Helioseismic Measurement of Solar Torsional Oscillations

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Bands of slower and faster rotation, the so-called torsional oscillations, are observed at the Sun's surface to migrate in latitude over the 11-year solar cycle. Here, we report on the temporal variations of the Sun's internal rotation from solar p-mode frequencies obtained over nearly 6 years by the Michelson Doppler Imager (MDI) instrument on board the Solar and Heliospheric Observatory (SOHO) satellite. The entire solar convective envelope appears to be involved in the torsional oscillations, with phase propagating poleward and equatorward from midlatitudes at all depths throughout the convective envelope.

Solar torsional oscillations were discovered in Doppler velocity measurements of the Sun's surface rotation as migrating bands of slower and faster rotation (zonal flows) (*1*). Helioseismic data from the ground-based [Global Oscillation Network Group (GONG)] and space-based (SOHO/MDI) projects (*2*) over a significant fraction of the solar activity cycle show that these banded zonal flows are not just a surface phenomenon but penetrate into the solar interior to at least some 8 to 10% of the solar radius (*3*).

Helioseismology uses the observed global resonant oscillations of the Sun, which are set up by acoustic-gravity waves that propagate inside the Sun, to infer conditions in the solar interior. In particular, information about the internal rotation comes from the rotational splittings of the frequencies of eastward- and westward-propagating modes. Inverting these effects of rotation, measured accurately for tens of thousands of different modes, allows us to infer the distribution of the rotational velocity in depth and latitude (*4*). Measurements performed at different times allow us to trace the variations of the internal differential rotation over the solar activity cycle. The spatial and temporal behavior of the 11-year torsional oscillations can thus be measured with the 5-min acoustic oscillations.

We analyzed the rotational splittings measured from the SOHO/MDI data collected between 1 May 1996 and 17 January 2002. The dependence of the oscillation frequencies on azimuthal order m is represented by an expansion in terms of the so-called a coefficients (*5*). The absolute value of m describes the longitudinal structure of the mode, and its sign determines whether the mode propagates eastward or westward. To trace the time variations, 27 individual 72-day data sets were used in the inversion. An additional 360-day data set, obtained by a coherent analysis of the first year of MDI observations, was used as a reference. We employed a differential inversion, by first subtracting the rotational splittings of the reference data set from each of the individual 72-day data sets. This allowed us to reduce the effects of systematic errors in the input data and to improve the stability of the results. With the reference data set measured near the solar activity minimum, we were thus measuring the variation of the internal rotation relative to the solar minimum.

The two-dimensional (2D) inversion for the angular velocity $\Omega(r,\theta)$ (r being the distance from the center of the Sun and θ the colatitude) was performed with an adaptive regularization technique (*6*), an iterative method where the number of the iterative descents plays the role of a regularization parameter. A principal feature of the inversion technique is that the sensitivity of the solution to random errors is nearly uniform over all of the approximation domain.

The results for the 27 individual data sets (Fig. 1) show migrating zonal flows, which move toward the equator at lower latitudes and toward the pole at higher latitudes. The results illustrate the development of the torsional oscillations over nearly 6 years of SOHO observations, which cover the rising phase of solar cycle 23. The zonal flows are relatively feeble, of the order of 5 nHz in rotational frequency compared to the total rotation (which we have subtracted out) which is of the order of 450 nHz. The low-latitude feature, migrating toward the equatorial plane, is limited to the outer 10% of the solar radius, consistent with previous findings (*3*). The high-latitude acceleration, which develops around 60° latitude, shows much deeper penetration (*7*). The magnitudes of the variations are small in the first frames, with data collected at nearly the same time as the reference data set.

To improve the signal-to-noise ratio, the 72-day rotational splittings were also averaged over 1-year periods, to produce five nonoverlapping data sets, and inverted (Fig. 2). The high-latitude acceleration around 60° latitude penetrates to the base of the convection zone, which occupies the outer 29% of the Sun by radius. The base of the convection zone also appears to be a probable lower boundary that limits this flow. At lower latitudes, a decelerating

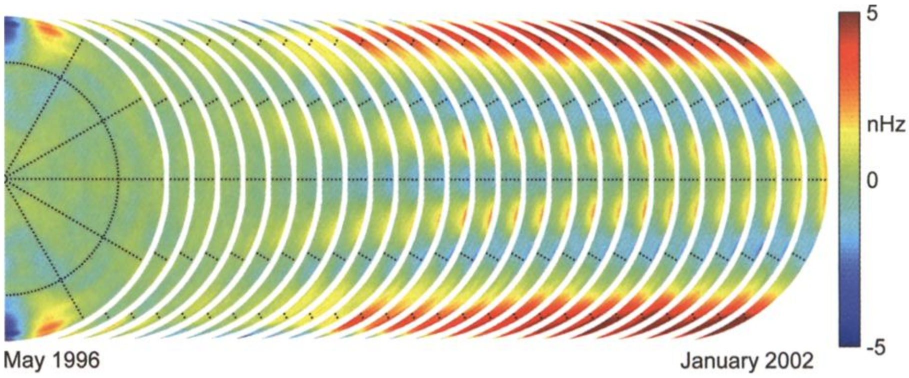

Fig. 1. Variation of the solar internal rotation relative to the first 360-day data set, over the first nearly 6 years of MDI measurements from SOHO. The equatorward migration of the low-latitude branch of the torsional oscillation and the strengthening of the high-latitude branch are visible. The inversions (with k = 3 iterations) are of successive 72-day sets of MDI/SOHO data. Dotted lines indicate the base of the convection zone and the 0°, 30°, and 60° latitudes.

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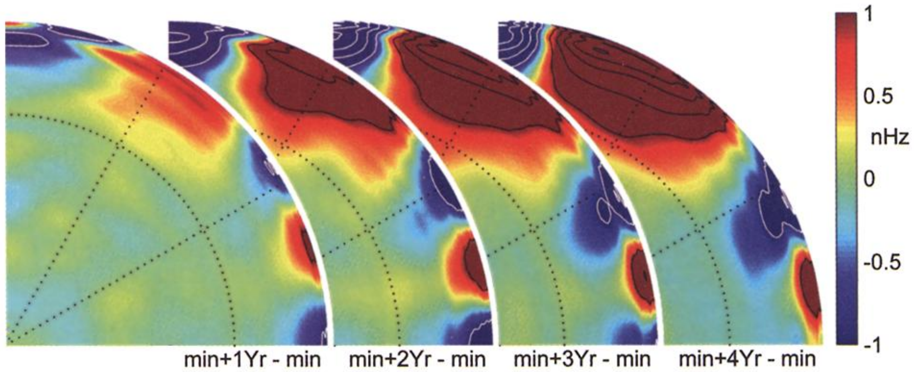

Fig. 2. Variation of the solar internal rotation over 1, 2, 3, and 4 years of increasing solar activity, relative to the first year (which was around the minimum of solar activity). The inversions used $k = 3$ iterations. The color scale is enlarged by a factor of 5 compared to Fig. 1 to make visible a small-amplitude signal (the colors saturate outside the ± 1 nHz interval). Positive (black) and negative (white) contours are shown at 1-nHz intervals. Note that the results shown in the second panel from the left are less stable than the others because only three, not five, 72-day subsets were used in data averaging, due to the interruption in SOHO measurements in 1998.

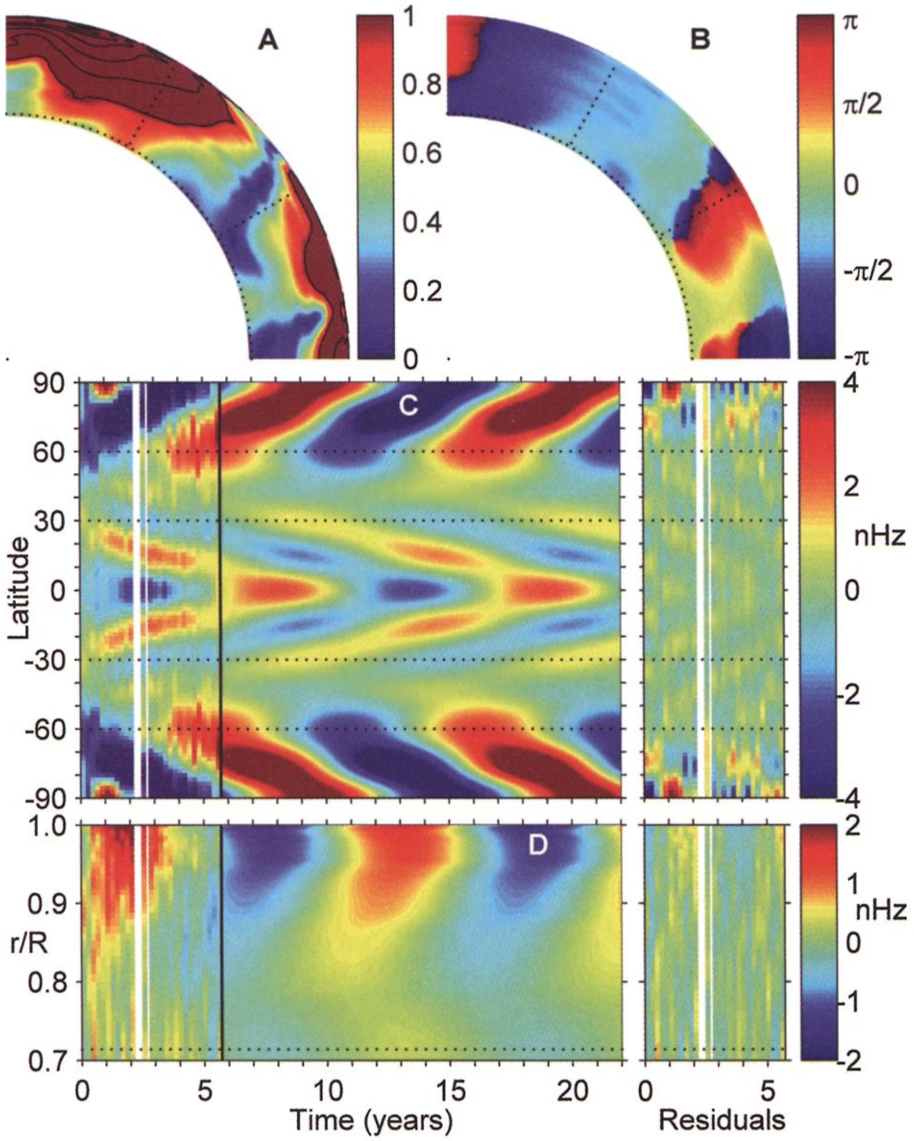

Fig. 3. (**A**) Amplitude $A(r,\theta)$ (left) and (**B**) phase $\Phi(r,\theta)$ (right) obtained by fitting an harmonic function $A(r,\theta)\sin[\omega_0 t + \Phi(r,\theta)] + B(r,\theta)$ with an 11-year period $2\pi/\omega_0$ to the variation of internal rotation (similar to what is shown in Fig. 1 but obtained from inversions with $k = 4$ iterations). The phase is defined to be zero when the harmonic variation is zero at the solar minimum and increases with time. Contours are shown at 1-nHz intervals in (A). (**C**) The rotational variation as a function of time and latitude at radius $r = 0.98R$ are shown for the first nearly 6 years, and thereafter the time series is continued by exhibiting the 11-year harmonic fit [based on (A) and (B)]. Shown to the right are the residuals from the fit, on the same color scale. (**D**) As for (C), but showing the rotation variation as a function of depth instead of latitude, at latitude 20°.

flow develops slightly above 30° latitude and migrates toward the equator; this too appears in our inversion to penetrate through much of the convection zone. The low-latitude faster flow is localized in the upper third of the convective envelope, and its depth profile can be resolved. Near-equatorial variations are seen again deep in the convection zone. Solar torsional oscillations thus are a global phenomenon, involving the entire convective envelope at all latitudes.

The sensitivity of the p-mode frequencies to the internal rotation degrades rapidly at greater depth: the acoustic waves, which constitute the acoustic oscillations, spend less time propagating in deeper regions where the sound speed is greater. The sensitivity also decreases at higher latitudes: in our global inversion, the rotation at the highest latitudes is measured by extrapolation. The inversion technique provides nearly uniform stability of the results to random errors (this property can be observed in the left-hand frames of Figs. 1 and 2; the noisy component in the input data results in the cloudy features covering the meridional plane with uniform amplitude). We do not see the prominent, well-structured variations near the base of the convection zone, as predicted by some recent dynamo models (*8*). It does not mean, however, that there are no such variations there, of an amplitude comparable to that in the higher layers; these variations could be present but unresolved by our inversion from the noise in the helioseismic data. Indeed, variations near the base of the convection zone on time scales short compared to the solar cycle have been reported (*9*).

The inversion technique is an iterative one: the regularizing properties are controlled by the number of iterations. The inversions of the solar data (Figs. 1 and 2) were obtained with $k = 3$ iterations. With increasing iteration number (relaxing the regularization parameter), the true features of the solution and the features induced by the random errors tend to exhibit a different behavior: the true features are gradually developed and stabilized, whereas those produced by errors grow rapidly. This property helps to distinguish the statistically significant features of the solution (*10*). Our conclusion that the entire convective envelope is involved in the torsional oscillations is a stable feature of the inversion.

Although the observations cover only half of the 11-year activity cycle, an analysis of the results indicates that the variations in rotational velocity inferred from the consecutive 72-day data sets, in the regions where they are better resolved, have a slow time dependence which is consistent with harmonic behavior. We can construct a phase-amplitude diagram, which should provide insight into the global behavior of the solar torsional oscillations (Fig. 3, A and B). The diagram was obtained by fitting, at each point of a 2D spatial grid, the time variation of the angular velocity with a harmonic function with 11-year period (plus an additive constant, because the rotation is measured relative to the solar minimum). We define the phase to be zero when the harmonic variation is zero at the solar minimum and increases with time (*11*).

The zero phase is seen at about 42° latitude, from which the zonal flows propagate toward the pole and toward the equator (Fig. 3, C and D). The phase difference between the 42° latitude of stationary phase and the equator is close to 3π, when measured not too deep below the surface. Because the inversion measures only a north-south symmetric average of the rotation, the equator must also be a location of stationary phase. But a (symmetrized) pair of traveling waves coming from opposite hemispheres and meeting at the equator would give an arbitrary phase at the equator: that the phase turns out to be to within the uncertainties an exact multiple of π is certainly suggestive that what we observe is a global coherently driven phenomenon. When the phase is measured in the deeper layers, below the prominent shallow low-latitude flow (Figs. 1 and 2), the phase difference between midlatitudes and the equator is π. It is not possible to accurately measure the polar phase. In the near-vicinity of 42°, the phase is stationary whereas the amplitude varies with position, being zero at about 42° (Fig. 3A); thus, it would appear that that we have a standing wave near that latitude and at the equator, and quite a complicated traveling wave with an 11-year period in between. The high phase coherence (Fig. 3B) supports the conclusion that the entire convection zone is involved in the torsional oscillations. This phase coherence is also a stable feature of the inversion.

Our discussion so far has focused on the 11-year harmonic signal; however, interestingly, the residuals from the harmonic fit (also shown in Fig. 3, C and D) show a shorter periodicity, roughly 11/3 years. Thus, the residuals suggest the presence of a third harmonic of the 11-year variation. The ridges in the plot of residuals as a function of latitude and time are also about three times steeper at low- and midlatitudes than the ridges of the dominant 11-year signal, indicating that the phase speed of the third harmonic is three times greater than that of the 11-year variation. Thus, when viewed as waves propagating toward the equator at low latitudes, the rotation variations are dispersive with the phase speed varying by a factor of three between the 11-year signal and its third harmonic. At high latitudes, the picture appears to be different. There, the slope of the residuals (Fig. 3C) is similar to that of the dominant signal (in fact, the latitudinal phase speed is similar to that of the low-latitude residuals but of opposite sign), so although higher harmonics are present at high latitudes, the data are consistent with nondispersive poleward migration.

The systematic behavior evidenced by the residuals is seen also at greater depths (Fig. 3D), though the signal in the residuals is overwhelmed rapidly by the noisy observational errors when deeper layers are measured. We need better or longer data sets to explore in detail the variations with depth in the possibly nonlinear dynamics of the torsional oscillations. Further, the variations of the solar interior are unlikely to be strictly periodic, so as long a time base as possible is required. But even as they are seen now, the nonlinearities provide important additional constraints on the physical models of the oscillations.

References and Notes

1. R. Howard, B. J. LaBonte, *Astrophys. J.* **239**, L33 (1980).
2. The MDI is the high-resolution helioseismic instrument on board SOHO; for details, see P. H. Scherrer *et al.*, *Solar Phys.* **162**, 129 (1995). GONG is a ground-based helioseismic network of six stations [J. W. Harvey *et al.*, *Science* **272**, 1284 (1996)].
3. R. Howe *et al.*, *Astrophys. J.* **533**, L163 (2000); H. M. Antia, S. Basu, *Astrophys. J.* **541**, 442 (2000).
4. Details of rotation inversions, in particular for MDI data, may be found in, e.g., J. Schou *et al.* [*Astrophys. J.* **505**, 390 (1998)]. The first-order frequency splitting allows only the north-south symmetrized component of the rotation to be inferred. Hence the results presented here, and in papers such as the one cited above, are symmetric about the solar equator.
5. M. H. Ritzwoller, E. M. Lavely, *Astrophys. J.* **403**, 810 (1991).
6. The adaptive regularization technique is described by V. N. Strakhov, S. V. Vorontsov, in *Helio- And Asteroseismology at the Dawn of the Millennium*, A. Wilson, Ed., ESA SP-464, p. 539 (European Space Agency, Noordwijk, 2001). The method is based principally on the use of conjugate gradients for the iterative solution of a linear system. We represent $\Omega(r,\theta)$ by polynomials in two variables [acoustic radius $\int_0^r c^{-1}\,dr$, $c(r)$ being the adiabatic sound speed, and $\cos^2\theta$]. The polynomial degree in the radial variable needs only to be sufficiently high (degree 80 was used in the results reported here). In the angular coordinate, the polynomial degree is 9, being determined by the highest *a* coefficients in the input data sets which we use (18th degree fit to the multiplet splittings). We use orthogonal polynomials with properly defined weighting function as described by Strakhov and Vorontsov (see above).
7. Our results extend to much higher latitudes than those derived from direct Doppler measurements of surface torsional oscillations [see, e.g., R. K. Ulrich, *Astrophys. J.* **560**, 466 (2001)]: such measurements are restricted by the viewing angle to lower than about 60° latitude. Our greater coverage is feasible because of the global nature of the modes and the large number of *a* coefficients measured by MDI. Our results are consistent with the near-surface helioseismic inferences of variations in the near-polar rotation rate by J. Schou [*Astrophys. J.* **523**, L181 (1999)], and H. M. Antia and S. Basu [*Astrophys. J.* **559**, L67 (2001)].
8. E. Covas, R. Tavacol, D. Moss, A. Tworkowski, *Astron. Astrophys.* **360**, L21 (2000). The comparison of such dynamo models with helioseismic deductions about the torsional oscillations provides new constraints on theories of solar dynamo action.
9. R. Howe *et al.*, *Science* **287**, 2456 (2000).
10. The feasibility of separating the principal true features of the rotation from artifacts introduced by random data errors (noise) may be related to the form of the spectrum of singular values of the problem in hand. In a model problem for inferring the rotation from helioseismic splittings, J. Christensen-Dalsgaard, P. C. Hansen, and M. J. Thompson [*Mon. Not. R. Astron. Soc.* **264**, 541 (1993)] found that the significant singular values of the problem sat on a long plateau, terminated by a sharp drop to small singular values. The existence of the plateau means that true features of the solution on larger scales are revealed and stabilize as the regularization in the inversion is reduced; only when the regularization is reduced enough for the small values beyond the plateau to come into play does the data noise rapidly come to dominate the solution.
11. We note that although the amplitude diagram can be distorted in the deeper layers by the regularized inversion of noisy data, as discussed above (the amplitude being systematically underestimated in the regions where the seismic measurement is most difficult), this effect is not expected to distort the phase diagram. The phase is less sensitive to the number *k* of iterations performed in the adaptive regularization technique, in a suitable range of *k*, and the resulting phase diagram provides a more robust measurement in a spatial domain that includes essentially the entire convective envelope.

12. The Solar Oscillations Investigation (SOI) involving MDI is supported by NASA grant NAG5-8878 to Stanford University. SOHO is a mission of international cooperation between the European Space Agency and NASA. Supported in part by the UK Particle Physics and Astronomy Research Council under grant PPA/G/O/1998/00576 and PPA/G/S/2000/00442, and by the Danish National Research Foundation through its establishment of the Theoretical Astrophysics Center.

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