On the Capacity Improvement of Ad Hoc Wireless Networks Using Directional Antennas

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ABSTRACT

The capacity of ad hoc wireless networks is constrained by the interference between concurrent transmissions from neighboring nodes. Gupta and Kumar have shown that the capacity of an ad hoc network does not scale well with the increasing number of nodes in the system when using *omnidirectional* antennas [6]. We investigate the capacity of ad hoc wireless networks using directional antennas. In this work, we consider arbitrary networks and random networks where nodes are assumed to be static.

In arbitrary networks, due to the reduction of the interference area, the capacity gain is proven to be $\sqrt{\frac{2\pi}{\alpha}}$ when using directional transmission and omni reception. Because of the reduced probability of two neighbors pointing to each other, the capacity gain is $\sqrt{\frac{2\pi}{\beta}}$ when omni transmission and directional reception are used. Although these two expressions look similar, the proof technique is different. By taking advantage of the above two approaches, the capacity gain is $\frac{2\pi}{\sqrt{\alpha\beta}}$ when both transmission and reception are directional.

For random networks, interfering neighbors are reduced due to the decrease of interference area when directional antennas are used for transmission and/or reception. The throughput improvement factor is $\frac{2\pi}{\alpha}$, $\frac{2\pi}{\beta}$ and $\frac{4\pi^2}{\alpha\beta}$ for directional transmission/omni reception, omni transmission/directional reception, and directional transmission/directional reception, respectively.

We have also analyzed hybrid beamform patterns that are a mix of omnidirectional/directional and a better model of real directional antennas.

Categories and Subject Descriptors

C.2 [**Computer-Communication Networks**]: Miscellaneous

MobiHoc'03, June 1–3, 2003, Annapolis, Maryland, USA.

General Terms

Theory

Keywords

Capacity, Ad hoc networks, Directional antenna

1. INTRODUCTION

Ad hoc wireless networks are wireless networks without fixed base stations or any wireline backbone infrastructure. The nodes use peer-to-peer packet transmissions and multihop routes to communicate with each other. Throughput capacity is a key characteristic of ad hoc networks. Consider an ad hoc network with n nodes randomly located in a domain of area one square meter. It was shown by Gupta and Kumar in [6] that under a Protocol Model of interference, such a network could provide a per node throughput of $\Theta(\frac{1}{\sqrt{nlogn}})$ bits/sec. It was also shown there that even under the best possible placement of nodes, such a network could not provide a per-node throughput of more than $O(\frac{1}{\sqrt{n}})$ bits/sec. In this case, the total end-to-end capacity is roughly $O(\frac{n}{\sqrt{n}})$, which is $O(\sqrt{n})$.

The key reasons why the overall capacity is reduced are:

a) interference in a zone around the receiver prevents any other node in the zone from receiving data from any transmitter.

b) as the number of hops increases, the "forwarding burden" of nodes increases; i.e., they spend a fraction of their capacity relaying other nodes' traffic rather than their own. Even if the interference zone around receivers is of area 0, due to range limits of any one-hop, the multi-hops necessary for a large network may in general grow as $\sqrt{n}$.

Several works study how these reasons affect the capacity of the network and try to find ways to complement these effects. Li et al.[9] examine interactions of the 802.11 MAC and ad hoc forwarding and the effect on capacity for several simple configurations and traffic patterns. It is shown that for total capacity to scale up with network size the average distance between source and destination nodes must remain small as the network grows. In [5], Grossglauser and Tse propose a scheme that takes advantage of the mobility of the nodes. By exploiting node mobility as a type of multiuser diversity, they show that the throughput can increase dramatically when nodes are mobile rather than fixed. Gastpar and Vetterli [4] study the capacity under a different traffic pattern. There is only one active source and destination pair, while all other nodes serve as relay, assisting the transmission between this source-destination pair. The capacity is shown to scale as $O(\log n)$. Liu et al. [10] study the throughput capacity of hybrid wireless networks formed by placing base stations in a ad hoc network. This is not a pure wireless ad hoc network since these base stations are connected by a high-bandwidth wired network. They show if the number of base stations m grows faster than $\sqrt{n}$, the throughput capacity increases linearly with the number of base stations.

Such research on the capacity of wireless ad hoc networks and the popular IEEE 802.11 protocol typically assume the use of *omnidirectional* antennas at all nodes. An outcome of this assumption is that all nodes lying in the vicinity of a pair of communicating nodes are required to stay silent. However, with *directional* antennas, more than one pair of nodes located in each other's vicinity may potentially communicate simultaneously, depending on the directions of transmission. This can increase spatial reuse of the wireless channel.

Nasipuri et al. [11] propose a MAC protocol for an ad hoc network of mobile wireless nodes that are equipped with multiple directional antennas. Their protocol uses a variation of the RTS/CTS exchange to let both source and destination nodes determine each other's directions. Simulation experiments indicate that by using four directional antennas in each node, the average throughput in the network can be improved 2∼3 times over that obtained by using CSMA/CA with omnidirectional antennas.

Ko et al. [7] present a DMAC protocol that exploits the characteristics of both directional and omnidirectional antennas to allow simultaneous transmissions that are not allowed in the 802.11 protocol. Choudhury et al. [3] designs another MMAC protocol which uses multi hop RTS's to establish links between distant nodes, and then transmit CTS, DATA and ACK over a single hop. Simulation results show that MMAC outperforms DMAC as well as 802.11 on most of the topology configurations and the traffic patterns.

Ramanathan [12] raises several interesting issues resulted from spatial reuse and larger transmission range of switched or steered beamforming antenna. He evaluates the effectiveness of a number of enhancements, including channel access approaches, link power control, and directional neighbor discovery. Simulation results show that beamforming can yield a 28% to 118% improvement in throughput and up to a factor-of-28 reduction in delay.

Bao and Garcia-Luna-Aceves [1] propose ROMA (Receiver-Oriented Multiple Access), a distributed channel access scheduling protocol for ad hoc networks with directional antennas that are capable of forming multiple beams to carry out several simultaneous data communication sessions. Unlike random access schemes that use on-demand handshakes or signal scanning to resolve communication targets, ROMA determines a number of links for activation in every time slot using only two-hop topology information.

Most of these works focus on designing MAC protocols to take advantage of the use of directional antennas and thus improve the performance of wireless ad hoc networks. Consequently, there is still a need to provide a theoretical framework to understand how much capacity improvement can be achieved.

We introduce an interference model for antennas to analyze the capacity improvement on using directional antennas. We consider two types of networks, *Arbitrary Networks*, where the node locations, destinations of sources, and traffic demands, are all arbitrary, and *Random Networks*, where the nodes and their destinations are randomly chosen. Basically random implies a statistical distribution and arbitrary means the distribution could be arranged to achieve best result in terms of capacity.

First we give a brief description of the antennna background in order to introduce our antenna model.

2. ANTENNA MODEL

The antenna family has more than twenty types of antennas which can be grouped in variable ways [8]. In the study of wireless networks, the antenna model is often grouped under omnidirectional and directional. Omnidirectional antennas, also known as isotropic antennas, radiate and receive equally well in all directions. This unfocused approach scatters signals, reaching desired users with only a small percentage of the overall energy sent out into the environment.

Given the limitation of omnidirectional antennas, directional antennas are used to overcome this inadequacy. Fig.1 shows a common directional pattern in two-dimension [2]. The main lobe is the direction of maximum radiation or reception. In addition to the main lobe, there are also sidelobes and backlobes. These lobes represent lost energy so good antenna designs attempt to minimize them. We always want the main lobe to extend toward a user with a null directed toward a co-channel interferer. In this paper, the beamwidth refers to HPBW (Half-Power Beamwidth), measured between the -3 dB points, i.e. the points on the main lobe where the signal strength drops off -3 dB (one-half) from the maximum signal point.

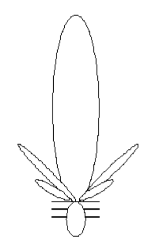


Figure 1: Radiation Pattern for Directional Antennas

In physical layer, the parameter to measure the antenna is the gain in each directional. Antenna *gain* is given in units of dBi (dB gain with respect to an isotropic source) or dBd (dB gain with respect to a half-wave dipole). In medium access technology, the transmission or receiving range is always used to describe the distance the antenna can reach. From two-ray pass loss model [13], it is easy to get that the distance is an inversely proportional function of the antenna gain, with the exponent factor 4. In our model, we use a circle to model the omnidirectional antenna, with the only parameter - radius r indicating the transmission or receiving range.

We approximate the directional antenna pattern as a circular sector with radius r and angle α or β depending on the mode of the antenna (α for transmitter and β for receiver). We use different parameters for antenna beamwidths of transmitters and receivers because we use a new conditional probability argument to handle the case of directional reception. Radius r represents the transmission range or receiving range according to the antenna mode. The angle of the sector approximates the beamwidth of the antenna pattern. The reasons we simplify the antenna model are as follows. It's difficult to model a real antenna with precise values from main lobes, sidelobes, and backlobes. As we will show in the following sections, the only thing that matters is the interference area of the nodes. Simplifying the shape of antenna pattern will not change this property and using a complex model will not result a fundamental change in our work on capacity analysis.

3. THE TRANSPORT CAPACITY FOR ARBITRARY NETWORKS

3.1 Interference Model

We build the *sender-based* interference model as follows: the interference zone is defined as the area that a transmission can cover. That is to say, this transmission will interfere with all the nodes except the intended receiver. The reason that we use the *sender-based* interference model instead of the *receiver-based* model in [6] is that this model can result in the same scaling law yet is easier to be extended to analyze directional antennas.

We assume the senders should be set apart in order to avoid collision in the intersection of the transmission zones. This is a conservative assumption of the real case, since things may be better in practice. All the bounds derived from this model hold for this assumption. Also the capacity improvement achieved by using directional antennas is feasible.

First let's consider the simplest case whose transmission and reception are both omnidirectional. Suppose node Tx_1 and node Tx_2 are transmitting over the mth subchannel, where their transmission ranges are r_1 and r_2, respectively. Then these two transmitters should be at least at a distance $(1+\Delta)(r_1+r_2)$ to avoid collision from each other, which is:

$$|Tx_1 - Tx_2| \geq (1+\Delta)(r_1+r_2) \tag{1}$$

The quantity $\Delta > 0$ models situations where a guard zone is specified by the protocol to prevent a neighboring node from transmitting on the same subchannel at the same time. We require the guard zone $\Delta > 0$.

The Interference Model is illustrated in Fig.2, which, for simplicity, uses the same transmission range for each node.

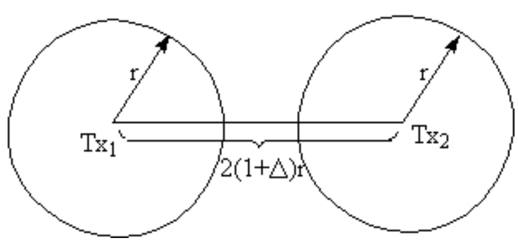


Figure 2: *Sender-based* Interference Model

We consider the setting on a planar disk of unit area. Consider the following assumptions:

(A1) There are n nodes arbitrarily located in a disk of unit area on the plane. These nodes are immobile.
(A2) The network transports λnT bits over T seconds.
(A3) The average distance between the source and destination of a bit is $\bar{L}$. So together with (A2), this implies that a transport capacity of $\lambda n\bar{L}$ bit-meters per second is achieved.
(A4) Each node can transmit over any subset of M subchannels with capacities W_m bits per second, $1 \leq m \leq M$, where $\sum_{m=1}^{M} W_m = W$.
(A5) Transmissions are slotted into synchronized slots of length τ seconds.

3.2 Omnidirectional Antennas

Now we get to the proof part for the upper bound on transport capacity using omnidirectional antennas. We adopt the reasoning introduced in [6] to get the upper bound for transport capacity of the network.

Consider bit b, where $1 \leq b \leq \lambda nT$. Let us suppose that it moves from its origin to its destination in a sequence of $h(b)$ hops, where the hth hop traverses a distance of r_b^h. Then from (A3)

$$\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} r_b^h \geq \lambda nT\bar{L} \tag{2}$$

Note now in any slot at most $n/2$ nodes can transmit. Hence for any subchannel m and any slot s

$$\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} 1(\text{The } h\text{th hop of bit } b \text{ is over subchannel } m \text{ in slot } s) \leq \frac{W_m \tau n}{2} \tag{3}$$

Summing over the subchannels and the slots, and noting that there can be no more than $\frac{T}{\tau}$ slots in T seconds, yields

$$H := \sum_{b=1}^{\lambda nT} h(b) \leq \frac{WTn}{2} \tag{4}$$

From the Interference Model introduced above, disks of radius $(1+\Delta)$ times the lengths of hops centered at the transmitters over the same subchannel in the same slot are essentially disjoint. Ignoring edge effects, all these disks are within the domain. Since at most $W_m\tau$ bits can be carried in slot s from a receiver to a transmitter over the mth subchannel, we have

$$\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} 1(\text{The } h\text{th hop of bit } b \text{ is over subchannel } m \text{ in slot } s)\pi(1+\Delta)^2(r_b^h)^2 \leq W_m\tau \cdot 1 \tag{5}$$

Here the unit in each side is bit-meter^2. Summing over the subchannels and the slots gives

$$\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} \pi(1+\Delta)^2(r_b^h)^2 \leq WT \tag{6}$$

This can be rewritten as

$$\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} \frac{1}{H}(r_b^h)^2 \leq \frac{WT}{\pi(1+\Delta)^2 H} \tag{7}$$

Note now that the quadratic function is convex. Hence

$$\left(\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} \frac{1}{H} r_b^h\right)^2 \leq \sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} \frac{1}{H}(r_b^h)^2 \tag{8}$$

Combining (7) and (8) yields

$$\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} r_b^h \le \sqrt{\frac{WTH}{\pi(1+\Delta)^2}} \tag{9}$$

Now substituting (2) in (9) gives

$$\lambda nT\bar{L} \le \sqrt{\frac{WTH}{\pi(1+\Delta)^2}} \tag{10}$$

Substituting (4) in (10) yields the result:

$$\lambda n\bar{L} \le \frac{1}{\sqrt{2\pi}}\frac{1}{(1+\Delta)}W\sqrt{n} \text{ bit-meters per second.} \tag{11}$$

Note that $\bar{L}$ is the average distance between the source and the destination in a unit area domain. It can be seen as a constant. So the throughput capacity is always proportional to the transport capacity. Also, it is more meaningful to compare the per-node throughput than the aggregate throughput. For this reason, *throughput capacity* is always used to refer *per-node throughput* in the later part of this paper.

3.3 Directional Transmission and Omnidirectional Reception

If we let the sender be directional, the transmission pattern is no longer a circle with uniform directivity in every direction. The transmission pattern for a directional antenna has a main lobe pointing to the receiver, as well as several side lobes which have much less power in those directions. In this paper, as mentioned before, we think of the directional antenna model as a sector characterized by (a) the transmission/reception range r and (b) the beamwidth α for transmission or β for reception mode.

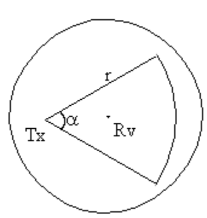


Figure 3: *Sender-based* Interference Model for Directional Transmission

For the sender-based interference zone defined in previous section, the zone becomes a sector with radius r and angle α, shown in Fig.3. Thus, the disks in the unit area domain become sectors with radius $(1+\Delta)$ (considering the guard zone) times the lengths of hops centered at the transmitters and the beamwidth α (assume all the transmitters have the same beamwidth). These sectors centered at the transmitters over the same subchannel in the same slot are essentially disjoint. Similar to the omnidirectional antenna case (except that α is introduced), the interference zone area is calculated as $\frac{\alpha}{2\pi}\pi(1+\Delta)^2 r^2$. Following the same procedure above ((2) to (11)), we change only (5) such that:

$$\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} 1(\text{The } h\text{th hop of bit } b \text{ is over subchannel } m \text{ in slot } s)\frac{\alpha}{2\pi}\pi(1+\Delta)^2(r_b^h)^2 \le W_m\tau \tag{12}$$

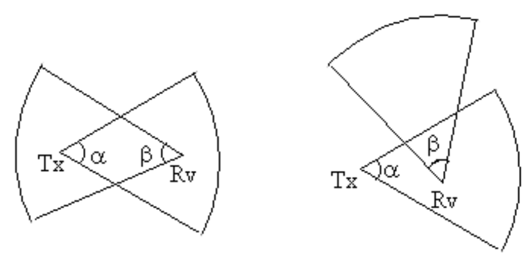
So the transport capacity becomes:

$$\lambda n\bar{L} \le \frac{1}{\sqrt{\alpha}}\frac{1}{(1+\Delta)}W\sqrt{n} \text{ bit-meters per second.} \tag{13}$$

Compared the result in the previous section (11), the directional transmission scales the capacity by $\sqrt{\frac{2\pi}{\alpha}}$.

3.4 Directional Transmission and Directional Reception

What will happen if the receiver antennas are also directional? Intuitively, the result should be more optimistic and the following analysis confirms that it is. Let's consider the sender-based interference zone. Unlike the omnidirectional reception, not all the receivers in this zone will be interfered with. Therefore the transmission zones (interference zone) are not necessarily disjoint. We propose the following modification to the interference zone concept. We introduce a new conditional probability argument for this case. This conditional probability is defined to be the probability that a specific receiver will expierence interference given it is in the transmission zone. Assume all the receivers have the same antenna characteristics and can point in any direction with equal probability. The probability that the antenna pattern of a receiver will cover the transmitter is $\frac{\beta}{2\pi}$. So the conditional probability of interference for a receiver within the transmission zone is $\frac{\beta}{2\pi}$, demonstrated in Fig.4.



(a) Interference (b) No Interference

Figure 4: Interference Model for Directional Antennas

On average, there are $\frac{\beta}{2\pi}$ proportion of the number of receivers inside the transmission zone will get interfered with. Thus the *conditional* interference zone area is:

$$\frac{\beta}{2\pi}[\pi(1+\Delta)^2(r_b^h)^2\frac{\alpha}{2\pi}] = \frac{\alpha\beta(1+\Delta)^2(r_b^h)^2}{4\pi} \tag{14}$$

In the left side of the equation, $\frac{\beta}{2\pi}$ is the conditional probability. The part inside the square brackets is the area of the sector (transmission area). Changing the inequality (5) with this conditional interference zone area we get:

$$\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} 1(\text{The } h\text{th hop of bit } b \text{ is over subchannel } m \text{ in slot } s)\frac{\alpha\beta(1+\Delta)^2(r_b^h)^2}{4\pi} \le W_m\tau \quad (15)$$

Following the steps from (6) to (11), we get the transport capacity using directional antennas at both transmitter and receiver's sides:

$$\lambda n\bar{L} \le \sqrt{\frac{2\pi}{\alpha\beta}}\frac{1}{(1+\Delta)}W\sqrt{n} \text{ bit-meters per second.} \quad (16)$$

Compare this capacity with what we get for the omnidirectional case in (11), the capacity gain is $\frac{2\pi}{\sqrt{\alpha\beta}}$, which is more than the gain using directional antennas only as transmitters.

We have skipped the omnidirectional transmission and directional reception case before this subsection, because the derivation of case directional transmission and reception has already included that of this case. The sender-based interference zone is a circle and the receivers are considered as sectors. The conditional interference area is $\frac{\beta}{2\pi}$ times the area of the disk, so the capacity gain is $\sqrt{\frac{2\pi}{\beta}}$.

Also, when the angles α and β both become smaller, the transport capacity will increase. The network transport capacity is $O(W\sqrt{\frac{n}{\alpha\beta}})$. Then each node will obtain a throughput capacity of $O(\frac{W}{\sqrt{n\alpha\beta}})$ bit-meters per second. When the beamwidth approaches 0, the wireless networks can be seen as the wired links.

From the view of scalability of networks, if the product of beamwidths $\alpha\beta$ decreases asymptotically as fast as $\frac{1}{n}$, the per node capacity will be unrelated with the number of nodes in the domain; thus, the maximum throughput capacity can scale and becomes a constant which is no longer $O(\frac{1}{\sqrt{n}})$.

In the argument of the directional reception case, we use conditional probability to model the direction of the receiving antennas to evaluate the average capacity of the networks based on the placement of nodes optimized by omnidirectional antennas. The results show a further improvement in capacity of ad hoc wireless networks using directional antennas. The results should not be considered as an *upper* bounds on capacity, but rather as *lower* bounds on the potential capacity *improvement* by using directional antennas.

3.5 Achievability of the Capacity Improvement

A question may arise when we look at the result of the capacity of the directional antennas, for example, capacity improvement indicated in (16). What will happen when the beamwidth α or β approaches to zero? Will the capacity grow arbitrarily high? The answer is no.

When the antenna beamwidth reaches a specific threshold, such that the transmission range conducted by all the transmitters just cover the whole domain, the per-node throughput will achieve a constant number related to W. This constant should be less or equal to $W/2$, because the wireless network link is half-duplex. When the angles of antenna beam get even smaller, the capacity will not increase any more. When we go through the inequalities from (2) to (11), equality can be achieved under some conditions for each inequality. (For example, for inequality (2), the equality can be achieved if and only if the routes for each source-destination pair are along a straight line.) So the capacity improvement for directional antennas is feasible. Now look at (12) and (15) where antenna angles are involved. When the angle α or β is too small, the aggregate transmission range conducted by all the transmitters will not cover the whole domain. Thus equality cannot hold due to the small area of the interference zone. That is to say, the interference has been fully reduced and we cannot get any improvement by narrowing the beam of the antennas.

Fig.5 presents a numerical example to illustrate the scaling law of the network capacity (disregarding the constant factor) when the antenna beamwidth is in different orders of the number of nodes n in network. Assuming $\alpha = \beta$, when α decreases slower than $\frac{1}{\sqrt{n}}$, the scaling law in (16) holds. The faster α decreases, the slower the per-node throughput decreases with the increase of the number of nodes in the domain. When α decreases as fast as $\frac{1}{\sqrt{n}}$, the throughput will be $O(W)$. That is the best result we can get with directional antennas; there is no change even if α decreases faster than $\frac{1}{\sqrt{n}}$. In this case, (16) no longer holds.

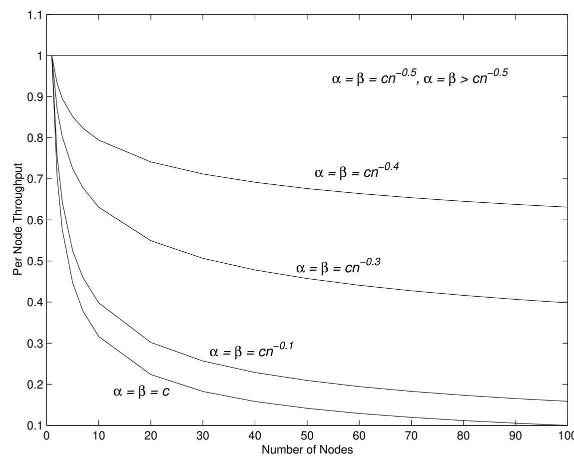


Figure 5: Per-node Throughput as a Function of n

4. THE THROUGHPUT CAPACITY FOR RANDOM NETWORKS

4.1 Throughput Capacity

Though the setting of the problem of random networks is very different from that of arbitrary networks, the analysis of capacity improvement by using directional antennas is very similar.

We use the important concept of Voronoi tessellation whose definition is: the partitioning of a plane with n points into n convex polygons such that each polygon contains exactly one point and every point in a given polygon is closer to its central point than to any central point of other polygons. An example of Voronoi tessellation is shown in Fig.6.

We briefly repeat the steps in [6] for completeness and omit the proof for most of the lemmas.

Definition: Adjacent Cells: Two cells are called *adjacent*, if they share a common point.

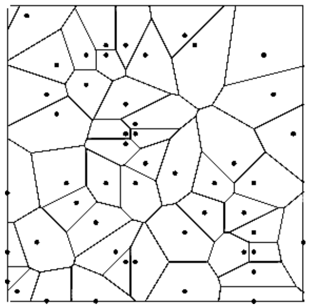


Figure 6: Voronoi Tessellation

Lemma 1: We can construct a Voronoi tessellation V_n in relation to the number of nodes n and the locations of nodes. In this tessellation:

(V1) Every Voronoi cell contains a disk of area $100 \log n/n$. Let

$$\rho(n) := \text{radius of a disk of area} \frac{100 \log n}{n} \tag{17}$$

(V2) Every Voronoi cell is contained in a disk of radius $2\rho(n)$.

(V3) Each Voronoi cell contains at least one node.

(V4) We can choose the range $r(n)$ of each transmission such that

$$r(n) = 8\rho(n)$$

This range allows direct communication within a cell and between adjacent cells. Every node in a cell is within this distance $r(n)$ from every other node in its own cell or adjacent cell.

We assume that power control is not used and the beam of each directional antenna can be steered to it's intended sender or receiver. So the transmission range needed for connectivity is the same as that of omnidirectional antennas. Thus the Voronoi tessellation V_n in *Lemma 1* is also usable for directional antennas.

Definition: Interfering Neighbors: Two cells are said to be *interfering neighbors* if there is a point in one cell which is within a distance $2(1 + \Delta)r(n)$ of some point in the other cell.

One important property of the constructed Voronoi tessellation is that the number of interfering neighbors of a cell is *uniformly bounded*. This property is the key factor for analyzing the capacity gain of using directional antennas.

Lemma 2: For the case that all the antennas are omnidirectional, every cell in V_n has no more than c_1 interfering neighbors. c_1 depends only on Δ and grows no faster than linearly in $(1 + \Delta)^2$.

Proof: Let V be a Voronoi cell. If V' is an interfering neighboring Voronoi cell, there must be two points, one in V and the other in V', which are no more than $2(1+\Delta)r(n)$ units apart. From (V2), the diameter of a cell is bounded by $4\rho(n)$. Hence V', and similarly every other interfering neighbor in the Interference Model, must be contained within a common large disk D of radius $6\rho + 2(1 + \Delta)r(n)$.

Such a disk D cannot contain more than $c_2 = \frac{(6\rho+2(1+\Delta)r(n))^2}{\rho^2(n)}$ $= (22+16\Delta)^2 \sim O((1+\Delta)^2)$ disks of radius $\rho(n)$. By (V2), there can therefore be no more than $c_1 = c_2 - 1$ cells within D. This therefore is an upper bound on the number of interfering neighbors of the cell V.

How can we extend this lemma to the case that antennas are directional? Considering the interference zone introduced in section 3, the same concept can be utilized here. First we must consider how to assess the number of interfering neighbors when the transmission antennas are directional. Whatever shape the transmission pattern is, from the perspective of interference and spatial reuse, the aspect that matters is the area of the interference zone. The nodes are randomly, i.e., independently and uniformly, located in the domain. So the number of interference neighbors is proportional to the area of the interference zone.

Like the antenna model we used to analyze arbitrary networks, we also assume the antennas are sectorized with the same parameters used in Section 3. So the area of interference zone for directional transmission is $\frac{\alpha}{2\pi}\pi r^2(n)$. So, virtually the interfering neighbors should not exceed $c_3 = \frac{\alpha}{2\pi}c_1$.

Similarly, when directional transmission and omnidirectional reception are used, the area of conditional interference zone derived using probability becomes $\frac{\beta}{2\pi}\pi r^2(n)$. Correspondingly the number of interfering neighbors is bounded by $c_4 = \frac{\beta}{2\pi}c_1$. Again, when antenna transmission and reception are both directional, the area of conditional interference zone is $\frac{\alpha}{2\pi}\frac{\beta}{2\pi}\pi r^2(n)$ and the number of interfering neighbors is no more than $c_5 = \frac{\alpha\beta}{4\pi^2}c_1$.

Lemma 3: Let c_6 be the number of interfering neighbors for each cell in tessellation V_n. There is a schedule for transmitting packets such that in every $(1+c_6)$ slots, each cell in the V_n gets one slot in which to transmit, and such that all transmissions are successfully received within the transmission and reception coverage.

Each node wishes to communicate with the node nearest to a randomly chosen location. Routing strategy is to choose the routes of packets to approximate the straight-line which is connecting the source and destination. So the routes actually are the cells the straight-line intersects.

Lemma 4: There is a $\delta'(n) \to 0$ such that

$$Prob(\sup_{V \in V_n} (\text{number of lines } L_i \text{ intersection } V) \leq c_7\sqrt{n \log n}) \geq 1 - \delta'(n)$$

Note the final destination forwarding inside a cell is at most one hop away. The traffic is relayed by the cells intersected by the routing straight-line. Hence the traffic handled by a cell is proportional to the number of lines passing through it. Since each line carries traffic of rate $\lambda(n)$ bits pe second, the following bound can be obtained.

Lemma 5: There is a $\delta'(n) \to 0$ such that

$$Prob(\sup_{V \in V_n} (\text{Traffic needing to be carried by cell } V) \leq c_7\lambda(n)\sqrt{n \log n}) \geq 1 - \delta'(n)$$

From Lemma 3 we know that there exists a schedule for transmitting packets such that in every $(1 + c_6)$ slots, each cell in tessellation V_n gets one slot to transmit. Thus the rate at which each cell gets to transmit is $W/(1 + c_6)$ bits per second.

On the other hand, the rate at which each cell needs to transmit is less than $c_7\lambda(n)\sqrt{n\log n}$ with high probability. This rate can be accommodated by all cells if it is less than the rate available, i.e., if

$$c_7\lambda(n)\sqrt{n\log n} \leq \frac{W}{1+c_6} \tag{18}$$

So we have proved the following theorem, noting the linear growth of c_1 in $(1+\Delta)^2$ in Lemma 2.

Theorem 1: For random networks, there is a deterministic constant $c > 0$ not depending on n, Δ, or W, such that

$$\lambda(n) = \begin{cases} \frac{cW}{(1+\Delta)^2\sqrt{n\log n}}, & \text{Omni Tx Omni Rv,} \\ \frac{2\pi}{\alpha}\frac{cW}{(1+\Delta)^2\sqrt{n\log n}}, & \text{Dir Tx Omni Rv,} \\ \frac{2\pi}{\beta}\frac{cW}{(1+\Delta)^2\sqrt{n\log n}}, & \text{Omni Tx Dir Rv,} \\ \frac{4\pi^2}{\alpha\beta}\frac{cW}{(1+\Delta)^2\sqrt{n\log n}}, & \text{Dir Tx Dir Rv.} \end{cases}$$

bit per second is feasible with high probability. Here we use the easy-to-read abbreviation: Omni = omnidirectional; Dir = directional; Tx = transmission mode; Rv = reception mode.

Comparing the different results among each antenna mode, we can clearly see the throughput gain factor when directional antennas are used. We get an ideal throughput gain of $\frac{4\pi^2}{\alpha\beta}$ using directional transmission and reception.

For random networks, unlike arbitrary networks, the per-node throughput will *not* be a constant with the increasing of the network size if equal power levels are chosen for each directional antenna. The multi-hop burden still exists in that the source and the destination may be far away from each other. In general directional antennas have the potential to reduce the multi-hop problem by per-node power control. Ideally the transmission range may be far enough to reach any node the sender wants to communicate with. So only when the antenna beamwidths are small enough and the transmission range is far enough, can the throughput capacity be a constant not exceeding $W/2$.

Note even the gain factor of capacity by using directional antennas is larger for random networks than for arbitrary networks, the scaling term determines the absolute capacity of arbitrary networks and random networks.

4.2 Hybrid Antenna Model

To achieve a better model of real directional antennas, now we use an antenna model whose beamforming patterns are a mix of omnidirectional and directional antenna models. We define *hybrid antenna model* as an antenna model whose main lobe is characterized as a sector and whose sidelobes and backlobe together form a circle. Shown in Fig. 7, the antenna pattern has a gain value g_m for the main lobe of beamwidth α. It also has a sidelobe of gain g_s of beamwidth $(2\pi-\alpha)$. (Use β if it is a receiver). In the 3-dimension space, the radiation pattern is like a conical main lobe plus a bulb-shaped sidelobe at the base of the cone.

In our work, the transmission range and the beamwidth are determined by the network. So g_m is also determined since it is dependent on the transmission range and vice versa. To estimate what is the radius of the sidelobe given the gain and the beamwidth of the main lobe, first we use the result from [12]. Given g_m and α, we have

$$g_s = \frac{\eta D - g_m}{D-1} \tag{19}$$

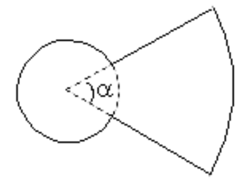


Figure 7: Radiation Pattern of Hybrid Antenna Model

where $D = \frac{4}{tan^2\frac{\alpha}{2}}$ is an often used quantity called the *directivity* of the antenna, and η is the efficiency of the antenna which accounts for losses.

The radius (transmission range) of the sidelobe can be calculated using the path loss equation [13]:

$$P_{RV}(dB) = P_{TX}(dB) + K + G_{RV} + G_{TX} - 10\gamma\log d \tag{20}$$

Where G_{RV} and G_{TX} are the receiver and transmitter antenna gains in dB. P_{RV} and P_{TX} are the received and transmit power respectively in dBm. K is a constant dependent on the environment and wavelength and γ is the path loss exponent. The path loss exponent is dependent on the environment. For free space, γ is 2 and for mobile environment γ is usually 4.

From the path loss equation, we can also estimate the ratio of the radius of the sidelobe to that of the main lobe. For example, if $g_s/g_m = x$, G_{TX} becomes $G_{TX}+10\log x$. To maintain equality, d changes to $dx^{\frac{1}{\gamma}} = d\sqrt[4]{x}$. So we can get the transmission range of the sidelobe given the transmission and the beamwidth of the main lobe. Let's further define the radius of the sidelobe of the transmitter as $s_\alpha \cdot r$ and the radius of the sidelobe of the receiver as $s_\beta \cdot r$.

Now we come to the calculation of the area of the interference zone for each combination of antenna modes. Define the interference zone area of the omnidirectional antennas as $A_{OO} = \pi r^2$.

For directional transmission and omnidirectional reception, the interference zone area is the area of the hybrid antenna radiation pattern, which is calculated as:

$$\begin{aligned} A_{DO} &= \frac{\alpha}{2}r^2 + \frac{2\pi-\alpha}{2}s_\alpha^2 r^2 \\ &= A_{OO}\frac{\alpha+(2\pi-\alpha)s_\alpha^2}{2\pi} \end{aligned} \tag{21}$$

For omnidirectional transmission and directional reception, the conditional interference area is the transmission area multiplied by the probability that the nodes inside the transmission range will experience interference. This conditional interference area is calculated as:

$$\begin{aligned} A_{OD} &= A_{OO}(Pr\{|Tx-Rv| \leq s_\beta r\} \\ &\quad +Pr\{|Tx-Rv| > s_\beta r\} \cdot \\ &\quad Pr\{\text{main lobe of Rv pointing to } Tx\}) \\ &= A_{OO}(s_\beta^2 + (1-s_\beta^2)\frac{\beta}{2\pi}) \end{aligned} \tag{22}$$

For directional transmission and directional reception, the case is more complicated. We divide the transmission area into two parts: one is a small circle with radius $s_\alpha r$, and the other is an annulus sector with radii $s_\alpha r$ and r. Since r is fixed, for the same total energy of the directional antenna, the larger the beamwidth, the smaller the gain of the sidelobe. So first consider $\alpha > \beta$, then $s_\alpha < s_\beta$. The conditional interference area is:

$$\begin{aligned}A_{DD} =& \pi s_\alpha^2 r^2 + \frac{\alpha}{2}(1 - s_\alpha^2) r^2 (Pr\{|Tx - Rv| \leq \\ & s_\beta r | \text{“}C_1\text{”}\} + Pr\{|Tx - Rv| > s_\beta r | \text{“}C_1\text{”}\} \frac{\beta}{2\pi}) \\ =& \pi s_\alpha^2 r^2 + \frac{\alpha}{2}(1 - s_\alpha^2) r^2 (\frac{s_\beta^2 - s_\alpha^2}{1 - s_\alpha^2} + \frac{1 - s_\beta^2}{1 - s_\alpha^2} \cdot \frac{\beta}{2\pi}) \\ =& A_{OO}(s_\alpha^2 + \frac{\alpha}{2\pi}(s_\beta^2 - s_\alpha^2) + \frac{\alpha\beta}{4\pi^2}(1 - s_\beta^2)) \qquad (23)\end{aligned}$$

where “C_1” is the condition statement “Rv is inside the annulus sector”. Note $\frac{\alpha}{2}(1 - s_\alpha^2) r^2$ is the area of the annulus sector.

Then consider $\alpha < \beta$, i.e. $s_\alpha > s_\beta$. The conditional interference area is:

$$\begin{aligned}A_{DD} =& \pi s_\alpha^2 r^2 (Pr\{|Tx - Rv| \leq s_\beta r | \text{“}C_2\text{”}\} + Pr \\ & \{|Tx - Rv| > s_\beta r | \text{“}C_2\text{”}\} \frac{\beta}{2\pi}) + \frac{\alpha}{2}(1 - s_\alpha^2) r^2 \frac{\beta}{2\pi} \\ =& \pi s_\alpha^2 r^2 (\frac{s_\beta^2}{s_\alpha^2} + (1 - \frac{s_\beta^2}{s_\alpha^2}) \frac{\beta}{2\pi}) + \frac{\alpha}{2}(1 - s_\alpha^2) r^2 \frac{\beta}{2\pi} \\ =& A_{OO}(s_\beta^2 + \frac{\beta}{2\pi}(s_\alpha^2 - s_\beta^2) + \frac{\alpha\beta}{4\pi^2}(1 - s_\alpha^2)) \qquad (24)\end{aligned}$$

where “C_2” is the condition statement “Rv is inside the small circle”.

For easy reference, we combine results (23) and (24) into one:

$$\begin{aligned}A_{DD} =& A_{OO}(\min(s_\alpha^2, s_\beta^2) + \frac{\max(\alpha, \beta)}{2\pi} |s_\alpha^2 - s_\beta^2| \\ & + \frac{\alpha\beta}{4\pi^2}(1 - \max(s_\alpha^2, s_\beta^2))) \qquad (25)\end{aligned}$$

The special case where $\alpha = \beta$ lets the conditional interference area become $A_{DD} = A_{OO}(s_\alpha^2 + \frac{\alpha^2}{4\pi^2}(1 - s_\alpha^2))$.

Using the concept of interfering neighbors, *Theorem 1* may be extended to *Theorem 2* representing the hybrid antenna model.

Theorem 2: For random networks, there is a deterministic constant $c > 0$ not depending on n, Δ, or W, such that

$$\lambda(n) = \begin{cases} \frac{cW}{(1+\Delta)^2 \sqrt{n \log n}}, & \text{Omni Tx Omni Rv,} \\ \frac{A_{OO}}{A_{DO}} \frac{cW}{(1+\Delta)^2 \sqrt{n \log n}}, & \text{Dir Tx Omni Rv,} \\ \frac{A_{OO}}{A_{OD}} \frac{cW}{(1+\Delta)^2 \sqrt{n \log n}}, & \text{Omni Tx Dir Rv,} \\ \frac{A_{OO}}{A_{DD}} \frac{cW}{(1+\Delta)^2 \sqrt{n \log n}}, & \text{Dir Tx Dir Rv.} \end{cases}$$

bit per second is feasible with high probability. Here the gain factor for each case can be derived from (21), (22), and (25). This result is based on the hybrid antenna model, which considers the effects of sidelobes and backlobes of directional antenna.

Note that A_{DO}, A_{OD}, and A_{DD} are all monotone increasing functions of s_α and s_β. This shows that smaller antennna sidelobes result in higher throughput gain.

5. CONCLUSIONS

Use of directional antenna in the context of ad hoc wireless networks can largely reduce radio interference, thereby improving the utilization of wireless medium. We study the throughput capacity for different combination of antenna modes. The approach to get the throughput capacity is mainly based on [6], but the interference model used is different.

Our work is focused on discovering the lower bounds in capacity *improvement* that directional antennas can provide relative to the traditional omnidirectional antennas. The conditions that the ad hoc network can scale well were discussed for both arbitrary networks and random networks. For instance, in arbitrary networks, with the reduction of the transmission area and the reduced probability of two neighbors pointing to each other, the capacity of networks using directional antennas will be improved by a factor of $\frac{2\pi}{\sqrt{\alpha\beta}}$. Here α and β are the beamwidths of transmission and receiving directional antennas, respectively. If the beamwidths of transmission and receiving antennas are decreased asymptotically as fast as $\frac{1}{\sqrt{n}}$, the throughput capacity will keep constant with the increase of number of nodes in the network.

For random network, due to the reduction of interfering neighbors, the throughput capacity with the use of directional antennas can achieve a gain as large as $\frac{4\pi^2}{\alpha\beta}$. The use of directional antennas can take advantage of decreasing both interference (local) and multi-hop relay burden (global) through the coordination of the transmission power and antenna directivity.

To model the sidelobes and backlobes of real directional antennas, a hybrid antenna model is used. By calculating the area of the interference zone, we get the capacity improvement gain factors for different transmission/reception cases.

The results for the capacity are based on a conservative model, so better bounds may exist, which need different approaches. Our future work includes finding a new approach to solve this problem. Also some new protocols may be proposed to accommodate the theoretical analysis.

6. ACKNOWLEDGEMENTS

Many thanks are due to Babak Azimi-Sadjadi, who gave us fruitful discussions and provided insights on many parts of our work.

7. REFERENCES

[1] L. Bao and J.J.Garcia-Luna-Aceves. Transmission scheduling in ad hoc networks with directional antennas. In *ACM MobiCom'02*, September 2002.

[2] J. J. Carr. *Directional or Omnidirectional Antenna?* Http://www.dxing.com/tnotes/tnote01.pdf.

[3] R. R. Choudhury, X. Yang, R. Ramanathan, and N. Vaidya. Using directional antennas for medium access control in ad hoc networks. In *ACM MobiCom'02*, September 2002.

[4] M. Gastpar and M. Vetterli. On the capacity of wireless networks: The relay case. In *IEEE INFOCOM'02*, June 2002.

[5] M. Grossglauser and D. Tse. Mobility increases the capacity of ad hoc wireless networks. In *IEEE INFOCOM'01*, April 2001.

[6] P. Gupta and P. R. Kumar. The capacity of wireless networks. *IEEE Transactions on Information Theory*, IT-46(2):388–404, March 2000.

[7] Y. Ko, V. Shankarkumar, and N. H. Vaidya. Medium access control protocols using directional antennas in ad hoc networks. In *IEEE INFOCOM'2000*, March 2000.

[8] J. D. Kraus and R. J. Marhefka. *Antennas: for All Applications, 3rd Ed.* McGraw-Hill, New York, 2002.

[9] J. Li, C. Blake, D. D. Couto, H. Lee, and R. Morris. Capacity of ad hoc wireless networks. In *ACM MobiCom'01*, July 2001.

[10] B. Liu, Z. Liu, and D. Towsley. On the capacity of hybrid wireless networks. In *IEEE INFOCOM'03*, March 2003.

[11] A. Nasipuri, S. Ye, and R. E. Hiromoto. A mac protocol for mobile ad hoc networks using directional antennas. In *Proceedings of the IEEE Wireless Communications and Networking Conference (WCNC 2000)*, September 2000.

[12] R. Ramanathan. On the performance of ad hoc networks using beamforming antennas. In *ACM MobiHoc'01*, October 2001.

[13] T. S. Rappaport. *Wireless Communications: Principles and Practice*. Prentice Hall, Upper Saddle River, New Jersey, 1996.