Robust Rate-Based Flow Controllers for High-speed Networks: The Case of Uncertain Time-varying Multiple Time-delays[1]

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Abstract

An $\mathcal{H}^\infty$ based robust controller is designed for a rate-feedback flow-control problem in single-bottleneck data-communication networks. The controller is robust to uncertain time-varying multiple time-delays in different channels. The controller forces the queue length at the bottleneck node to the desired steady-state value aymptotically and also satisfies a pricing/fairness condition. Lower bounds for stability margins for uncertainty in the time-delays and for the rate of change of the time-delays are derived. In the final version of this paper, a number of simulations will be included to demonstrate the time-domain performance of the controller. Trade offs between the uncertainty levels and time-domain performance will also be discussed.

Keywords: Communication networks, Flow control, $\mathcal{H}^\infty$ control, time-delay systems, uncertain time-varying multiple time-delays.

1 Introduction

High speed data communication networks require resource management methods in order to provide good quality of service to its users. One typical resource management tool is flow control, which is aimed at avoiding traffic congestion by regulating the rate of data packets sent from the sources. This problem has been studied widely in computer networks and communications literature, see for example [6, 10, 16, 17] and their references. Feedback schemes used for flow control may be classified into two groups: "rate-based" and "window-based" (also called "credit-based") flow control. The rate-based control is chosen as the standard flow control scheme in Asynchronous Transfer Mode (ATM) switching networks, by the ATM forum, [1]. This framework has three main components: source end-systems, switches (or network nodes) and destination end-systems. Once every N packets (called "cells") the sources send a control cell which can be used by switches to convey feedback. The control cells travel to the destination and are returned to the source in the same path. Feedback signal may be in the form of a single bit or an explicit rate value, and can be written in the forward or reverse direction of

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travel of the control cell. In this framework the key elements of congestion control in a network are flow controllers at the bottleneck nodes, that determine the feedback signal. Indeed many papers in the literature deal with this basic problem, see for example [2, 3, 4, 9, 12, 18, 21] and their references.

A challenging aspect of flow control, as far as controller design is concerned, is the existence of time-delays in the data-flow. Since the controller is to be implemented at the bottleneck node, which regulates the data rates of the sources, a time-delay occurs between the time a command signal for a rate is issued and the actual time this rate is set (time-delay from the bottleneck node to the source node). Furthermore, the effect of the new rate is seen only after a time-delay which is required for the data to reach the bottleneck (time-delay from the source node to the bottleneck node). Therefore, the total delay in the process (from the control input to the regulated output) is the sum of these two delays, i.e., the round-trip delay. Any computational delays may also be included in this round-trip delay. To further complicate the situation, these time-delays are usually uncertain and are time-varying. Furthermore, since there usually are more than one source affecting a bottleneck, there are multiple time-delays. There are several controller design methods for different classes of systems with time-delays, see for example [11, 13, 19] and their references. The techniques developed in [8, 20] are used in this paper.

One of the design goals considered here is "pricing" or "fairness". Pricing means allocating different percentages of the available capacity to different sources. When equal capacity is to be allocated to all the sources, this term may be replaced by fairness. Another design objective is to keep the queue size close to a certain desired size. By choosing this level sufficiently larger than zero and sufficiently smaller than the buffer size, nonlinear effects may also be avoided and the outgoing flow rate may be kept close to the full capacity (thus achieving the maximum utilization of the network). However, the most important design specification is *stability robustness* with respect to uncertainties in the values of time-delays in each flow path [5].

In a recent work, [14], an $\mathcal{H}^\infty$ based flow controller was designed for an explicit rate feedback based congestion control in high speed networks. Internally robust implementation of this controller was discussed in [15], where some simulations demonstrating the time-domain performance of the controller were also presented. One disadvantage of that controller, however, is that, it was obtained by equalizing the nominal time-delays in all the channels. This may result in an underutilization of the network if the difference between the time-delays of different sources is large. Another shortcomming was that, only time-invariant time-delay was considered. Therefore, there was no guarantee of robustness when the time-delays vary in time. In the present work, we alleviate both of these drawbacks. Precisely, we design a controller which is robust to uncertain time-varying multiple time-delays. A multivariable approach is undertaken here, as opposed to the single-input single-output approach undertaken in [14, 15]. Therefore, different time-delays in different channels are dealt with appropriately. The controller forces the queue length at the bottleneck node to the desired steady-state value aymptotically and also satisfies a pricing/fairness condition.

2 A Test for Robust Stability

The plant is described by (see [14])

$$\dot{q}(t) = \sum_{i=1}^{n} r_i(t - \tau_i(t)) - c(t) \tag{1}$$

where $q(t)$ is the queue length at the bottleneck node, $r_i(t)$ is the rate command for the i^{th} source issued at the bottleneck node, $c(t)$ is the outgoing flow rate from the bottleneck node, and $\tau_i(t)$ is the round-trip time-delay between the i^{th} source and the bottleneck node. We assume that the queue size always remains between zero and the buffer capacity and that each source i can send data at a rate d_i and $r_i(t) \leq d_i$. It is also assumed that each time-delay $\tau_i(t)$ is time varying and is given by:

$$\tau_i(t) = h_i + \delta_i(t) \tag{2}$$

where h_i is the nominal time-invariant known delay and $\delta_i(t)$ is the time-varying time-delay uncertainty, which is assumed to satisfy

$$|\delta_i(t)| < \delta_i^+ \tag{3}$$

for some $\delta_i^+ > 0$. To avoid time-advance, we assume that $\delta_i^+ \leq h_i$.

From (1), $q(t)$ is given by:

$$q(t) = \int_0^t \left[\sum_{i=1}^n r_i(\nu - \tau_i(\nu)) - c(\nu) \right] d\nu \ + q(0) \tag{4}$$

Let

$$q_0(t) := \int_0^t \left[\sum_{i=1}^n r_i(\nu - h_i) - c(\nu) \right] d\nu \ + q(0) \tag{5}$$

and

$$\delta_q(t) := q(t) - q_0(t) = \sum_{i=1}^n \int_0^t \left[r_i(\nu - \tau_i(\nu)) - r_i(\nu - h_i) \right] d\nu \tag{6}$$

Let

$$\lambda_i := \nu - \tau_i(\nu) = \nu - h_i - \delta_i(\nu) =: f_i(\nu) \tag{7}$$

Then

$$\frac{d\lambda_i}{d\nu} = 1 - \frac{d\delta_i}{d\nu} = 1 - g_i(\lambda_i) \tag{8}$$

where $g_i(\lambda) := \left. \frac{d\delta_i}{d\nu} \right|_{\nu = f_i^{-1}(\lambda)}$. Note that, the inverse function $\nu = f_i^{-1}(\lambda)$ is assumed to exist (otherwise, the data sent from source i at a particular time instant would be received by the bottleneck node more than once). This implies that $\frac{d\lambda_i}{d\nu} > 0$, which, by (8), requires $g_i(\lambda_i) < 1$. We will indeed assume,

$$|g_i(\lambda)| < \beta_i \quad \forall \lambda \geq -h_i \tag{9}$$

for some known bound β_i, $0 < \beta_i < 1$, which simply means putting a bound on the rate of variations of the time-delay uncertainty.

Now, noting that, from (8),

$$d\nu = \frac{d\lambda_i}{1 - g_i(\lambda_i)} \tag{10}$$

and assuming that $\delta_i(0) = 0$, (6) can be rewritten as:

$$\begin{aligned}
\delta_q(t) &= \sum_{i=1}^{n} \left[\int_{-h_i}^{t-h_i-\delta_i(t)} r_i(\lambda_i) \frac{d\lambda_i}{1 - g_i(\lambda_i)} \; - \; \int_{-h_i}^{t-h_i} r_i(\nu) d\nu \right] \\
&= \sum_{i=1}^{n} \left[\int_{-h_i}^{t-h_i-\delta_i(t)} r_i(\lambda_i) \frac{g_i(\lambda_i)}{1 - g_i(\lambda_i)} d\lambda_i \; + \; \int_{-h_i}^{t-h_i-\delta_i(t)} r_i(\lambda_i) d\lambda_i \; - \; \int_{-h_i}^{t-h_i} r_i(\nu) d\nu \right] \\
&= \sum_{i=1}^{n} \left[\int_{-h_i}^{t-h_i-\delta_i(t)} r_i(\lambda_i) \frac{g_i(\lambda_i)}{1 - g_i(\lambda_i)} d\lambda_i \; - \; \int_{t-h_i-\delta_i(t)}^{t-h_i} r_i(\nu) d\nu \right]
\end{aligned} \tag{11}$$

where we also used $\frac{1}{1-g_i(\lambda_i)} = \frac{g_i(\lambda_i)}{1-g_i(\lambda_i)} + 1$. Thus, assuming that $r_i(t) = 0$, for $t < 0$, and using (3), (9), and the facts that $h_i + \delta_i(t) \geq 0$ and $r_i(t) \geq 0$, we obtain

$$|\delta_q(t)| \leq \sum_{i=1}^{n} \left[\frac{\beta_i}{1 - \beta_i} \int_0^t r_i(\nu) d\nu \; + \; \int_{t-h_i-\delta_i^+}^{t-h_i} r_i(\nu) d\nu \right] =: \tilde{\delta}_q(t) \tag{12}$$

Now, consider the system shown in Figure 2, where $P_0(s) = \frac{1}{s}\left[e^{-h_1 s} \; \ldots \; e^{-h_n s}\right]$ is the transfer function matrix of the nominal plant (see [14]),

$$\bar{W}(s) = [\bar{W}_1(s) \; \ldots \; \bar{W}_n(s)] \tag{13}$$

with

$$\bar{W}_i(s) := \frac{1}{s} \left(\frac{\beta_i}{1 - \beta_i} + e^{-h_i s} - e^{-(h_i + \delta_i^+) s} \right) \tag{14}$$

and $\|\Delta^0_{\mathrm{LTV}}\| < 1$ (we take the operator norm $\|\cdot\|$ as the induced $\mathcal{L}_2$ norm). A block diagram for the actual closed-loop system, on the other hand, is as shown in Figure 1.

From Figure 2, we see that $\left\|\hat{\delta}_q\right\|_2 < \left\|\tilde{\delta}_q\right\|_2$, where $\hat{\delta}_q$ is as shown in Figure 2. However, by a proper choice of $\|\Delta^0_{\mathrm{LTV}}\| < 1$, $\left\|\hat{\delta}_q\right\|_2$ may be arbitrarily close to $\left\|\tilde{\delta}_q\right\|_2$ (e.g., let $\Delta^0_{\mathrm{LTV}} = 1 - \epsilon$; then $\hat{\delta}_q \to \tilde{\delta}_q$ as $\epsilon \to 0$). Thus, (12) implies that the upper bound on the norm of the uncertainty block Δ_{LTV}, shown in Figure 1, is not greater than the upper bound on the norm of the uncertainty block $\hat{\Delta}_{LTV}$, shown in Figure 2. Thus, any fixed LTI controller $K(s)$ which robustly stabilizes the system shown in Figure 2 also robustly stabilizes the actual system shown in Figure 1.

To find a fixed LTI controller $K(s)$ which robustly stabilizes the system shown in Figure 2, let us consider the following coprime factorizations of the nominal plant in $\mathcal{H}^\infty$:

$$P_0(s) = \frac{1}{s}\left[e^{-h_1 s} \; \ldots \; e^{-h_n s}\right] = N(s) M^{-1}(s) = \tilde{M}^{-1}(s) \tilde{N}(s) \tag{15}$$

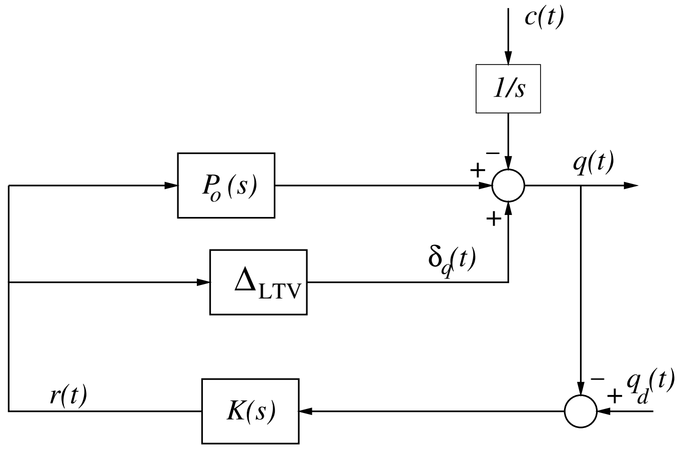

Figure 1: Actual system with time-varying time-delay uncertainty.

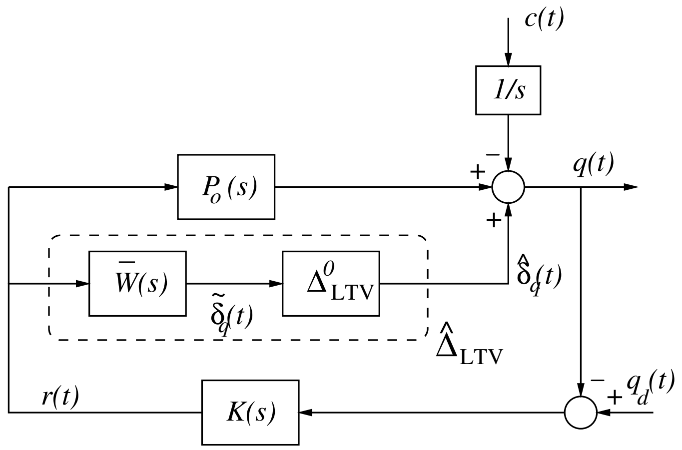

Figure 2: Ficticious system with larger uncertainty.

where $N(s) = \tilde{N}(s) = \dfrac{1}{s+\epsilon}\left[e^{-h_1 s} \ \ldots \ e^{-h_n s}\right]$, $M(s) = \dfrac{s}{s+\epsilon} I_n$, and $\tilde{M} = \dfrac{s}{s+\epsilon}$, where I_n denotes the $n \times n$ identity matrix and $\epsilon > 0$ is arbitrary. Now a parametrization of all controllers $K(s)$ which stabilize $P_0(s)$ can be obtained in terms of $Q(s) \in \mathcal{H}^\infty$, [22]:

$$K(s) = [X(s) + M(s)Q(s)]\,[Y(s) - N(s)Q(s)]^{-1} \tag{16}$$

where $X(s)$ and $Y(s)$ satisfy:

$$\tilde{M}(s)Y(s) + \tilde{N}(s)X(s) = 1 \tag{17}$$

Since $\lim_{s\to 0}\tilde{M}(s) = 0$, to satisfy (17), we must have $\lim_{s\to 0}\tilde{N}(s)X(s) = 1$, equivalently, $\frac{1}{\epsilon}[1 \ \ldots \ 1]\,X(0) = 1$. Thus, we choose

$$X(s) = \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{bmatrix} \epsilon \tag{18}$$

where real numbers α_i are such that $\sum_{i=1}^{n} \alpha_i = 1$. The entries of $X(s)$, α_i's, bring in an extra freedom in choosing the controller. Later in the paper we will show that this freedom may be used in determining the steady-state weights given to different sources, which may be used as a pricing tool. Once $X(s)$ is chosen as in (18), we must choose

$$Y(s) = \tilde{M}^{-1}(s)\left[1 - \tilde{N}(s)X(s)\right] = \frac{s+\epsilon}{s} - \frac{\epsilon}{s}\sum_{i=1}^{n} \alpha_i e^{-h_i s} \tag{19}$$

The closed-loop system shown in Figure 2 is robustly stable for all $\|\Delta^0_{\mathrm{LTV}}\| < 1$ (hence, the actual closed-loop system is robustly stable for all time-delay variations satisfying (3) and (9)) if (e.g., see [22]) $K(s)$ stabilizes $P_0(s)$ (i.e., $K(s)$ is as given in (16) with $Q(s) \in \mathcal{H}^\infty$) and

$$\|W(s)K(s)S(s)\|_\infty \le 1 \tag{20}$$

where

$$S(s) = \tilde{M}(s)\left[Y(s) - N(s)Q(s)\right] = \left[1 + P_0(s)K(s)\right]^{-1} \tag{21}$$

and

$$W(s) = [W_1(s) \ \ldots \ W_n(s)] \tag{22}$$

is such that

$$|W_i(j\omega)| \ge |\bar{W}_i(j\omega)| \ , \quad \forall \omega \in \mathbb{R} \ , \quad i = 1, \ldots, n \ . \tag{23}$$

Noting that $|\bar{W}_i(j\omega)| = \frac{1}{|\omega|}\left|\frac{\beta_i}{1-\beta_i} + e^{-jh_i\omega}\left(1 - e^{-j\delta_i^+\omega}\right)\right| \le \frac{1}{|\omega|}\left|\frac{\beta_i}{1-\beta_i}\right| + \frac{1}{|\omega|}\left|1 - e^{-j\delta_i^+\omega}\right| \le \frac{1}{|\omega|}\left|\frac{\beta_i}{1-\beta_i}\right| + \delta_i^+ \le \frac{1}{|\omega|}\left[\left(\frac{\beta_i}{1-\beta_i} + \delta_i^+\omega_i\right)^2 + \omega^2\left(\delta_i^+ + \frac{\beta_i}{\omega_i(1-\beta_i)}\right)^2\right]^{1/2} = \frac{1}{|\omega|}\left|\left(\frac{\beta_i}{1-\beta_i} + \delta_i^+\omega_i\right) + j\omega\left(\delta_i^+ + \frac{\beta_i}{\omega_i(1-\beta_i)}\right)\right|$ for any $\omega_i > 0$, we let

$$W_i(s) = \frac{1}{s}\left[\left(\frac{\beta_i}{1-\beta_i} + \delta_i^+\omega_i\right) + \left(\delta_i^+ + \frac{\beta_i}{\omega_i(1-\beta_i)}\right)s\right], \tag{24}$$

where $\omega_i > 0$ ($i = 1, \ldots, n$) are additional design parameters, which must be chosen to minimize the $\mathcal{H}^\infty$ cost for given uncertainty levels δ_i^+ and β_i (see Section 5).

3 Performance Issues

3.1 Tracking

One of the performance objectives of rate-based congestion control is to keep the queue size, $q(t)$, as close to its desired value, $q_d(t)$, as possible. In this section we assume that the limit $\lim_{t\to\infty} c(t) =: c_\infty$ exists, for example $c(t)$ can be a step-like function. Consider $q_d(s) = \frac{1}{s}\widehat{q}(s)$ where $\widehat{q}$ is an arbitrary bounded energy signal. For example if $\widehat{q}$ is a pulse of finite duration, then q_d is a saturating ramp. Considering the nominal plant, the tracking error, $e(t) = q_d(t) - q(t)$, satisfies the following frequency domain identity:

$$e(s) = S(s)\ \frac{1}{s}\ [\widehat{q}(s) + c(s)]$$

Since K is a stabilizing controller, and P_0 has a pole at $s = 0$, we have that $S(0) = 0$, and by the final value theorem the steady state value of the error $e_{ss} = \lim_{s\to 0} se(s)$ is

$$e_{ss} = \lim_{s\to 0} \left([e^{-h_1 s}, \ldots, e^{-h_n s}]K(s)\right)^{-1} s\,[\widehat{q}(s) + c(s)] = ([1, \ldots, 1]K(0))^{-1}\, c_\infty$$

(note that the signal $\widehat{q}(t)$ is assumed to have finite energy, so its final value is zero). Thus, at least one of the entries of the controller must have a pole at $s = 0$, in order to have zero steady state error.

3.2 Pricing and fairness

Also note that the rate feedback signals are given by

$$[\, r_1(s) \quad \cdots \quad r_n(s)\,]^{\mathsf{T}} = K(s)S(s)\ \frac{1}{s}\ (\widehat{q}(s) + c(s)). \tag{25}$$

According to the pricing policy, it may be desired to give different steady-state weights (say $\kappa_1, \ \ldots, \ \kappa_n$) to different sources. That is, it is desired that

$$\lim_{t\to\infty} r_i(t) = \kappa_i c_\infty \tag{26}$$

where real numbers $\kappa_i > 0$ are such that $\sum_{i=1}^{n} \kappa_i = 1$. This means that the entries, $K_i(s)$, of the controller $K(s)$ must satisfy

$$\lim_{s\to 0} K_i(s)\left([e^{-h_1 s}, \ldots, e^{-h_n s}]K(s)\right)^{-1} = \kappa_i\ , \qquad i = 1, \ldots, n. \tag{27}$$

If there is no pricing policy, then the steady-state weights must be distributed equally among the sources (i.e., $\kappa_i = \frac{1}{n}$, $i = 1, \ldots, n$) to satisfy fairness.

3.3 Transient response

Besides the steady-state behavior, it is also desired to control the transient response of the system. For this purpose the $\mathcal{H}^\infty$ norm of the weighted sensitivity function can be minimized (this corresponds to worst energy minimization for the tracking error, see for example [7, 8]). More precisely, the problem is to

$$\text{minimize} \quad \|W_s(s)S(s)\|_\infty \tag{28}$$

over all controllers $K(s)$ stabilizing $P_o(s)$, where $W_s(s)$ is the sensitivity weighting filter. By examining the formula given in [20], it is seen that the poles of this filter appear as the poles of the open-loop system $P_0(s)K(s)$. Since $P_0(s)$ has one pole at $s=0$, in order for $K(s)$ to have a pole at $s=0$ (which was found to be a requirement for tracking) $W_s(s)$ must have double poles at $s=0$. Thus we take:

$$W_s(s) = \frac{1}{s^2}. \tag{29}$$

4 An $\mathcal{H}^\infty$ Optimization Problem

We can combine the robust stability (20) and robust performance (28) conditions to define a two-block $\mathcal{H}^\infty$ optimization problem:

$$\inf \left\| \begin{bmatrix} W_s(s)S(s) \\ W(s)K(s)S(s) \end{bmatrix} \right\|_\infty =: \gamma_{\text{opt}} \tag{30}$$

where the infimum is taken over all $K(s)$ stabilizing $P_0(s)$ subject to the pricing/fairness condition (27). Note that, due to the choice of $W_s(s)$ in (29), the tracking condition (that $K(s)$ must have a pole at $s=0$) will be satisfied automatically.

Note that

$$\begin{aligned} W_s(s)S(s) &= W_s(s)\tilde{M}(s)\left[Y(s) - N(s)Q(s)\right] \\ &= W_s(s)\frac{s}{s+\epsilon}\left[\frac{s+\epsilon}{s} - \frac{\epsilon}{s}\sum_{i=1}^{n}\alpha_i e^{-h_i s} - \frac{1}{s+\epsilon}\sum_{i=1}^{n} e^{-h_i s} Q_i(s)\right] \\ &= \sum_{i=1}^{n}\frac{1}{n} W_s(s)M_i(s)\left[Y_i(s) - N_i(s)Q_i(s)\right] \end{aligned} \tag{31}$$

where

$$Y_i(s) := \frac{s+\epsilon}{s} - \frac{n\epsilon}{s}\alpha_i e^{-h_i s} \tag{32}$$

$$N_i(s) := \frac{n}{s+\epsilon} e^{-h_i s} \tag{33}$$

$$M_i(s) := \frac{s}{s+\epsilon} \tag{34}$$

and $Q_i(s)$ is the i^{th} element of $Q(s)$. Let us also define

$$X_i(s) := \alpha_i \epsilon \tag{35}$$

which satisfies

$$M_i(s)Y_i(s) + N_i(s)X_i(s) = 1 \; . \tag{36}$$

Then

$$\begin{aligned} W(s)K(s)S(s) &= W(s)\left[X(s) + M(s)Q(s)\right]\tilde{M}(s) \\ &= \sum_{i=1}^{n} W_i(s)\left[\alpha_i\epsilon + \frac{s}{s+\epsilon}Q_i(s)\right]\frac{s}{s+\epsilon} \\ &= \sum_{i=1}^{n} W_i(s)\left[X_i(s) + M_i(s)Q_i(s)\right]M_i(s) \quad . \end{aligned} \tag{37}$$

Using (31) and (37), problem (30) can be rewritten as:

$$\inf_{Q(s)\in\mathcal{H}^\infty} \left\| \sum_{i=1}^{n} \left[\begin{array}{c} \frac{1}{n}W_s(s)\left[Y_i(s) - N_i(s)Q_i(s)\right]M_i(s) \\ W_i(s)\left[X_i(s) + M_i(s)Q_i(s)\right]M_i(s) \end{array} \right] \right\|_\infty =: \gamma_{\text{opt}} \tag{38}$$

where the infimum is taken over all $Q(s) \in \mathcal{H}^\infty$, subject to the pricing/fairness condition (27).

Let

$$\inf_{Q_i(s)\in\mathcal{H}^\infty} \left\| \left[\begin{array}{c} \frac{1}{n}W_s(s)\left[Y_i(s) - N_i(s)Q_i(s)\right]M_i(s) \\ W_i(s)\left[X_i(s) + M_i(s)Q_i(s)\right]M_i(s) \end{array} \right] \right\|_\infty =: \gamma_i \tag{39}$$

where the infimum is taken over all $Q_i(s) \in \mathcal{H}^\infty$, subject to the pricing/fairness condition (27). Clearly, $\gamma_{\text{opt}} \le \sum_{i=1}^{n}\gamma_i$. Since it is very complicated, if not impossible, to find an optimal solution to problem (38) subject to (27), we propose a suboptimal solution $Q(s) = [Q_1(s) \;\ldots\; Q_n(s)]^T$, where each $Q_i(s)$ satisfies (39) such that the pricing/fairness condition (27) is also satisfied. For this, we will first find a solution to (39) for each i without considering the condition (27); and then show that, by a proper choice of α_i, this solution also satisfies the pricing/fairness condition (27).

Let us define

$$C_i(s) := \left[X_i(s) + M_i(s)Q_i(s)\right]\left[Y_i(s) - N_i(s)Q_i(s)\right]^{-1} \tag{40}$$

So that

$$Q_i(s) = \left[M_i(s) + C_i(s)N_i(s)\right]^{-1}\left[C_i(s)Y_i(s) - X_i(s)\right] \tag{41}$$

Then (39) can be rewritten as:

$$\inf_{C_i(s) \text{ stabilizing } P_i(s)} \left\| \left[\begin{array}{c} \frac{1}{n}W_s(s)\left[1 + P_i(s)C_i(s)\right]^{-1} \\ W_i(s)C_i(s)\left[1 + P_i(s)C_i(s)\right]^{-1} \end{array} \right] \right\|_\infty =: \gamma_i \tag{42}$$

where

$$P_i(s) := N_i(s)M_i^{-1}(s) = \frac{n}{s}e^{-h_i s} \; . \tag{43}$$

Now, let us define

$$\tilde{W}_i(s) := \frac{1}{n} W_s(s) = \frac{h_i^2}{n} \frac{1}{(h_i s)^2} \tag{44}$$

$$\hat{W}_i(s) := \frac{s}{n} W_i(s) = \tilde{\beta}_i + \tilde{\delta}_i (h_i s) \tag{45}$$

where

$$\tilde{\beta}_i := \frac{1}{n}\left(\frac{\beta_i}{1-\beta_i} + \delta_i^+ \omega_i\right) \quad \text{and} \quad \tilde{\delta}_i := \frac{1}{nh_i}\left(\delta_i^+ + \frac{\beta_i}{\omega_i(1-\beta_i)}\right) = \frac{1}{\omega_i h_i}\tilde{\beta}_i \tag{46}$$

$$\hat{P}_i(s) := \frac{1}{nh_i} P_i(s) = \frac{1}{h_i s} e^{-h_i s} \tag{47}$$

and

$$\hat{C}_i(s) := nh_i C_i(s) \tag{48}$$

so that (42) can be rewritten as

$$\inf_{\hat{C}_i(s) \text{ stabilizing } \hat{P}_i(s)} \left\| \begin{bmatrix} \tilde{W}_i(s)\left[1+\hat{P}_i(s)\hat{C}_i(s)\right]^{-1} \\ \hat{W}_i(s)\hat{P}_i(s)\hat{C}_i(s)\left[1+\hat{P}_i(s)\hat{C}_i(s)\right]^{-1} \end{bmatrix} \right\|_\infty =: \gamma_i \tag{49}$$

which is now in terms of the normalized frequency $\hat{s}_i := h_i s$. In deriving (49) from (42) we also made use of the fact that $|e^{-jh_i\omega}| = 1$ for all $\omega \in \mathbb{R}$.

By defining

$$\hat{\beta}_i = \frac{n\tilde{\beta}_i}{h_i^2}\ , \qquad \hat{\delta}_i = \frac{n\tilde{\delta}_i}{h_i^2} \qquad \text{and} \qquad \hat{\gamma}_i = \frac{n\gamma_i}{h_i^2}$$

and applying the formulae given in [20] and [15], the optimal solution to (49) is found as:

$$\hat{C}_i(s) = \frac{\hat{\gamma}_i}{\hat{\delta}_i}\left(\frac{\hat{s}_i - k_i}{\hat{s}_i}\right)\frac{1}{1+F_i(\hat{s}_i)} \tag{50}$$

where

$$F_i(\zeta) = \frac{(\zeta+k_i)(\zeta+a_i)(\zeta^2+b_i\zeta+c_i) - (\zeta^4 - \hat{\gamma}_i^{-2}) - \frac{\hat{\gamma}_i}{\hat{\delta}_i} e^{-\zeta}\zeta^2(\zeta-k_i)}{\zeta^4 - \hat{\gamma}_i^{-2}} \tag{51}$$

where

$$c_i := \sqrt{x_i}\ , \qquad a_i := \frac{1}{c_i\hat{\delta}_i}\sqrt{1-\frac{\hat{\beta}_i^2}{\hat{\gamma}_i^2}}$$

$$b_i := \sqrt{\frac{\hat\beta_i^2}{\hat\gamma_i^2} + 2c_i - a_i^2} \ , \quad k_i := \frac{\rho_i - 1}{\sqrt{\hat\gamma_i}(\rho_i + 1)}$$

where $x_i > 0$ is the unique positive root of

$$x_i^3 + \frac{1}{\hat\gamma_i^2} x_i^2 + \left(1 - \frac{\hat\beta_i^2}{\hat\gamma_i^2}\right) \frac{\hat\beta_i^2}{\hat\delta_i^4} x_i - \frac{\left(1 - \frac{\hat\beta_i^2}{\hat\gamma_i^2}\right)^2}{\hat\delta_i^4} = 0$$

and

$$\rho_i := \frac{e^{-\frac{1}{\sqrt{\hat\gamma_i}}}}{\hat\delta_i \left(\frac{1}{\sqrt{\hat\gamma_i}} + a_i\right)\left(\frac{1}{\hat\gamma_i} + c_i + \frac{b_i}{\sqrt{\hat\gamma_i}}\right)}.$$

The optimum $\mathcal{H}^\infty$ performance cost $\hat\gamma_i$ is determined as the largest root of the equation

$$1 - \frac{\hat\gamma_i}{\hat\delta_i} e^{-s} s^2 \frac{(s - k_i)}{(s + k_i)(s + a_i)(s^2 + b_i s + c_i)}\bigg|_{s = j/\sqrt{\hat\gamma_i}} = 0. \tag{52}$$

A time domain realization of $F_i(\hat s_i)$ can be obtained by noting that

$$F_i(\zeta) = C_i \left(e^{-\zeta} I - e^{-A_i}\right) (\zeta I - A_i)^{-1} B_i \tag{53}$$

where

$$A_i = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \hat\gamma_i^{-2} & 0 & 0 & 0 \end{bmatrix} \qquad B_i = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad C_i = \frac{\hat\gamma_i}{\hat\delta_i} \begin{bmatrix} 0 & 0 & k_i & -1 \end{bmatrix}.$$

This in turn shows that the impulse response of $F_i(\zeta)$ is restricted to the time interval $[0\ ,\ 1]$, and hence, $F_i(h_i s)$ may be realized as a finite impulse response (FIR) filter of duration h_i. Thus, a discrete-time realization of $F_i(h_i s)$ requires only h_i/T_s states, where T_s is the sampling period.

Once $\hat C_i(s)$ is obtained from (50), from (48), $C_i(s)$ can be obtained simply as $C_i(s) = \frac{1}{nh_i}\hat C_i(s)$ and, in turn, $Q_i(s)$, thus $Q(s)$, can be obtained from (41). Once $Q(s)$ is obtained, the sought controller $K(s)$ is given by (16). Now, it remains to be shown that, the parameters α_i used in $X(s)$ can be chosen so that the resulting controller satisfies the pricing/fairness condition (27). Using (16), it can be shown that (27) can be written as:

$$\lim_{s \to 0} \frac{\alpha_i \epsilon + \frac{s}{s+\epsilon} Q_i(s)}{\sum_{l=1}^n e^{-h_l s}\left[\alpha_l \epsilon + \frac{s}{s+\epsilon} Q_l(s)\right]} = \kappa_i \ , \qquad i = 1, \ldots, n. \tag{54}$$

However, since $Q_i(s) \in \mathcal{H}^\infty$ (thus $\lim_{s\to 0} Q_i(s)$ is finite; indeed, $\lim_{s\to 0} Q_i(s) = \frac{\epsilon}{n}(1 + h_i\epsilon)$) for all $i = 1, \ldots, n$ and $\sum_{l=1}^n \alpha_l = 1$, the left hand side of (54) simply reduces to α_i. Thus, to satisfy the pricing/fairness condition, we simply need to choose:

$$\alpha_i = \kappa_i \ , \qquad i = 1, \ldots, n. \tag{55}$$

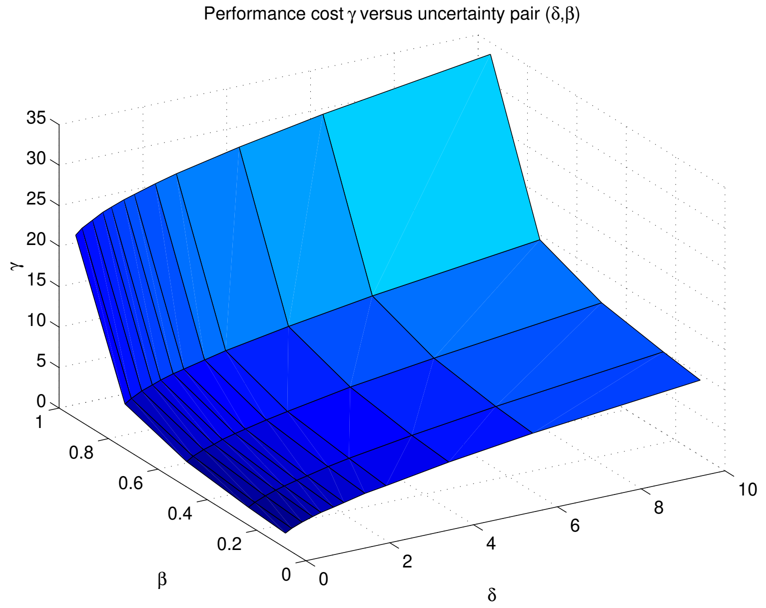

Figure 3: γ_i versus (δ_i^+, β_i), for $n = 1$ and $h_i = 1$.

5 Computational Issues and Stability Margins

The mixed sensitivity minimization problem (49) posed in the previous section can be solved as described earlier for each fixed uncertainty level (β_i, δ_i^+). Recall that the problem data also depends on free parameters ω_i ($i = 1, \ldots, n$) introduced in the weighting functions. By closely examining the robustness weight $\hat{W}_i(s)$ we see that as $\omega_i \to 0$, or $\omega_i \to \infty$, the $\mathcal{H}^\infty$ performance cost $\gamma_i \to \infty$. We would like to minimize the performance cost γ_i by properly choosing ω_i. Clearly, once β_i and δ_i^+ are fixed, we can compute the best ω_i and the corresponding γ_i from a line search. Figure 3 shows the smallest γ_i for several feasible values of β_i and δ_i^+.[6] As expected, when β_i is close to 1, the smallest $\mathcal{H}^\infty$ cost γ_i is large, which means that fast variations in time delay cannot be tolerated as far as robust stability is concerned. Large values of δ_i^+ also lead to large γ_i.

Robust stability is achieved when (20) is satisfied. By using the arguments leading to (49), a sufficient condition for (20) is

$$\left\| \hat{W}_i(s)\hat{P}_i(s)\hat{C}_i(s) \left[1 + \hat{P}_i(s)\hat{C}_i(s)\right]^{-1} \right\|_\infty \leq \psi_i \tag{56}$$

where $\psi_i > 0$ are such that $\sum_{i=1}^n \psi_i = 1$. One naive choice is $\psi_i = \frac{1}{n}$, $i = 1, \ldots, n$. However, if there are differences among the channels about the uncertainty levels, it makes more sense to choose larger ψ_i for channels with higher uncertainty levels. By (49), condition (56) is satisfied if $\tilde{\beta}_i$ is replaced by $\frac{\psi_i}{\gamma_i}\tilde{\beta}_i$ and $\tilde{\delta}_i$ is replaced by $\frac{\psi_i}{\gamma_i}\tilde{\delta}_i$. Therefore, using (46), lower bounds (sufficient conditions) on the

[6]For simplicity we chose $n = 1$; however, since the subproblems are decoupled, the value of n only affects the numerical value of γ_i, the general shape of the graph is the same for $n > 1$.

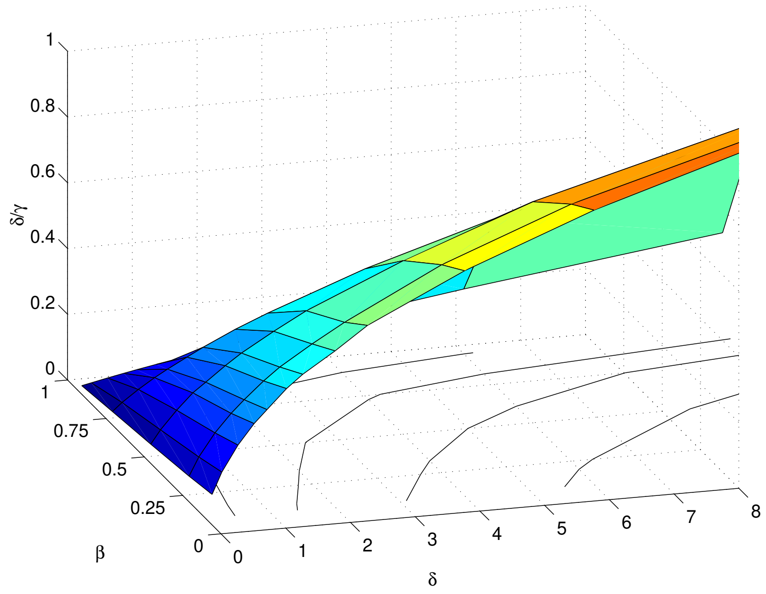

Figure 4: δ_i^+/γ_i versus (δ_i^+, β_i), for $n = 1$ and $h_i = 1$.

actual stability margins for $|\delta_i(t)|$ (call it $\bar{\delta}_i$) and for $\left|\dfrac{d\delta_i(t)}{dt}\right|$ (call it $\bar{\beta}_i$) may be calculated by solving

$$\left(\frac{\bar{\beta}_i}{1-\bar{\beta}_i} + \bar{\delta}_i\omega_i\right) = \frac{\psi_i}{\gamma_i}\left(\frac{\beta_i}{1-\beta_i} + \delta_i^+\omega_i\right) \tag{57}$$

where β_i and δ_i^+ are the uncertainty levels used in deriving the controller and ω_i is the design parameter which must be calculated to minimize γ_i as explained above. Note that, for any given β_i and δ_i^+, there are infinitely many solutions $(\bar{\beta}_i, \bar{\delta}_i)$ to (57). There is also a clear trade off between the stability margin $\bar{\delta}_i$ for the magnitude of the time-delay uncertainty and the stability margin $\bar{\beta}_i$ for the rate of change of the time-delay. This means that if the time-delay uncertainty is small, then faster variations in its rate of change can be tolerated and vice versa.

One possible solution to (57) is

$$\bar{\delta}_i = \frac{\psi_i}{\gamma_i}\delta_i^+ \qquad \text{and} \qquad \bar{\beta}_i = \frac{\psi_i\beta_i}{\psi_i\beta_i + \gamma_i(1-\beta_i)} \; .$$

These quantities are plotted in Figures 4 and 5 respectively, for the case $n = 1$ (thus we take $\psi_i = 1$) and $h_i = 1$. It is seen that, for a fixed β_i, $\bar{\delta}_i$ increases with increasing δ_i^+ while $\bar{\beta}_i$ decreases slightly. Similarly, for a fixed δ_i^+, $\bar{\beta}_i$ increases with increasing β_i while $\bar{\delta}_i$ decreases slightly. These figures also indicate that, for large stability robustness margins $\bar{\delta}_i$ and $\bar{\beta}_i$, the uncertainty levels δ_i^+ and β_i must be chosen as large as possible. Such a choice, however, may adversely affect the time-domain performance of the controller (see [15] for a similar case). These effects will be discussed in the final version of this paper, where a number of simulation results will be presented.

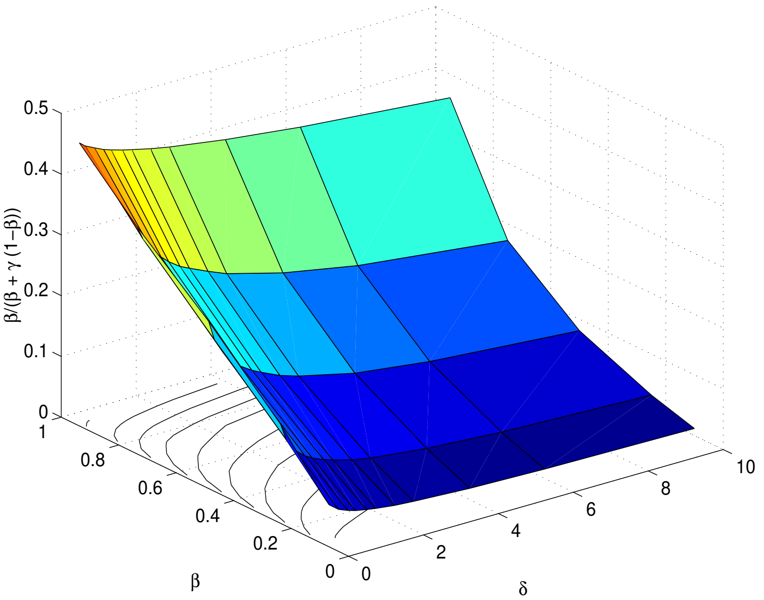

Figure 5: $\beta_i / [\beta_i + \gamma_i(1 - \beta_i)]$ versus (δ_i^+, β_i), for $n = 1$ and $h_i = 1$.

6 Conclusions

Robust controller design for a flow control problem in data-communication networks with a single bottleneck node has been considered. A controller has been designed which is robust to uncertain time-varying multiple time-delays. The controller forces the queue length at the bottleneck node to the desired steady-state value aymptotically and also satisfies a pricing/fairness condition. Stability margins for uncertainty in the time-delays and for the rate of change of the time-delays have also been discussed and their lower bounds have been derived.

In the final version of this paper, a number of simulations will be included to demonstrate the time-domain performance of the controller. Trade offs between the uncertainty levels and time-domain performance will also be discussed.

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