

ular situations, but it is not at all a formidable breakthrough in fundamental physics, as assumed by some. Rather, Tsallis's valuable contribution has been to bring the ideas of mid-20th-century statisticians into physics.

In statistical mechanics, Tsallis entropy takes a convenient form, appropriated for physics, of the entropy used in mathematical

statistics and information theory, namely, Havrda-Charvat structural entropy ( $I$ ), one of infinitely many that can be introduced [e.g., (2)]. These entropies produce what can be called unconventional statistics, which are useful for fitting purposes (they depend on adjustable parameters). They can be used in the most general and universal entropy [Kullback-Leibler entropy (3),

whose physical counterpart is the Boltzmann-Gibbs-Shannon entropy] when a researcher is unable to satisfy the principle of sufficiency in statistics, as stated by Fischer in 1922 (3, 4). This means that the researcher does not have access to all information on the characteristics of the system relevant for the problem at hand.

The choice of the unconventional statistics to be used depends on each particular experimental/theoretical situation. For example, Renyi statistics appears to be appropriate for dealing with the so-called fractal systems (5, 6). The utility of such unconventional entropies is that they provide a way to generate probability distributions, in the context of statistical physics based on information theory (7). In that sense, they are what can be called statistical entropies, and it is utterly wrong to identify Havrda-Charvat-Tsallis "entropy" with the physical entropy of systems in nature. There is only one situation in which informational entropies can be related to the classical Clausius-Boltzmann entropy of thermodynamics, namely, Kullback-Leibler-Shannon statistical entropy in the case of strict equilibrium (5, 8). Tsallis statistics does not supersede Boltzmann-Gibbs statistics; it is one of infinitely many that can be used to "patch" the inconvenience, noted above, that arises when one is unable to satisfy Fischer's principle of sufficiency in Boltzmann-Gibbs statistics.

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## ADRIAN CHO'S ARTICLE ON TSALLIS ENTROPY

("A fresh take on disorder, or disorderly science," *News Focus*, 23 Aug., p. 1268) emphasizes the importance of nonextensive energies when analyzing complex systems. To complement his picture, I would like to draw attention to an alternative way of treating nonextensive energies, developed by Terrell Hill about 40 years ago (1–3). Hill's approach is based on the fundamental foundation of Gibbs' ensembles and does not involve modifying the definition of entropy. To my knowledge, Hill's work remains the only comprehensive treatment of finite-size effects in thermostatics.

The heart of Hill's approach, now known as "nanothermodynamics," can be understood by tracing the development of the first law of thermodynamics (4). In 1850, Clausius made his clear statement that the change in internal energy of a system is equal to the added heat minus the work done. In 1876, Gibbs extended the first law by including the chemical potential,  $\mu$ .  $\mu$  comes from the change in energy when a single particle (e.g., electron, atom, or molecule) is added to a system of particles. Use of  $\mu$  allows the formal treatment of equilibria between different substances or between different phases. In 1962, Hill extended the first law by including a subdivision potential,  $E$ .  $E$  comes from the change in energy when a single small system is added to an ensemble of small systems. Use of  $E$  allows the formal treatment of finite systems that have nonextensive energies, such as clusters with nonlinear interactions or surface terms.

Some experts have said that the success of Tsallis entropy may come from mathematical flexibility in the empirical parameter  $q$ . Hill's nanothermodynamics has additional flexibility with no new parameters. In the usual thermodynamic limit, only two intensive variables can be independent. For example, the grand canonical ensemble has  $\mu$  and  $T$  independent, but the volume is fixed by the size of the sample. However, nanothermodynamics has a well-defined "generalized ensemble" where the system is described by three independent variables, e.g.,  $\mu$ ,  $T$ , and pressure for molecules, or  $\mu$ ,  $T$ ,

and field for magnets. Furthermore, the generalized ensembles allow unrestricted thermal fluctuations, which provide an explanation for the measured response from several complex systems, including glass-forming liquids (5) and ferromagnetic materials (6).

It is legendary how Gibbs' work remained relatively unknown for 15 to 20 years, until it was translated into German and used by the pioneers in physical chemistry. Even now, after 40 years, Hill's work is still relatively unknown, possibly because, after completing his article and books on the subject (1, 2), he switched his main interest to molecular biology and did not have the time or inclination to promote his many contributions to fundamental thermostatics.

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## CORRECTIONS AND CLARIFICATIONS

**REPORTS:** "Gene expression during the life cycle of *Drosophila melanogaster*" by M. N. Arbeitman *et al.* (27 Sept., p. 2270). Eileen E. M. Furlong should have been listed as co-first author. Her affiliations were also listed incorrectly. She is at the Department of Developmental Biology and Department of Genetics, Stanford University, Stanford, CA 94305, USA, and the Developmental Biology Program, European Molecular Biology Laboratory, 69117 Heidelberg, Germany.

**SPECIAL ISSUE ON MAPPING CELLULAR SIGNALING: VIEWPOINTS:** "Phosphorelay and transcription control in cytokinin signal transduction" by J. Sheen (31 May, p. 1650). In Fig. 1, all labels reading "APH1/2" should instead read "AHP1/2." Also, the volume number in reference (18) is incorrect; it should be 129.

## Letters to the Editor

Letters (~300 words) discuss material published in *Science* in the previous 6 months or issues of general interest. They can be submitted by e-mail ([science\\_letters@aaas.org](mailto:science_letters@aaas.org)), the Web ([www.letter2science.org](http://www.letter2science.org)), or regular mail (1200 New York Ave., NW, Washington, DC 20005, USA). Letters are not acknowledged upon receipt, nor are authors generally consulted before publication. Whether published in full or in part, letters are subject to editing for clarity and space.