraints, and antenna directionality. This research, therefore, defines the broadcast capacity of ~~VANET~~ as the *maximum successful concurrent transmissions*.[1] With this definition, we further investigate the effect of traffic flow network features and communication network configurations on the capacity of ~~VANET~~ on freeway segments. Our research objective is to explore the explicit relationship between the broadcast capacity of ~~VANET~~ and the significant factors in both transportation and wireless communication networks. Specifically, two integer programming (IP) models are developed which incorporate traffic flow features to optimize the capacity of ~~VANET~~. Since the IP models cannot be solved efficiently in real-time, we develop an advanced statistical model to predict the network capacity by the significant parameters in both transportation networks an communication networks. In the description that follows, the performance of VANET refers to the capacity of VANET and nodes are also referred as vehicles.

The rest of this paper is organized as follows: Section 2 defines the network model used in this paper. Section 3 describes our IP models to investigate the capacity under our definition. Section 4 discuss our methodology to develop the statistical models, which are used to predict the capacity of ~~VANET~~ by significant parameters in both the road networks and the communication networks. Section 5 summarizes our main results, insights, and further identifies the future work.

2. Network model

We begin our study with some assumptions about the network model. We first briefly introduce our specifications about the network topology, the medium access control, and the antenna of the communication device. After those, we study in depth the application of signal-interference-plus-noise threshold in our study.

2.1. Network topology

The VANETs of interest in this study are comprised of a fleet of n vehicles moving on the *sample freeway segment* defined in Chapter 3 and with length L. Moreover, vehicles on the entire road segment are numbered from the entrance to exit.

2.2. Medium access control (MAC)

The time varying network topology and the lack of centralized control in VANET make the use of coordinated MAC protocols difficult, but render a random access schema to be a preferred choice.

[1] A transmission includes a receiving node and a transmitting node.

Therefore, in this study, we assume the medium access control in VANET applies the spirit of a random access scheme. That is to say, vehicles independently broadcast messages to the nodes in the entire network without cooperation (Koskinen and Virtamo, 2005).

2.3. Antennas

We assume individual vehicles are equipped with directional antennas. In addition, if two or more messages are sent to the same vehicle simultaneously, they will result in message collisions and failed transmissions. But, one message can be received by multiple receiving nodes simultaneously.

2.4. Signal-to-interference-and-noise ration (SINR)

Given by Gupta and Kumar (2000), SINR is one of the conditions of a successful transmission: in any time slot, for a given receiver node k at y_k, and a transmitting node j at y_j, if node j can successfully transmit message to node k then the SINR at receiver is greater than some common threshold, TH, i.e.

$$\frac{p_j\|y_j - y_k\|^{-\alpha}}{N_0 + I} \geqslant \mathrm{TH}, \tag{1}$$

where N_0 is the background noise on the frequency channel utilized by network, p_j is the transmission power of node j, and I denotes the sum of the interference power. Defining the transmission state of each node by e_i,

$$e_i = \begin{cases} 1 & \text{node } i \text{ is transmitting packets,} \\ 0 & \text{o.w.,} \end{cases}$$

the sum of interference can be measured by

$$I = \sum_{1 \neq i,j} e_i p_i \|y_i - y_k\|^{-\alpha}. \tag{2}$$

In order to simplify the problem, we ignore the background noise and assume that all nodes transmit messages with the same power, $p_i \equiv p$, then the SINR condition for a successful transmission is reduced to

$$\sum_{i \neq i,j} e_i \|y_i - y_k\|^{-\alpha} \leqslant \frac{1}{\mathrm{TH}} \|y_j - y_k\|^{-\alpha}. \tag{3}$$

Eq. (3) demonstrates that two key factors impact the success of a transmission: (1) the distances between the transmitters and receivers and (2) the interference around the receivers. In other words, the distribution of vehicles, the transmission range, and the SINR threshold of the wireless communication device will influence the success of the transmission in VANET significantly. This observation serves as a starting point for our study in following sections.

3. Modeling the capacity of VANET by IP models

The number of successful concurrent transmissions in VANET reflects how many packets can be successfully transmitted simultaneously in VANET at any given snapshot time t. In other words, it demonstrates how fast an information can be disseminated in the VANET. Based on this, we define the capacity of VANET in this study as the maximum number of successful concurrent transmissions in VANET. Considering each successful transmission includes one transmitter as well as one receiver. To capture this objective, clearly, involves two aspects. On the one hand, we can maximize the number of successful concurrent receiving nodes (receivers). On the other hand, we can maximize the number of successful concurrent transmitting nodes (transmitters). A successful receiving node is governed by four conditions: (1) the node should be at the receiving state, (2) since the signal power degrades in terms of the distance, only the nodes in the transition range are possible to obtain the information packages successfully, (3) there must be a corresponding transmitting node, which sends packets to the receiving node and (4) the signal power at the receiving node should be higher enough than the interference. In other words, SINR at the receiving node should be higher than its required threshold. Correspondingly, a successful transmitting node is identified by other two conditions: (1) the node is in transmitting state and (2) there is at least one other node, which is successfully receiving its packets. In the following study, we build the IP models for: (i) maximum concurrent receivers and (ii) the maximum concurrent transmitters. Before we discuss the IP models, we first add two more notations:

- R: a parameters that denotes the transmission range and
- z^t_{jk}: a decision variable which is defined below:

$$z^t_{jk} = \begin{cases} 1 & \text{node } k \text{ successfully receives packets of node } j \text{ at time } t, \\ 0 & \text{o.w.} \end{cases}$$

The IP model which maximizes the number of concurrent receiving nodes are provided in IP model 1, where the vehicular location variables, $y^t_i,\ i = 1, \ldots, n$ are considered as input parameters. After changing the constraint (10) and the objective function in Eq. (4), we obtain IP model 2, which maximizes the number of concurrent transmitting nodes. The two IP models are provided below separately.

IP model 1: maximizing the number of successful concurrent receiving nodes

$$\max \sum_{j=1}^{n} \sum_{k=1}^{n} z_{jk}^{t}, \tag{4}$$

$$\sum_{i=1(i\neq j)}^{k-1} e_i^t (y_i^t - y_k^t)^{-\alpha} - \mathrm{TH}^{-1} e_j^t (y_j^t - y_k^t)^{-\alpha} \leqslant M(1 - z_{jk}^t), \quad j \neq k \tag{5}$$

$$R^{-\alpha} - e_j^t * (y_j^t - y_k^t)^{-\alpha} \leqslant (1 - z_{jk}^t), \quad k = 1,\ldots,n, \quad j = 1,\ldots,k-1, \tag{6}$$

$$e_k^t + \sum_{j=1}^{k-1} z_{jk}^t \leqslant 1, \tag{7}$$

$$\sum_{k=1}^{n} e_k^t \geqslant 1, \tag{8}$$

$$\sum_{j=k}^{n} z_{jk}^t = 0, \quad k = 1,\ldots,n, \tag{9}$$

$$\sum_{k=j+1}^{n} z_{jk}^t \leqslant n e_j, \quad j = 1,\ldots,n, \tag{10}$$

$$e_k^t = 0, \text{ or } 1, \quad k = 1,\ldots,n, \tag{11}$$

$$z_{jk}^t = 0, \text{ or } 1, \quad k = 1,\ldots,n, \quad j = 1,\ldots,n, \tag{12}$$

where

- Constraint (5) presents the condition that if the information packets can be successfully received at a receiver then the signal power at the receiver is higher enough than the interference around.
- Constraint (6) presents the condition that if information packets can be successfully received at a receiver then the receiver should be in the transmission range of the corresponding transmitter.
- Constraint (7) presents the condition that a node can either be a transmitter or a receiver, but not both at the same time.
- Constraint (8) presents one more requirement for the network that at least one node is transmitting information at each time.
- Constraint (9) presents that each node can only get information from one direction (forward or backward).
- Constraint (10) presents the condition that if a node is successfully receiving information packets, then there must be a corresponding transmitter.
- Constraint (11) and Constraint (12) show e and z are binary variables.

IP model 2: maximizing the number of successful concurrent transmitting nodes

$$\max \sum_{j=1}^{n} e_j^t, \tag{13}$$

$$\text{Eq. (5)}\ldots \text{ to } \ldots\text{Eq. (9)},$$

$$\text{Eq. (11)}\ldots \text{ to } \ldots\text{Eq. (12)},$$

$$\sum_{k=j+1}^{n} z_{jk}^t \geqslant e_j^t, \quad j = 1,\ldots,n, \tag{14}$$

where

- Objective function (13) maximizes the number of successful concurrent transmitters in VANET.
- Constraint (14) presents the condition that if a node is considered as a successful transmitting node then there is at least one other node which is successfully receiving its packets.

When VANET is used to build decentralized advanced traveler information systems (ATIS), the capacity of VANET directly influences the information quality and quantity reaching to individual vehicles, which consequently affect driver's routing decision en-route. Therefore, it is necessary to integrate the capacity of VANET into the routing decision policy. To realize it, the capacity of VANET must be evaluated in real-time. Even though the above IP models can provide the capacity of a VANET with the given network topology (i.e. vehicular positions) at each time instant, two limitations of the IP models limits their potential applications for real-time traffic operation managements. First of all, based on our computational experiences, for most topology of VANET, it will take a relative long time to obtain the optimal value. This is not consistent with the rapid topology change in VANET, since for each topology, the capacity of the VANET need to be re-computed. Second, the vehicular trajectory data used in IP models is the microscopic traffic data, which cannot explicitly capture the relationship between traffic flow features and the capacity of VANET. Motivated by the above points, we next investigate the statistical models which predict the capacity of VANET by the significant parameters in communication networks as well as macroscopic traffic flow characteristics.

4. Modeling the capacity of VANET by statistical models

Our main idea is to estimate the capacity of VANET by using the statistical models. As presented in Fig. 1, we consider the IP models as a black box which has input data such as the transmission range, the SINR threshold, the network size, and the vehicular locations. The corresponding output data is indicated to VANET capacity. For a given road segment with fixed traffic flow characteristics, we solve the IP models at each time instant, and then build a relationship between the capacity of the VANET and the given parameters in the communication layer and the transportation layer. In the following paragraphs, we first discuss the experiment design, and then we demonstrate that the design helps in building an efficient statistical model.

4.1. Experiment design

Our experiment procedure to investigate the statistical models is described in Fig. 2. In this experiment, we characterize the transportation network by parameters, such as traffic demand, the number of lanes, and free flow speed limitation, which are also the necessary parameters used to specify the uninterrupted road segments in MITSIMLab (MIT, 2002). At the same time, the communication network is featured by transmission range and SINR threshold. Based on the features in both transportation and communication network mentioned above, we develop our statistical models. In order to obtain the statistical model with high prediction capability, we have the following requirements for the experimental procedure:

1. Vehicular trajectory data is required as the input parameters for the IP models 1 and 2. A well tested, open source dynamic traffic simulation framework – MITSIMLab (Ben-Akiva et al., 2002) is used as the simulation platform. It is developed by MIT Intelligent Transportation System Program.
2. Various combination among lanes, speed limit and traffic flow should be setup in the MITSIMLab simulation so that the vehicular trajectory data under different traffic flow conditions can be collected.
3. Various reasonable transmission ranges and SINR thresholds should be involved in the experiments.

4.1.1. Response surface methodology

Since a multitude of ~~factor~~ considered in the statistical model, it is hard to have a limited data set covering all possible situations. Therefore, we use response surface methodology (RSM) to design a set of experiments.

RSM is a technique employed before the regression analysis so that the prediction capability of the relationship can be guaranteed even though limited experiments are conducted. That is, both the input variables and their values must be selected carefully so that the output obtained represents a good approximation of the population. Thus, the RSM involves the

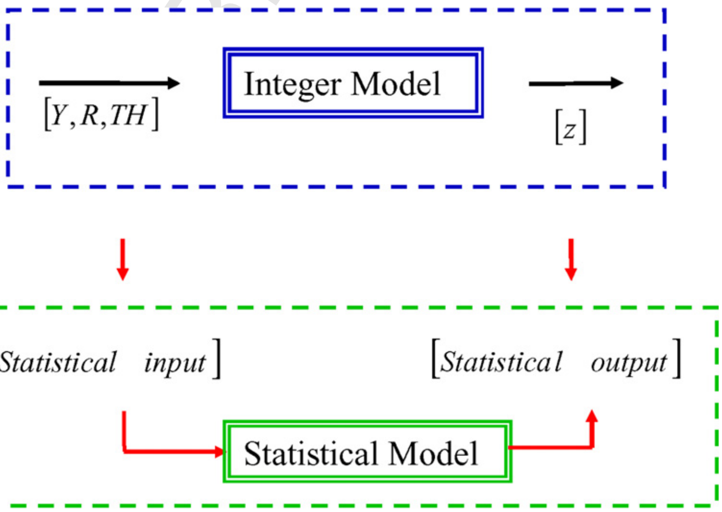

Fig. 1. Exploring the relationship between the network characteristics and VANET performance.

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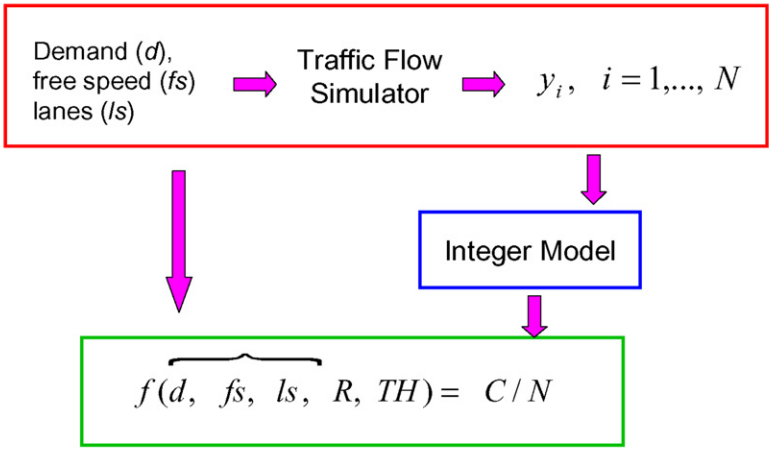

Fig. 2. Experiment procedure for developing the statistical models.

experiment design and the regression analysis to gain the understanding of the response variable using specific inputs. Regression analysis of RSM usually uses orthogonal arrays in the computation of the parameters (the most popular of these designs are the factorial 2^k) so that the coefficient estimates are uncorrelated and robust to small changes in the observed data. Moreover, RSM uses designs that must accomplish some specific properties (see Box and Draper (1987)). One of the principal properties is the rotatability or the distribution of information in the experimental region. A rotatable design has the same prediction variance in the response at any points that are equidistant from the design center. For more detail information about RSM experiment design, the reader is mainly referred to the books by Box and Draper (1987), Khuri and Cornell (1987), or Myers and Montogomery (2002).

In addition, before explaining the RSM approach to the VANET capacity problem, we will briefly introduce a class of RSM model called the central composite design which has been chosen because of its good rotatability and orthogonality properties for our experiment.

4.1.1.1. Central composite design CCD. The central composite design (CCD) is a design used in RSM and contains an imbedded factorial or fractional design with center points. To improve the experiment design space, it is augmented with a group of "star points" or "axial points" (α) that allow estimation of curvature. If the distance from the center of the design space to a factorial point is $\geqslant$ 1 unit for each factor, the distance from the center of the design space to the axial point is $\pm\alpha$ with $|\alpha| > 1$. The precise value of α depends on certain properties desired for the design and on the number of factors involved:

$$\alpha = (\text{number of runs})^{1/4}. \tag{15}$$

For example, for a three factor experiment at two levels $\alpha = (2^3)^{1/4} = 1.682$. The number of runs ($=2^k$) depends on the number of factors (k). Similarly, the number of center points to contain also depends on certain properties required for the design.

4.1.2. Description of RSM procedure in this study

The experimental design for the VANET capacity problem is a half CCD with five factors at two levels: demand (d), free speed (fs), the number of lanes (ls), SINR threshold (TH) and transmission range (R). Reference values used to generate the experiment data set is shown in Table 1. The design (combinations) will be used to predict the ratio of the successful receiving nodes (C) to the network size (N) (or the ratio (S/N) of the successful transmitting nodes (S) to the network size). Then the ratio (C/N) can be adjusted by a response surface using the five factors chosen, say $C/N = f(d, \text{fs}, \text{ls}, \text{TH}, R)$.

Based on the reference data, we obtain the experiments shown in Table 2. Each parameter has five possible values corresponding to the levels +1, −1, the center point 0 and the axial points +2, −2 ($\alpha = [2^{5-1}]^{\frac{1}{4}}$). For example, the corresponding traffic flows are 1000 (−2), 1750 (−1), 2500 (0), 3250 (+1), 4000 (+2) v/h. The CCD consists in two separate components: the factorial design which allows the estimation of the linear and interaction terms, and the additional points (axial and central points) to estimate coefficients for the second-order variables. Therefore, the number of runs for the CCD design is a function of the number of runs of the factorial experiment. For instance, in our experiment, the total number of runs for the half CCD is

Table 1
Reference values of CCD experiment design

	Demand	Lanes	Free speed	R	TH
Lower level (−1)	1750	2	72.5	112.5	17.5
Upper level (+1)	3200	4	77.5	237.5	22.5

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Table 2
CCD experiment design

d	ls	fs	R	TH	C/N	S/N	d	ls	fs	R	TH	C/N	S/N
1000	3	75	175	20	0.596511	0.272059	2500	3	75	175	20	0.684063	0.270510
1750	2	72.5	112.5	22.5	0.545913	0.311828	2500	3	75	175	20	0.694276	0.279070
1750	2	72.5	237.5	17.5	0.780279	0.258065	2500	3	75	175	20	0.688819	0.292683
1750	2	77.5	112.5	17.5	0.565577	0.258242	2500	3	75	175	20	0.692019	0.250000
1750	2	77.5	237.5	22.5	0.754056	0.291777	2500	3	75	175	20	0.676968	0.301394
1750	4	72.5	112.5	17.5	0.580614	0.272487	2500	3	80	175	20	0.692202	0.271646
1750	4	72.5	237.5	22.5	0.750006	0.254598	2500	5	75	175	20	0.662521	0.239687
1750	4	77.5	112.5	22.5	0.548842	0.266963	3250	2	72.5	112.5	17.5	0.604998	0.274279
1750	4	77.5	237.5	17.5	0.783397	0.250000	3250	2	72.5	237.5	22.5	0.784760	0.250223
2500	1	75	175	20	0.671612	0.240015	3250	2	77.5	112.5	22.5	0.569831	0.250000
2500	3	70	175	20	0.704380	0.275172	3250	2	77.5	237.5	17.5	0.838229	0.260882
2500	3	75	50	20	0.458840	0.296341	3250	4	72.5	112.5	22.5	0.563773	0.270468
2500	3	75	300	20	0.883917	0.278883	3250	4	72.5	237.5	17.5	0.810913	0.268519
2500	3	75	175	15	0.738219	0.258583	3250	4	77.5	112.5	17.5	0.601975	0.272156
2500	3	75	175	25	0.677866	0.268293	3250	4	77.5	237.5	22.5	0.785274	0.269330
2500	3	75	175	20	0.676876	0.269231	4000	3	75	175	20	0.708620	0.260714

Table 3
Range of parameters considered in the experiment design

	Demand	Lanes	Free speed	R	TH
Minimum	1000	1	70	50	15
Maximum	4000	5	80	300	25

32 which is obtained by $2^{5-1}+10+6=32$ where 2^{5-1} is the runs of the half factorial design, 10 corresponds to the axial points (2 level times 5 factors) and 6 corresponds to the number of replicates of the center point. The replicates in the center point are used to estimate the curvature and provide a stable measure of the variability (see Myers and Montgomery (2002)). As a rough guide a number of (3–5) center points are usually added in factorial experiments. However, for CCD designs Myers et al. (1992) have recommendations about the number of center points. In our experiment we decided to use 6 which uses one more run than the recommendation.

For each input data set: d, fs, ls, TH and R, we will run simulation models and IP models to obtain the vehicular trajectory data and the corresponding successful concurrent transmissions under different time instants. Finally we obtain the average output ratio: C/N or S/N for each input data set (see Table 2).

Based on the data set in Table 2, we will develop our statistical models. Due to the limitation of the experiment data set, the applicability of the developed statistical models are restricted to the range of values presented in Table 3. In other words, within the values in Table 3 we can assure that the adjusted R-square will represent a good measure of "goodness of fit" of the model. For values out of these ranges, the measure would be the R-square of prediction (included in the regressions also) but in some cases these are good and in others it is poor.

4.2. Statistical model for maximum successful concurrent receivers

Based on the CCD experiment data, we first conduct a simulation by MITSIMlab to get the vehicle trajectory data at each time ~~instance~~ under given network configurations. Combining the vehicle trajectory data, signal-to-interference-plus-noise threshold and transmission range in the CCD experiments, IP model 1 strictly explores the maximum number of successful concurrent receivers under each network configurations. Finally, we develop the statistical model to estimate the ratio of the maximum number of concurrent receivers C to the network size N. We have evaluated two different statistical models: a linear model and a second-order polynomial model.

We first use the best subsets algorithm to find the best linear model evaluated by the C_p Mallows and the highest adjusted R-sq criterions (Neter et al., 2004). The corresponding results in Table 4 show that the best linear model corresponds to the one presented in Table 5, which includes the independent variables: d, R and SINR. Its R-sq and R-sq (~~adg~~) are 97.7% and 97.5%, respectively.

Employing the same procedure above and further including the second-order variables, we obtain the final nonlinear model presented in Table 6. It includes the same variables in the linear model but with the addition of d^2. This improves the adjusted R-sq up to 98.2%.

Examining the statistics pertinent to predict the performance of the linear and nonlinear models above, we find that the linear model in Table 5 is not significant since the P-value indicates a fitness test value[2] which is less than the significant size

[2] $H_0: E\{y\} = \beta_0 + \beta_1 x_1 + \cdots + \beta_{p-1} x_{p-1}$; $H_a: E\{y\} \neq \beta_0 + \beta_1 x_1 + \cdots + \beta_{p-1} x_{p-1}$

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Table 4
Best subsets for C/N

Vars	R-sq	R-sq (adj)	Mallows C-p	S	d	l_s	f_s	R	TH
1	92.4	92.2	59.1	0.027246				×	
1	3.2	0.0	1086.2	0.097464	×				
2	95.6	95.3	24.3	0.021057	×			×	
2	94.5	94.1	37.0	0.023576				×	×
3	97.7	97.5	2.2	0.015477	×			×	×
3	95.7	95.2	26.1	0.021382	×	×		×	
4	97.7	97.4	4.0	0.015694	×	×		×	×
4	97.7	97.4	4.2	0.015761	×		×	×	×
5	97.7	97.3	6.0	0.015992	×	×	×	×	×

Table 5
Regression analysis: *avgratio* versus d, R, TH

Predictor	Coefficients	SE coefficients	T	P	VIF
The regression equation is avgratio = 0.445 + 0.000026d + 0.00170R − 0.00640TH					
Constant	0.44474	0.02890	15.39	0.000	
d	0.00002640	0.00000421	6.27	0.000	1.0
R	0.00170370	0.00005055	33.70	0.000	1.0
TH	−0.006404	0.001264	−5.07	0.000	1.0

R-sq = 97.7 and R-sq (adj) = 7.5%
PRESS = 0.00938855 and R-sq (pred) = 96.81

Source	DF	SS	MS	F	P
Analysis of variance					
Regression	3	0.287681	0.095894	400.33	0.000
Residual error	28	0.006707	0.000240		
Lack of fit	11	0.004612	0.000419	3.40	0.012
Total	31	0.294388			

Table 6
Regression analysis: *avgratio* versus d, R, TH, d^2

Predictor	Coefficients	SE coefficients	T	P	VIF
The regression equation is avgratio = 0.358 + 0.000101d + 0.00170R − 0.00640TH − 0.00000001d^2					
Constant	0.35817	0.03474	10.31	0.000	
d	0.00010067	0.00002151	4.68	0.000	36.6
R	0.00170370	0.00004269	39.91	0.000	1.0
TH	−0.006404	0.001067	−6.00	0.000	1.0
d^2	−0.00000001	0.00000000	−3.50	0.002	36.6

R-sq = 98.4% and R-sq (adj) = 98.2%
PRESS = 0.00824379 and R-sq (pred) = 97.20

Source	DF	SS	MS	F	P
Analysis of variance					
Regression	4	0.289775	0.072444	424.03	0.000
Residual error	27	0.004613	0.000171		
Lack of fit	10	0.002518	0.000252	2.04	0.094
Total	31	0.294388			

$\alpha = 0.05$. Therefore, we select the nonlinear model as our final statistical model to predict the ratio of the successful concurrent receivers to the network size of the VANET. Based on the regression model, we obtain the following insights, which are consistent with our intuition:

- The negative sign of SINR threshold, TH demonstrates that increasing SINR threshold degrades the proportion of successful concurrent receivers in VANET. This is consistent with our intuition that high SINR threshold results in less successful receivers in VANET.
- The positive sign of transmission range, R shows that large transmission range improves the proportion of successful receiving nodes in VANET. If the system only improves the transmission range, R of individual vehicles, then each transmitting node will cover more nodes. Due to the interference between concurrent transmissions, the optimal situation is that more nodes are in receiving state, consequently, the optimal ratio of receiving nodes in VANET will be improved. Therefore, the positive sign of R in the regression model is reasonable.

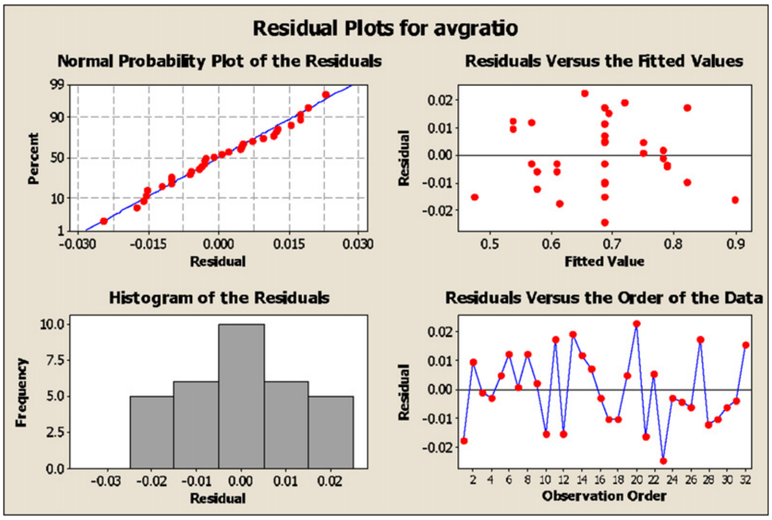

Fig. 3. Residual plot for average receiver ratio in VANET.

- The influence of traffic demand, *d* on the ratio of the successful receiving nodes in VANET to the network size has two sides. On the one hand, if only the traffic demand of VANET increases, and any other configurations in both the road network and the communication network do not change, then the traffic density will increase. This will make each transiting node cover more receiving nodes. Consequently, successful concurrent receiver ratio in the VANET will increase. This is consistent to the positive sign of demand variable in our regression model. On the other hand, if the traffic demand is increased a lot, then the vehicles will be distributed very densely. Consequently, the interference between concurrent transmission will degrade successful concurrent receiving rate. This phenomenon supports the negative sign of demand square in the selected regression model.

4.2.1. *Regression model test*

To examine the suitability of the final regression model in Table 6, we further test the model from several other aspects. Both graphic and formal statistical tests are applied.

- *Test for constancy of residual variance: the* plot of "residuals vs. fitted values" in Fig. 3 shows that the residuals randomly fluctuate around the reference line (residual = 0), which indicates that the residuals have constant variance.
- *Test for independence of residuals: the* plot of "residuals vs. the order of the data" in Fig. 3 indicates the residual is independent on the order of the data since the residuals are randomly distributed around the reference line. We further apply Durbin–Watson statistic to test the correlation between adjacent residuals. The results in Table 6 show Durbin–Watson statistic is equal to 2.35494, which is greater than the upper bound, 1.67.[3] Hence, there is no correlation between adjacent residuals.
- *Test for normality: the* normal probability plot in Fig. 3 shows an approximately linear pattern consistent with a normal distribution. Fig. 4 shows the result of the standard normality test (Kolmogorov–Smirnov test[4]) which also indicates a normal distribution of the residuals.
- *Lack of fit test: the P-value* (=0.0984) of lack of fit test shown in Table 6 is lager than usual test size, 0.05, so we accept H_0 that this model is appropriate.

4.2.2. *Calibrating the prediction capability*

In this study, we finally select the nonlinear model to predict the maximum ratio of successful concurrent receivers in VANET. The estimated regression coefficients, variance analysis and other statistics pertaining to the fitted model are shown in Table 6. Now, we further demonstrate the validity of the nonlinear model.

We first note that in the nonlinear model, the PRESS (prediction sum of squares) is equal to 0.00824379 is reasonably close to the SSE (=0.004613). This supports the validity of the fitted model. It also demonstrates that the MSE is a valid indication of the prediction capability of the regression model.

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[3] Upper bound = 1.67 and lower bound = 1.46. Refer to http://www.csus.edu/indiv/j/jensena/mgmt105/durbin.htm.

[4] Refer to http://en.wikipedia.org/wik/Kolmogorov-Smirnov_test.

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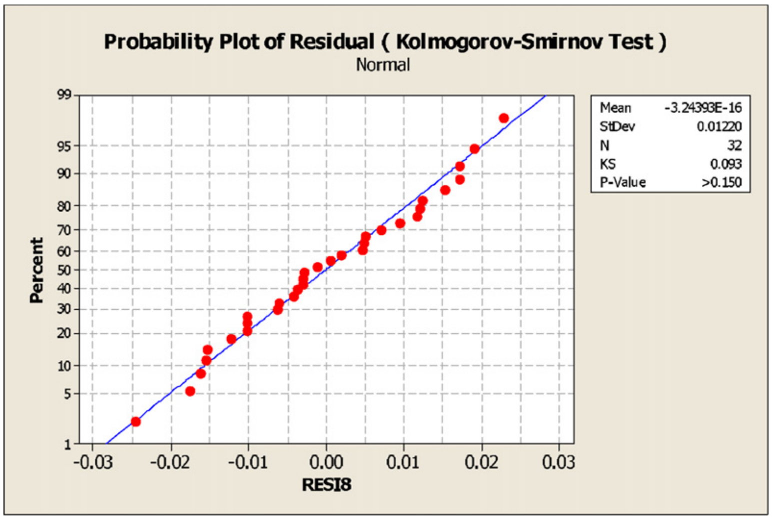

Fig. 4. Probability plot of residual for the regression model used to predict the maximum receiver ratio in VANET.

In addition, we collected another data set with 15 combinations in Table 7 to test the predictive capability of the nonlinear regression model. For each data set, the capacity of the VANET measured by the regression model and the IP model are denoted by $\hat{c}$ and c, respectively, in Table 7. Comparing the values of $\hat{c}$ and c, we find that the average absolute error is about 8% and the average relative error is 15%.

Therefore, we arrive at the conclusion that the prediction performance of the selected nonlinear regression model is reasonably good.

The validation results strongly support our idea that the significant parameters in both the transportation networks and the communication networks, such as the average traffic demand, the transmission range, and the signal-to-interference-plus-noise ratio are good indictors to predict the performance of VANET, for instance, the maximum ratio of the successful concurrent receivers in VANET.

4.3. Statistical model for maximum successful concurrent transmitters

Using the same idea and experiments shown in Table 2, we further explore the corresponding statistical model used for predicting the ratio (S/N) of the maximum successful concurrent transmitters to the network size in VANET.

IP model 2 is used to rigorously investigate the maximum number of successful concurrent transmitters in VANET and further provides the ratio (S/N). The results for the cases in the experiments are demonstrated in Table 2. Our regression

Table 7
New data for validation

Sample	Demand	Lanes	Free speed	R	TH	$\hat{c}_i$	c_i	$\hat{s}_i$	s_i
1	1550	2	72	195	24	0.668425	0.666872	0.298544	0.269231
2	3450	2	72	165	19	0.746325	0.716518	0.265129	0.259797
3	1500	2	77	160	16	0.656600	0.772115	0.263517	0.256250
4	1400	2	80	260	19	0.800200	0.881280	0.311925	0.275000
5	2075	2	76	60	18	0.511319	0.556250	0.290483	0.268750
6	3000	3	72	200	23	0.763800	0.594118	0.269055	0.286925
7	1500	4	73	225	19	0.747900	0.618889	0.257963	0.268750
8	3500	3	78	145	16	0.733100	0.615409	0.282924	0.269097
9	3100	2	72	200	18	0.799800	0.725000	0.266810	0.268750
10	2475	4	76	190	17	0.760919	0.818750	0.266506	0.281250
11	3600	5	70	140	16	0.727600	0.698903	0.293254	0.290025
12	1375	3	73	130	25	0.538969	0.428475	0.312075	0.279070
13	3150	1	76	60	25	0.518925	0.337500	0.240653	0.21875
14	1850	3	74	180	22	0.675825	0.587762	0.283980	0.227325
15	2500	4	79	70	16	0.564600	0.518750	0.263423	0.276741

$\hat{c}_i$ is the predicted ratio of receivers in the VANET with the configuration in the ith new data set using the regression model.
c_i is the ratio of receiver in the VANET with the configuration in the ith new data set. c_i is calculated by IP model 1.
$\hat{s}_i$ is the predicted ratio of senders in the VANET with the configuration in the ith new data set using the regression model.
s_i is the ratio of senders in the VANET with the configuration in the ith new data set. s_i is calculated by IP model 2.

Table 8
Response surface regression: *avgratio* versus coded variables, such as *d*-code and ls-code

Term	Coefficients	SE coefficients	T	P
Estimated regression coefficients for avgratio				
Constant	0.268330	0.002308	116.251	0.000
*d*_code	−0.005899	0.003398	−1.736	0.098
ls_code	−0.002619	0.003398	−0.771	0.450
fs_code	−0.004014	0.003398	−1.181	0.251
*R*_code	−0.005244	0.003398	−1.543	0.138
TH_code	−0.000118	0.003398	−0.035	0.973
ls_code * ls_code	−0.029835	0.006161	−4.842	0.000
*R*_code * *R*_code	0.029181	0.006161	4.736	0.000
*d*_code * ls_code	0.030238	0.008322	3.633	0.002
*d*_code * TH_code	−0.030547	0.008322	−3.670	0.002
fs_code * *R*_code	0.030572	0.008322	3.673	0.002

R-sq = 83.0%, *R*-sq = 74.5% and PRESS = 0.00361149

Source	DF	Seq SS	Adj SS	Adj MS	*F*	*P*
Analysis of variance for avgratio						
Regression	10	0.006777	0.006777	0.000678	9.78	0.000
Residual error	20	0.001385	0.001385	0.000069		
Lack of fit	16	0.001173	0.001173	0.000073	1.38	0.413
Total	30	0.008162				

Term	Coefficients	Term	Coefficients
Estimated regression coefficients for avgratio using data in uncoded units			
Constant	0.85413	*d*	0.000047
ls	0.018245	fs	−0.009363
R	−0.004364	TH	0.010159
ls * ls	−0.007459	*R* * *R*	0.000002
d * ls	0.00001	*d* * TH	−0.000004
fs * *R*	0.000049		

The analysis was done using coded units.

analysis concludes with the final regression model with *R*-sq = 83.0% and *R*-sq (adj) = 74.5%. The coefficients, variance analysis, and related statistics are shown in Table 8. In the following section, we briefly discuss about the signs of the coefficients and the residual test results.

4.3.1. Regression model test

The coefficients of the first order variables in the regression model are intuitive. For example, the sign of variable ls is negative. The reason is that when there are more lanes on the road, the vehicles are distributed more sparse. If all other configurations, such as transmission range and traffic demand, do not change, then the proportion of the successful concurrent transmitters will degrade since some transmitters cannot have receivers in its transmission range. In addition, the regression model shows that the combinations, $d * \text{ls}$, $d * \text{TH}$, and $\text{fs} * R$ significantly impact the successful concurrent transmitters in VANET since they have the relatively large coefficients. It is hard to present the intuitive meaning of the second-order variables in the regression models.

Besides checking the sign of the variables in the regression model, we further evaluate the regression model by testing several statistical features of the residuals. The plots in Fig. 5 show that the residuals have good performance in constancy variance, independence, and normality. For instance, the normal probability plot in Fig. 5 demonstrates an approximately linear pattern, which is consistent with a normal distribution. In addition, Fig. 6 shows the results of the standard normality test (Anderson–Darling test), which also indicates a normal distribution of the residuals (P-value > 0.05).[5]

4.3.2. Calibrating the regression model

The selected regression model also demonstrates good validation features, such as PRESS = 0.00361149, and SSE = 0.001385. They are reasonably similar to one other. This indicates that the MSE is an valid indictor of the prediction capability of the regression model. We also validate the regression model by the new data sets shown in Table 7. Each data set represents a configuration in both the road network and the communication network for the VANET. The capacity of the VANET with the given configurations is estimated by the selected regression models as well as the rigorous IP model 2, respectively. The results are reported in Table 7 by column $\hat{s}$ (the results of the regression model) and s (the results of IP model 2). We compare the values of $\hat{s}$ and s, and find that on the average, the absolute error between $\hat{s}$ and s is about 2%, and the average relative error between $\hat{s}$ and s is 7%. These validation results show that our selected regression model performs reasonable good.

[5] H_0: the data follows a normal distribution and H_a: the data does not follow a normal distribution.

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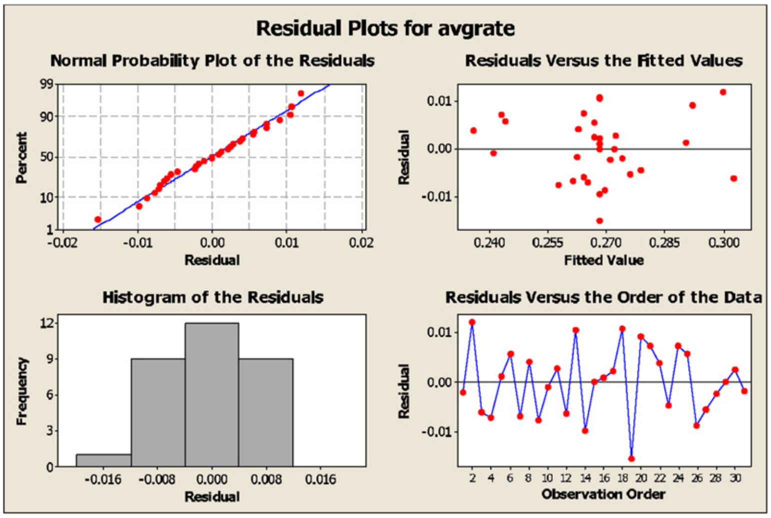

Fig. 5. Probability plot of residual for the regression model used to predict the maximum transmitter ratio in VANET.

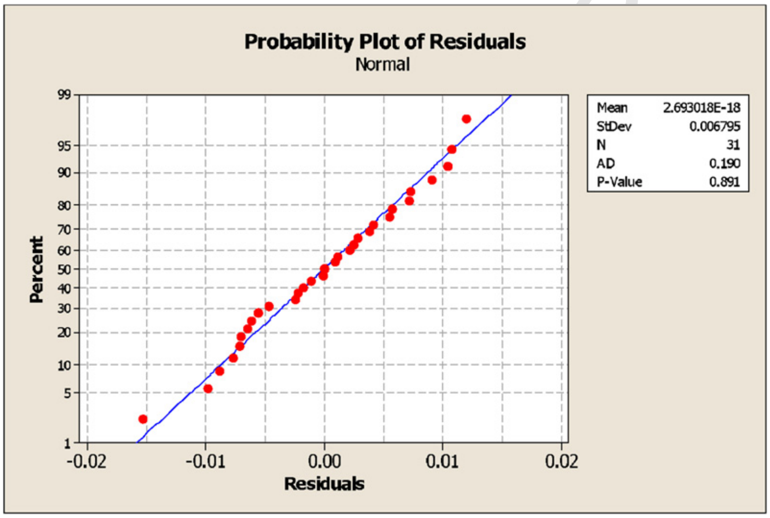

Fig. 6. Residual plot for average transmitter ratio in VANET.

The above two final statistical models integrate the parameters in both the transportation networks and the communication networks to estimate the broadcast capacity of VANET. For a given VANET with known network size, applying the reported statistical models we can approximate the maximum number of concurrent transmissions and therefore, know how many messages can be successfully transmitted in the network. This provides us more insight about the information propagation opportunities in VANET. They bring two significant benefits for VANET application in transportation. On the one hand, the explicit relationship between the broadcast capacity and the network configurations of VANET provides a guidance to design more efficient VANET for traffic managements by considering the traffic flow features. On the other hand, compared with the rigorous IP models, the statistical models provide a rapid way to approximate the broadcast capacity. Therefore, they have great potential to be used in the real-time application in the near future.

5. Conclusion

Due to the vehicular mobility, VANET is a special case of MANET. The simulation results in literature (Artimy et al., 2004, 2005a,b, 2006; Blum et al., 2004; Moreno et al., 2005) have shown that the performance of VANET is strongly impacted by the traffic flow features such as traffic density and traffic speed. Therefore, to fully understand the properties of information propagation in the systems, it is important to study the relationship between the measurements which are used to evaluate the performance of VANET and the parameters which is used to characterize the features of the traffic flow networks.

In this study, we are interested in exploring the concurrent transmissions in VANET since it reflects how fast a new message such as a traffic incident can be disseminated among vehicles in VANET. Specifically, we wish to estimate the maximum successful concurrent transmissions in VANET, which is defined as the capacity of VANET in this study. Technically, the maximum successful concurrent transmissions are represented by the maximum successful concurrent receivers or the maximum successful concurrent transmitters. IP models are first built to rigorously explore the maximum concurrent transmissions in both ways. However, the IP models cannot been solved efficiently as the network size increases. We, therefore, further develop the statistical models to approximate the concurrent transmissions. MITSIMLab is used as the simulation tool to generate the traffic flow data. CCD experiment design and linear regression are applied to develop the statistical models.

Our final regression models show that the ratio of maximum successful concurrent receivers in VANET is the function of average traffic demand and transmission range; the ratio of maximum successful concurrent transmitters in VANET is the function of average traffic demand, traffic speed limit, the number of lanes, SINR threshold, and transmission range. Furthermore, we obtain several insights about interaction between traffic flow characteristics and the capacity of VANET. For example, large vehicular transmission range will improve the ratio of successful receivers and degrade the ratio of successful transmitters in VANET. Increasing traffic demand will improve the ratio of successful receivers and degrade the ration of successful transmitter in VANET.

The main contributions of this work are in three-folders: first, this study investigates the interweave between the successful concurrent transmissions in ~~VANST~~ and traffic flow features such as traffic demand, the number of lanes and free speed limit. To our knowledge, this is the first time to study the capacity of VANET from the transportation points of view. Second, the methodology developed in this study can be further applied to investigate other relationships, in which the traffic flow features are represented by other parameters set or the configuration of the communication network is modified. For example, once we change the antenna features in VANET to be two-directional or omnidirectional, one immediate extension of this study is to address the new relationship between the traffic flow features and the capacity of VANET by the provided methodology. Finally, the provided statistical models in this study can be involved to building optimal online routing policies, in which the capacity of VANET is considered as one of significant factors to evaluate the information quality. In conclusion, the entire study in this paper strongly supports the implementation of the efficient decentralized ATIS in the near future.

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