BOOKS: MATHEMATICS

Infintesimal Steps

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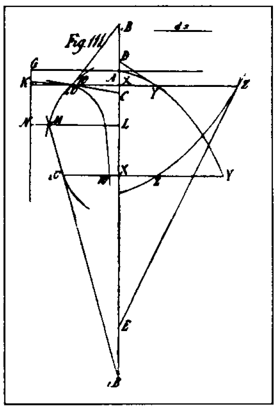
The Greeks were the first to confront the problem of the infinite mathematically, in the guise of Zeno's famous paradoxes of motion and the Pythagoreans' discovery of incommensurable magnitudes. If they failed to come to successful terms with the concept, they also knew that it was best avoided until mathematics could handle the concept without the uncertainties the infinite seemed destined to entail. Several millennia later, European mathematicians were no better off, as Galileo realized when he regarded the infinite as paradoxical. Similarly, George Berkeley leveled his criticism against the Newtonian calculus and its infinitesimals as nothing more than "ghosts of departed quantities." Nevertheless, successful analysis of motion or continuity cannot help but involve such problems as the infinite divisibility of space and time, and hence the infinite could not be ignored. The best-known part of the early history of the infinite and its counterpart, infinitesimals, is the development of the calculus by Newton, Leibniz, and their contemporaries. They created a useful tool for mathematics and for the newly emerging subject of mathematical physics. Progress in the 18th and 19th centuries deepened the understanding of the technical details of the calculus, as mathematical physics developed to an extraordinary degree in the hands of such adepts as Euler, Lagrange, and Laplace.

Reasoning with the Infinite
From the Closed World to the Mathematical Universe
by Michel Blay
University of Chicago Press, Chicago, 1998. 226 pp. $30, £25.95. ISBN 0-226-05834-4.

It is against this background of the struggle mathematics has waged with the infinite that Michel Blay's book must be considered. Its title adapts that of a well-known work by another French author, Alexandre Koyré's *From the Closed World to the Infinite Universe* (1957). Both books consider the major shifts in perspective required by the Scientific Revolution, although Blay focuses on a specific part of this story. Primarily, he is concerned with the emergence of mathematical physics—how investigators intent upon unlocking the secrets of nature eventually gave up an early, largely geometrical approach (which Blay terms the project of Galileo and Descartes "in its initial ontologico-geometrical purpose") in preference for what he calls "mathematization" (by which he means the analysis of nature in terms of "quantitative laws which can be exploited for the purpose of predicting the course of nature by means of mathematical reason").

In characterizing the result of this mathematization as "motion algorithmized," Blay explains Leibniz's application of the calculus to questions of maxima and minima. He shows how, through the Bernoullis' efforts (especially those of Jean Bernoulli), the French community quickly adapted its thinking to the Leibnizian calculus. This meant that "questions related to the science of motion could now be reduced ... to simple analytical procedures governing differentiation and integration." Further development by Varignon led to what Blay calls "the construction of the algorithmic science of motion." Blay then credits Fontenelle for "one of the first and most profound meditations on the meaning of mathematical physics and on the requirement of total mathematicity that was to govern the further development of this branch of knowledge." Unfortunately, Fontenelle himself was not so clear, and Blay may read more into Fontenelle's understanding of the mathematical infinite than is warranted.



Blay argues that Fontenelle's attention to "laws," "rules," and "general formulas" shows he truly understood the meaning of mathematical physics. And Blay finds that through the introduction of the mathematical instrument of indeterminables, Fontenelle "managed to transcend its instrumental status to acquire an explanatory value." But what, then, is one to make of Blay's own admission that Fontenelle's project was a failure? Or that a new science of motion was really the product of Fontenelle's contemporaries, including Jean Bernoulli, Leonhard Euler, A. C. Clairaut, and Jean d'Alembert? At the end of the 18th century, Blay's hero is Lagrange, whose aim it was "to dispose once and for all of the reasoning necessary to resolve mechanical problems, by embodying as much as possible of it in a single formula." This approach also impressed the English mathematician and logician George Boole, who nevertheless realized its shortcomings: "By the labours of Lagrange, the motions of a disturbed planet are reduced with all their complication to a pure mathematical question. It then ceases to be a physical problem; the disturbed and disturbing planet are alike vanished; the ideas of time and force are at an end; the very elements of the orbit have disappeared or only exist as arbitrary characters in a mathematical formula." (*1*, p. 35) While Boole was impressed with what a mathematical physics of the sort advanced by Lagrange represented, he was also aware of what had been lost in the process of a mathematization that was perhaps too abstract and too disembodied.

Blay does not really consider the consequences of excessive mathematization, or what this may have cost both physics and mathematics. Instead, he concludes his discussions with a look at the views of the French mathematician Hoen Wronski, the "first Kantian" in France. Questioning Lagrange's approach and drawing on Fontenelle's ideas, Wronski sought to give the calculus a speculative foundation by identifying what he termed "objective" laws dealing with finite quantities and "subjective" laws dealing with infinitesimals. Confusing these, he claimed, was to blame for the apparent inexactitude "that is felt to be attached to the infinitesimal Calculus." Wronski took the infinite to be "an exact instrument of mathematical investigation," although what he might have meant by this Blay does not explain. Blay's summary refers to a "silence imposed on infinity" in the period leading from Fontenelle to Wronski. It seems hard to agree that there was such a silence, given all that Blay's book has to say about what happened to the concept of infinity in this very period.

Regrettably, *Reasoning with the Infinite* does not examine in greater detail the true progenitors of mathematical physics who established the field on its modern footing—d'Alembert and Euler, or those later in the 18th century including Lagrange, Laplace, and Cauchy. In focusing attention instead on the likes of Fontenelle and Wronski, Blay illuminates figures likely to be little-known to most readers. This is the book's real strength, even if Fontenelle and Wronski's approaches to the infinite were not as satisfactory as those of more influential practitioners.

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Notes

1. Quoted in D. MacHale, *George Boole: His Life and Work* (Boole, Dublin, 1985).