Superfluidity Within a Small Helium-4 Cluster: The Microscopic Andronikashvili Experiment

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The infrared spectrum of single oxygen carbon sulfide (OCS) molecules was measured inside large superfluid pure helium-4 droplets and nonsuperfluid pure helium-3 droplets, both consisting of about 10⁴ atoms. In the helium-4 droplets, sharp rotational lines were observed, whereas in helium-3 only a broad peak was found. This difference is interpreted as evidence that the narrow rotational lines, which imply free rotations, are a microscopic manifestation of superfluidity. Upon addition of 60 helium-4 atoms to the pure helium-3 droplets, the same sharp rotational lines were found; it appears that 60 is the minimum number needed for superfluidity.

The phenomenon of superfluidity, which so far has only been observed for ⁴He below the transition temperature $T_{\lambda} = 2.12$ K and for ³He below $T_{\lambda} = 3 \times 10^{-3}$ K, has been extensively studied both experimentally and theoretically since it was first discovered in ⁴He between 1936 and 1938 (1). To account for the peculiar observations of the unusually high heat conductivity, the fountain effect, and the vanishing flow resistance in narrow capillaries, Tisza (2) and Landau (3) proposed that liquid helium below T_{λ} , designated as He^{II}, was in fact composed of two fluids, one normal and the other superfluid. When the temperature is lowered substantially below T_{λ} , the entire liquid is converted into the superfluid fraction. One of the most important of the early investigations was the 1946 experiment of Andronikashvili (4), which provided impressive confirmation of the two-fluid model. Using a cleverly designed compact disk torsional oscillator immersed in He^{II}, he observed that the moment of inertia of the disks surprisingly did not increase on reduction of the temperature, as expected for a viscous classical liquid, but decreased sharply below T_{λ} . This decrease was interpreted as indicating that the superfluid fraction made no contribution to the moment of inertia. The deviation from the classical value of the moment of inertia thus provided evidence for superfluid behavior. Whereas superfluidity in the bulk is now quite well understood, there are still some fundamental questions concerning superfluidity in small, finite-size systems, such as clusters, where the above macroscopic manifestations are not observable (5). Feynmann path integral Monte Carlo calculations provide evidence for superfluid behavior of pure ⁴He clusters with only 64 atoms (6) as revealed by weak anomalies in the heat

capacity and a reduction in the effective moment of inertia (7). There is considerable interest in finding new microscopic manifestations of superfluidity.

To address this question, researcher have recently used high-resolution spectroscopy to study small single molecules attached to large ⁴He droplets consisting of between 10³ and 10^4 He atoms (8–12). One of the most remarkable findings was the observation of sharp rotational lines in the ν_3 vibrational band of SF_6 molecules (8). The entire spectrum could be fitted very accurately by assuming a free molecule Hamiltonian (12), implying that the symmetry of the molecule is not disturbed by the medium. The fit provided a rotational temperature of $T_{\rm rot} =$ 0.38 K in excellent agreement with theoretical predictions for the droplet temperature (13) but revealed that the moment of inertia was a factor of 2.8 larger than that for the free molecule. This increase was interpreted as indicating some type of coupling to the liquid He environment, presumably due to the anisotropy of the potential describing the interaction of the molecule with the individual He atoms. Subsequent high-resolution laser spectroscopy experiments in the visible region of glyoxal molecules (14) revealed a well-resolved phonon wing structure that could be explained in terms of the sharp dispersion curve for the elementary excitations, a characteristic feature of the superfluid state. Thus, one possible explanation for the free rotations of SF_6 in He droplets is a superfluid environment.

To test for this possibility, we performed infrared (IR) absorption experiments on single molecules inside both large ³He and ⁴He droplets. The ³He droplets, which at 0.15 K (*12*, *13*) are colder than the ⁴He droplets, are still much too warm for ³He to be superfluid. At low temperatures, the viscosity of ³He is known to increase sharply (*15*) in a manner quite similar to that of the normal component of liquid ⁴He (*16*). Thus, the ³He droplets provide an environ-

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ment very similar to that of normal liquid ⁴He. Earlier attempts to measure the IR spectra of SF_6 in ³He droplets (12, 17) revealed no change in the characteristic features of the spectra. This lack of change was explained by considering the effect of the small ($\approx 10^{-3}$) amount of ⁴He impurities in the large (the average number of 3 He atoms $\overline{N}_3 = 45,000$) ³He droplets. Because of their high diffusivity (18) and their lower zero point energy, the ⁴He atoms quickly coated the SF₆ molecule, producing a layer sufficiently large to insulate the SF₆ molecules from the ³He environment as recently confirmed theoretically (19). To avoid these problems, we used highly purified ³He with a stated isotopic impurity level of 10^{-6} (Isotec; A. Matheson, Miamisburg, Ohio) and took great care to reduce all sources of contamination. For our experiments, OCS was chosen because its spectrum in ⁴He droplets exhibits better resolved rotational structure (17, 20) than that of SF_6 .

The cluster molecular beam apparatus is the same as that used and described in the earlier SF_6 IR spectroscopy studies (8, 9). The ³He gas was expanded through a $5-\mu m$ nozzle at a source pressure of $P_o = 5$ bar at a temperature of $T_o = 6.6$ K and continuously recycled. The resulting cluster beam then passed through an 8-cm-long scattering chamber before being detected 80 cm from the source by the electron beam ionizer of the mass spectrometer. The absorption of a laser photon (21) resulted in vibrational excitation of the molecule, which was followed by rapid energy transfer to the host droplet and the subsequent evaporation of about 1200 atoms. This absorption led to a substantial (≈ 5 to 10%) decrease in the mass spectrometer signal (8). The purity of the ³He was monitored in the mass spectrum, and no impurities to within 10 parts per million of the ³He peak could be de-tected. The \overline{N}_3 of the ³He droplets was measured to be 12,000 by deflection scattering in another apparatus (22).

The OCS IR spectrum in the pure ⁴He droplets (Fig. 1A) exhibits a series of sharp lines corresponding to the expected P- and R-branches for the rotational $\Delta J = -1$ and +1 transitions, respectively. At higher resolution, the linewidths were found to depend somewhat on the initial rotational state J in the ground vibrational state, increasing from about 0.005 \mbox{cm}^{-1} (~150 MHz) for J = 0 to about 0.02 cm⁻¹ (≈ 600 MHz) for J = 3 in both the P- and Rbranches (17, 20). The dependence on rotational state suggests some very weak coupling of the rotations with the liquid (7). This broadening, which is barely apparent in Fig. 1A, is, however, of no direct importance for the interpretation to be discussed below because it is about a factor of 20 to 40

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smaller than the rotational energy. As for SF_6 , the spectra can be fitted by a Hamiltonian for a free rotor, with a molecular moment of inertia that is a factor of 2.7 larger than that of the free molecule, compared with the factor of 2.8 found in SF_6 (12). The best fit rotational temperature is 0.37 K compared with 0.38 K for SF_6 (12). The absence of a Q-branch in the OCS spectrum indicates that, as in the case of SF_6 , the symmetry of the rotating molecule in the ⁴He droplet is the same as that of the free molecule (17, 20).

The OCS spectrum in the pure ³He droplets (Fig. 1B) shows an entirely different profile, which is characterized by a rather broad (linewidth, 0.1 cm^{-1}) peak at 2061.8 cm^{-1} . This featureless spectrum is typical for heavy molecules in normal liquids (23). As a result of thermal excitations, the molecule undergoes a process involving continuous changes of its angular momentum and rotational energy known as rotational diffusion (24). If the associated relaxation time τ_{rot} is less than the inverse of the line splitting, then the spectrum is expected to lose its rotational structure. With further decreases in the relaxation time, the spectrum develops into a single line at the band origin, with a linewidth that may be much less than the rotational envelope. This phenomenon, known as the collapse of the rotational structure or motional narrowing, has been observed frequently (23). From the bandwidths of the spectra in Fig. 1, the lifetimes for rotational diffusion are estimated to be 50 ps in ³He, compared with more than 1.5 ns for the sharpest R(0) line in the liquid ⁴He droplets.

The interaction potentials of OCS with ³He and ⁴He atoms are expected to be virtually identical, and therefore the observed differences in the two spectra are attributed to the different environments; in the case of 3 He, the OCS is in a normal fluid, and in the case of ⁴He it is in a superfluid. If the free molecular rotations in ⁴He were not due to superfluidity but were only a consequence of the weak van der Waals potential peculiar to He and the large zero point motion resulting from the low mass of He, then the effect should be even more extreme in the case of the colder, lighter ³He atom environment (25). We propose therefore that the appearance of sharp rotational spectra of a single molecule is a microscopic indication of superfluidity.

Additional evidence is provided by experiments in which a small amount of ⁴He was added to the scattering gas. As mentioned above, the embedded ⁴He atoms segregated from the ³He atoms and surrounded the impurity molecule (12, 19). The average number of ⁴He atoms, \overline{N}_4 , captured was

estimated assuming a Poisson distribution (9, 26):

$$P(N_4) = \frac{(\alpha L)^{N_4}}{N_4!} \exp(-\alpha L)$$
 (1)

where $\alpha = n_4 \sigma \overline{g} / v_{\rm Cl} (n_4$ is the scattering chamber density of ⁴He, σ is the coagulation cross section, \overline{g} is the average relative collision velocity, and $v_{\rm Cl}$ is the cluster beam velocity) (25) and L is the length of the scattering region. The coagulation cross section for ⁴He was assumed to be identical to the cross-sectional size of the He droplet in accord with a recent study of the mass dependence of capture cross sections (26). According to Eq. (1), the n_4 needed to obtain a distribution peaked at a given \overline{N}_4 is defined by

$$n_4 = \frac{\overline{N}_4}{\sigma \bar{g} L / v_{\rm CL}} \tag{2}$$

The standard deviation of the Poisson distribution is given by $s = \sqrt{\overline{N_4}}$.

The evolution of the adsorption spectra was monitored for an increasing number of ⁴He atoms (Fig. 2, A to F). Even with the addition of about seven atoms, the peak in Fig. 2B was substantially shifted by about 0.1 cm⁻¹ to the blue relative to the peak in Fig. 1A. With 25 ⁴He atoms, another shift back to the red was found (Fig. 2C). With 35 ⁴He atoms (Fig. 2D), the spectrum shows



Fig. 1. An expanded view of the OCS IR spectrum in pure ⁴He droplets with $\overline{N}_4 = 6000$ atoms (**A**) and in pure ³He droplets with $\overline{N}_3 = 12,000$ atoms (**B**). The depletion is plotted versus the change in wave number with respect to the origin of the spectrum in (A) at $\nu_{\rm o} = 2061.64$ cm⁻¹.

the first resemblance of the final spectrum that is well developed by the time 60^{4} He atoms have been added (Fig. 2E). The increase in the moment of inertia was about the same as that obtained in pure ⁴He droplets. From the relative intensities of the three peaks (Fig. 2, E or F), a $T_{\rm rot} = 0.11$ K was obtained that was somewhat lower than in our earlier measurement for SF_6 in mixed ³He-⁴He droplets (12, $1\underline{7}$). The further addition of ⁴He beyond $\overline{N}_4 = 100$ did not change the spectrum substantially. The appearance of a sharp rotational spectrum is an indication of superfluidity, and therefore these experiments indicate that superfluidity sets in gradually with the addition of about 60 atoms. A cross-sectional view of a large ³He droplet containing an OCS molecule surrounded by a shell of 60 ⁴He atoms is shown in Fig. 3.

The present experiments provide clear evidence that OCS molecules do not rotate freely in ³He droplets. Only with the addition of about 60 ⁴He atoms, which corresponds to about two shells around the OCS molecule, do the spectra reveal sharp rotational peaks. Thus, two effects contribute to the observed transformation of the spectral features. On the one hand, the ⁴He atoms decouple the molecule from the impeding effect of the ³He collisions. The ³He on the outside is not expected to affect the ⁴He atoms substantially if the inner core of the atoms is sufficiently large. This is in agreement with calculations on mixed clusters, which indicated that the ⁴He density at the



Fig. 2. A series of OCS IR spectra similar to Fig. 1B but with increasing average numbers \overline{N}_4 of added ⁴He atoms [$\overline{N}_4 = 0$ (**A**), 7 (**B**), 25 (**C**), 35 (**D**), 60 (**E**), and 100 (**F**)]. The change in wave number in the abscissa is with respect to $\nu_{\rm o} = 2061.71$ cm⁻¹.

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⁴He-³He interface falls off over a distance similar to that at the surface of a free ⁴He cluster of the same size (19, 27). The second effect is the buildup of a superfluid shell that enables the molecule to rotate freely. Recently, Feynmann path integral calculations in which the potential anisotropy was neglected have been carried out for SF6 inside small ⁴He clusters. The results revealed that superfluidity persists to a certain extent even in the highly compressed layers nearest the impurity molecule (28). These calculations are also consistent with measurements of healing lengths in thin films for which the superfluid fraction is suppressed at the interface and builds up to its full value within a distance of a few angstroms (29). Similarly, in the vicinity of the OCS molecules, we expect a substantial normal fluid fraction.

The observed increase in the moment of inertia of the molecule about which we speculated earlier (8) now has a straightforward explanation. If we neglect any interaction with the outer shell of ³He, then, because of its superfluid character, the moment of inertia of the shell of ⁴He atoms surrounding the molecule is expected to be

greatly reduced to below its rigid body value, just as in the case of Andronikashvili's experiment (4) or as found in the Feynmann path calculations (6, 28). Thus, the moment of inertia of a single OCS molecule in a superfluid cluster will have three contributions:

$$I = I_{OCS}$$

+ $\int_{0}^{R_{max}} \int_{0}^{\pi} \int_{0}^{2\pi} \rho_n(R, \vartheta, \varphi) R^4 dR \sin^3 \vartheta \, d\vartheta d\varphi$
+ hydrodynamic terms (3)

where I_{OCS} is the moment of inertia of the free molecule. The second term is integrated over the entire anisotropic distribution of the normal liquid density $\rho_n(R, \vartheta,$ $\varphi)$ (R, ϑ , and φ are spherical coordinates with respect to the center of mass and axis of the rotation) out to a distance R_{max} of about 7 Å corresponding to about 60 ⁴He atoms. $\rho_n(R, \vartheta, \varphi)$ is expected to be largest near the "surface" of the molecule (29). The last term accounts for the perturbation of the surrounding liquid by the ro-



Fig. 3. Simple schematic picture of an OCS molecule (yellow, black, and red) with a surrounding cluster of 60 ⁴He atoms (purple) inside a large ³He droplet (blue). This model corresponds to the spectrum shown in Fig. 2E, where superfluidity becomes apparent.

tating molecule that depends on the anisotropy of the molecule. Because the superfluid undergoes irrotational flow, we refer to this term as a hydrodynamic (backflow) correction. Simple estimates (30) suggest that this third term is quite small so that the factor 2.7 increase in the moment of inertia comes mainly from the normal liquid fraction near the molecule. Although the overall moment of inertia is larger than that of the free molecule, it is much smaller than if the surrounding shell of ⁴He atoms were a normal fluid and nearly rigidly coupled. A more complete microscopic understanding of the phenomena observed here will require detailed simulations with careful accounting of the anisotropic interactions.

These experiments provide convincing evidence for a microscopic manifestation of superfluidity in a system consisting of a small number of ⁴He atoms. We propose that this new phenomenon should therefore be designated "molecular superfluidity." The second major observation is the critical size of about 60⁴He atoms needed to permit molecular superfluidity to occur. When a molecule is coated with about two layers of ⁴He at temperatures well below T_{λ} , then the molecule is able to rotate nearly unimpeded but with an increased moment of inertia compared with that of the free molecule due to both the normal component dragged along and a hydrodynamical superfluid "backflow" correction. Unfortunately, we were not able to change the temperature of our droplets, which maintained a constant temperature determined by their heat of evaporation (13). Otherwise, it would be possible to observe the change in superfluid fraction with temperature. In most other respects, however, the new phenomenon is the microscopic analog of the famous Andronikashvili experiment (4).

REFERENCES AND NOTES

- 1. P. L. Kapitza, *Nature* **141**, 74 (1938); J. F. Allen and A. D. Misonor, *ibid.* p. 75
 - A. D. Misener, *ibid.*, p. 75
- L. Tisza, *ibid.*, p. 913; C. R. Acad. Sci. Paris 207, 1035 (1938); *ibid.*, p. 1186.
- 3. L. Landau, J. Phys. U.S.S.R. 5, 185 (1941); ibid. 11, 91 (1947).
- 4. E. L. Andronikashvili, *ibid*. **10**, 201 (1946).
- K. B. Whaley, Int. Rev. Phys. Chem. 13, 41 (1994).
 P. Sindzingre, M. L. Klein, D. M. Ceperley, Phys. Rev.
- Lett. 63, 1601 (1989). 7. L. Pitaevskii and S. Stringari, Z. Phys. D16, 299
- (1990).
 8. R. Fröchtenicht, J. P. Toennies, A. Vilesov, *Chem. Phys. Lett.* **229**, 1 (1994); M. Hartmann, R. E. Miller,
- J. P. Toennies, A. Vilesov, *Phys. Rev. Lett.* **75**, 1566 (1995).
 9. M. Hartmann, R. E. Miller, J. P. Toennies, A. F.
- Vilesov, *Science* 272, 1631 (1996).
- S. Goyal, D. L. Schutt, G. Scoles, *Phys. Rev. Lett.* 69, 933 (1992).
- 11. J. Higgins *et al.*, *Science* **273**, 629 (1996). 12. J. Harms, M. Hartmann, B. Sartakov, J. P. Toennies,
- A. F. Vilesov, J. Mol. Spectrosc. 185, 204 (1997).
- 13. D. M. Brink and S. Stringari, Z. Phys. D15, 257

(1990); A. Guirao, M. Pi, M. Barranco, *ibid.* **D21**, 185 (1991).

- M. Hartmann, F. Mielke, J. P. Toennies, A. Vilesov, G. Benedek, *Phys. Rev. Lett.* **76**, 4560 (1996).
- D. S. Betts, D. W. Osborne, B. Welber, J. Wilks, *Philos. Mag.* 8, 977 (1963).
- A. D. B. Woods and A. C. Hollis-Hallet, *Can. J. Phys.* 41, 596 (1963).
- M. Hartmann, thesis, University of Göttingen, Göttingen, Germany (1997) (available as Bericht 10/1997 from Max-Planck-Institut für Strömungsforschung, Göttingen, Germany).
- D. O. Edwards, M. S. Pettersen, T. G. Culman, J. Low Temp. Phys. 89, 831 (1992).
- 19. M. Barranco, M. Pi, S. M. Gatica, E. S. Hernandez, J. Navarro, *Phys. Rev. B* **56**, 8997 (1997).
- 20. S. Grebenev, M. Hartmann, M. Havenith, J. P. Toennies, A. F. Vilesov, in preparation.
- 21. The laser beam, produced by a Mütek MSD 1100

diode laser ($\lambda \simeq 5\mu$, resolution $\delta\nu \lesssim 100$ MHz), was directed coaxially but antiparallel to the droplet beam through a BaF₂ window at the far end of the apparatus.

- J. Harms and J. P. Toennies, in preparation. The method used is described by M. Lewerenz, B. Schilling, and J. P. Toennies [*Chem. Phys. Lett.* **206**, 381 (1993)].
- J. H. R. Clarke, in Advances in Infrared and Raman Spectroscopy 4 (Heyden, London, 1978), chap. 4, pp. 109–193.
- 24. R. G. Gordon, J. Chem. Phys. 44, 1830 (1966).
- In quantum solids such as H₂, the rotors behave as if they were free, and sharp rotational spectral features have been observed [P. J. Berkhout and I. F. Silvera, *Commun. Phys.* 2, 109 (1977); M. Okumara, M. C. Chan, T. Oka, *Phys. Rev. Lett.* 62, 32 (1989)].
- M. Lewerenz, B. Schilling, J. P. Toennies, *J. Chem. Phys.* **102**, 8191 (1995).

Elevation Change of the Southern Greenland Ice Sheet

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Seasat and Geosat satellite altimeter measurements for the Greenland ice sheet (south of 72°N latitude) show that surface elevations above 2000 meters increased at an average rate of only 1.5 \pm 0.5 centimeters per year from 1978 to 1988. In contrast, elevation changes varied regionally from –15 to +18 centimeters per year, seasonally by \pm 15 centimeters, and interannually by \pm 8 centimeters. The average growth rate is too small to determine if the Greenland ice sheet is undergoing a long-term change due to a warmer polar climate.

Understanding the current state of the polar ice sheets is critical for determining their contribution to sea-level rise and predicting their response to climate change. Current estimates from decades of tidegauge data indicate an increase in global sea level of 10 to 20 cm over the past century (1). It is uncertain, however, what the individual contributions of the polar ice sheets are to sea-level rise at this time. The Greenland ice sheet is of particular interest in climate change studies because it is significantly warmer than the Antarctic ice sheet, where temperatures remain well below freezing over the majority of its surface. Also, the potential for polar amplification of a global warming trend in the Northern Hemisphere is very likely (2). Thus, the Greenland ice sheet is likely to undergo more dramatic change in response to a global warming trend.

Using satellite radar altimeter data from

the Seasat and Geosat missions, Zwally et al. (3) estimated that the southern part of the Greenland ice sheet (south of 72°N latitude) grew by 23 ± 6 cm/year from 1978 to 1986. Zwally (4) suggested an increase in precipitation rates caused by a warmer polar climate as a possible cause of the volume growth. However, concerns have been raised about the effect of orbit errors, retracking errors, and systematic biases on these results (5-8). We reexamined elevation change of the Greenland ice sheet, using Seasat and Geosat altimeter data through 1988 after incorporating recent technical advancements in ice-sheet retracking, orbit computation, and orbit error reduction.

The Seasat and Geosat altimeters were designed primarily for measuring sea-surface height. Altimeter data collected over the ice sheets must be postprocessed to produce accurate surface elevation measurements. This is called "retracking" and is required because the leading edge of the reflected radar signal deviates from the tracking gate on the satellite, causing an error in the range measurement. Comparison of the repeatability of surface elevations produced from different ice-sheet retracking algorithms (7) showed that the retracking algorithm (9) used by Zwally *et al.* (3), hereafter referred to as the NASA algorithm, intro-

- The weak interaction at the ⁴He-³He interface is consistent with the very small surface tension measured at the interface [H. M. Guo, D. O. Edwards, R. E. Sarwinski, J. T. Tough, *Phys. Rev. Lett.* **27**, 1259 (1971)].
- Y. Kwon, D. M. Ceperley, K. B. Whaley, J. Chem. Phys. 104, 2341 (1996).
- 29. M. Chester and L. C. Yang, *Phys. Rev. Lett.* **31**, 1388 (1973).
- 30. H. Lamb, *Hydrodynamics* (Dover, New York, ed. 6, 1945), p. 154.
- 31. We thank J. Harms, B. Sartakov, and Y. Kagan for valuable discussions on ³He droplets; G. Benedek, J. Close, Y. Kagan, and B. Whaley for reading and commenting on the manuscript; and S. Bartolomei and M. Lewerenz for preparing Fig. 3. A.V. thanks the Deutsche Forschungsgemeinschaft for a "habilitation stipendium."

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duced larger errors in the elevation data than did three other retracking algorithms. Also, the NASA algorithm produced icesheet growth rate estimates 30 to 50% larger than those derived from three competing algorithms, which all produced nearly identical results. Although several refinements of the NASA algorithm have now been made (10), it still introduces significant random error in ice-sheet datasets. We used a threshold retracking algorithm developed specifically for measurement and detection of ice-sheet elevation change (11). The threshold algorithm reduces random errors in ice-sheet data by up to 35% compared to the current NASA algorithm (11).

For comparing Seasat and Geosat data, Zwally et al. (3) used orbit solutions accurate to about 1 m in the radial component (12). The solutions for Seasat and Geosat were derived from different gravity models. Because of this and other factors, these authors used a 40 \pm 40 cm systematic correction in their analysis. Since then, several consistent sets of orbit solutions have been developed for both datasets. We used Joint Gravity Model-3 (JGM-3) (13) orbit solutions that are now available for Seasat, Geosat-Geodetic Mission (GM), and the Geosat-Exact Repeat Mission (ERM) satellite datasets. The radial component of Geosat JGM-3 orbits is accurate to 10 cm (14). The Seasat JGM-3 orbits, while not as accurate as their Geosat counterparts, represent significant improvement over previous solutions.

Most studies of ice-sheet elevation change correct altimeter radial orbit error using a reference ocean surface in the vicinity of the ice sheet (for example, the North Atlantic for Greenland) (15). However, the predominant radial orbit error is a long-wavelength signal concentrated at a frequency of 2π /orbital period (1/rev frequency). Within each continuous orbit solution, the phase and amplitude of the 1/rev error change gradually over large distances, where a high level of correlation is main-

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