throughout the geologic record, and that is raising hopes that this glimpse of Precambrian animals will soon turn into a panorama. "Both the *Nature* and *Science* papers show we can map out early animal evolution," says Bengtson. "There are other phosphorites that we should go back to and look at with our new eyes. It's a new world waiting."

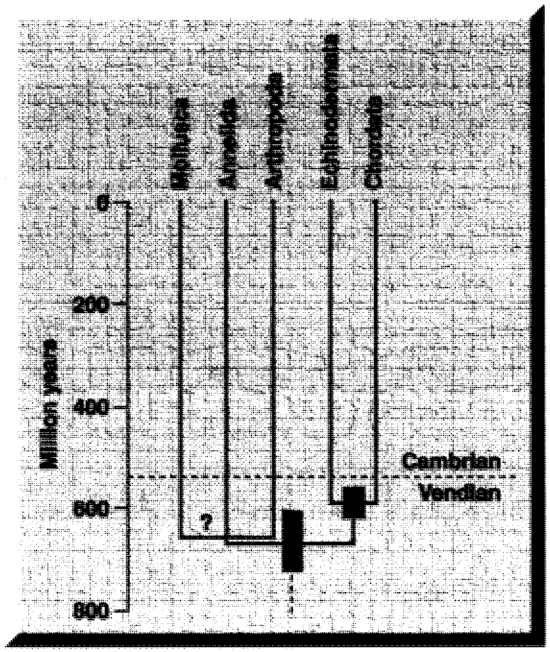
"The big question," says Conway Morris, "is how much farther back this will go." It might go quite a ways. Developmental biologist Eric Davidson of the California Institute of Technology in Pasadena and his colleagues have suggested that life might have spent many tens, even hundreds, of millions of years as tiny, simple assemblages of cells—"squishy little larvalike things"—that were more sophisticated than sponges but were prevented from evolving into the larger forms preserved at the Cambrian explosion by a developmental barrier (*Science*, 24 November 1995, p. 1300). Davidson's team suggested that paleontologists would therefore miss all the early action, but the new fossils raise the possibility that such tiny life-forms may be in the record after all, says Knoll. Davidson agrees that this is the most direct fossil evidence yet for an extended history for his larvalike animals.

Meanwhile, other biologists have also been theorizing about the origins of animals, and some have been pushing for very early dates for these life-forms. In 1996, for example, molecular evolutionist Gregory Wray and colleagues at the State University of New York, Stony Brook, analyzed the sequences and evolution of eight genes across the living animal phyla (*Science*, 25 October 1996, p. 568). Assuming that mutation rates were constant, they used the amount of gene-sequence difference among various organisms to calculate how long ago the groups split. They concluded that a deep division among these animals—between the mollusks, annelid worms, and arthropods (called the protostomes) on one hand and the echinoderms and chordates (called the deuterostomes) on the other—happened a whopping 1.2 billion years ago. The split between bilaterally symmetric animals (like people) and radially symmetric ones (like jellyfish) would have been even earlier.

But other molecular evolutionists aren't convinced. In the 25 January issue of the *Proceedings of the National Academy of Sciences*, Francisco Ayala of Pennsylvania State University (PSU) in University Park, statistical analyst Andrey Rzhetsky of Columbia University, and Ayala's father—evolutionary biologist Francisco José Ayala of the University of California, Irvine—reanalyze the gene data used by Wray and his colleagues. They use rates for 12 additional genes and throw out data from genes whose rates of evolution appeared to change over time. Their result: The protostome/deuterostome split occurred 670 million years ago, give or take 60 million years (see diagram).

That's at least drawing closer to dates from the fossil record, says evolutionary biologist Charles Marshall of the University of California, Los Angeles. "Seven hundred million years is a pretty long way out" for life to have existed without leaving a trace, he says. "But [at least] it's not a billion." The numbers from genetic studies may eventually converge, for more studies from more genes are on the way. But for now, their estimates for the protostome/deuterostome split range from 670 million to 1500 million years, according to a limited survey by *Science*. As molecular evolutionist Xun Gu of PSU notes, with some understatement, "This problem needs more data."

AYALA ET AL./PNAS



Looking back. The latest molecular study dates a key branch point in the tree of life to about 670 million years ago.

The new paleontological findings show that the additional data won't be coming from genes alone. Knoll predicts that "over the next few years, you are going to see people finding comparable fossils in yet older rocks." And finding whole animals—perhaps even the common ancestor of vertebrates and invertebrates at the root of the animal family tree—is a real possibility. Paleontologists may be able to test some of biologists' ideas on what the ancestral animal looked like, says Doug Erwin of the National Museum of Natural History in Washington, D.C., and "that's pretty exciting."

–Richard A. Kerr

MATHEMATICS

Proving a Link Between Logic And Origami

BALTIMORE—If you have ever tried to fold an origami boat or swan, you know it's not as easy as the step-by-step instructions suggest. But try doing it with just an unfolded square of paper and nothing but the crease lines. Even the most dedicated amateur origamist might give up in frustration when faced with the challenge of making the folds in the proper order. Predicting the properties of the finished origami model is still more daunting: It involves solving a problem that's as hard as anything known to computer scientists.

A pair of computer scientists has now shown that a well-known origami problem called the flat-folding problem belongs to a class of tasks that mathematicians call NP-complete: problems so difficult that each contains the key to solving—or being unable to solve—all the rest. (The initials "NP" stand for "nondeterministic polynomial time," which is computer science jargon for "may take an awfully long time unless you make a lucky guess.") The problem, predicting whether an origami model can be folded flat between the pages of a book without creating any new creases or otherwise damaging the model, has long tantalized the small community that studies the mathematics underlying origami. Now Barry Hayes, a computer scientist at the Mountain View, California, software firm Placeware Inc., and Marshall Bern of Xerox's Palo Alto Research Center have shown that this question is equivalent to a famous "hard" problem in computer science known as not-all-true 3-SAT.

The proof, which Hayes presented at a meeting here* in a session devoted to mathematical aspects of origami, is "a capstone of the type of research that has been emerging in origami mathematics," says Thomas Hull, a mathematician at Merrimack College in Andover, Massachusetts, who organized the session. Even though Hayes and Bern's proof is of purely theoretical interest, Hull says he is "delighted by the result," because the glimpse of a connection between paper sculpture and a much-studied problem in computer science may help bring other researchers into the fold.

The computer-science problem, not-all-true 3-SAT, involves determining whether a given logical expression, created by stringing together simple, three-term clauses, can be sat-

* Annual meetings of the American Mathematical Society and the Mathematical Association of America, 7–10 January.

isfied. "Satisfied" means, in this case, that the expression can be made true by assigning each term—which may appear in multiple clauses—a single value, True or False, throughout the expression. The constraint, as the "not-all-true" sobriquet suggests, is that you're not allowed to assign the value True to all three variables in any clause. For example, the two-clause expression "(A or B or C) and (A or C or D)" can be satisfied under the constraint by letting A, B, and D be true and C be false.

Simple though this example sounds, longer examples, where the same term occurs many times in different contexts, are much harder to satisfy. It's easy to show that a given example is satisfiable if you happen to know the proper True/False values, but there seems to be no way to find a solution short of trying all possible combinations. That's what makes the 3-SAT problem NP-complete. "Complete," in this context, means that every other NP problem, from factoring products of large primes to the famous Traveling Salesman problem, can be translated into a not-all-true 3-SAT problem.

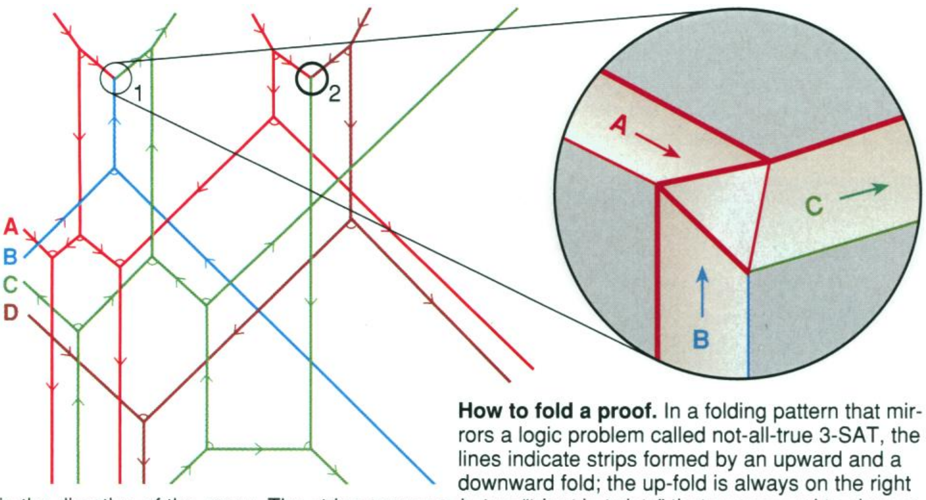
By translating 3-SAT expressions into crease patterns, Hayes and Bern were able to prove that flat-folding, too, is NP-complete. The key ingredient in their proof is a crease pattern consisting of an equilateral triangle with strips radiating from its three sides—a pattern well known to origamists as a "triangle twist." The triangle twist can't be folded into a flat sheet of paper if all three strips are creased in the same way—for example, with an upward fold (what origamists call a mountain fold) on the left and a downward fold (a valley fold) on the right of each one as it enters the triangle. It is flat-foldable only if one strip is folded in the opposite way (see figure). This folding constraint suggests that the triangle twist can stand for one clause in a not-all-true 3-SAT expression. The strips represent the three variables participating in the clause, and the True/False value carried by each variable depends on which side of the strip is mountain- or valley-folded as it enters the triangle.

The key to showing that origami folding is the equivalent of a 3-SAT problem is to fold a bunch of these triangle twists, each one representing a clause, connected by strip-folds that carry variables from triangle to triangle. Hayes and Bern showed that the entire pattern can be folded flat only if the strips carry the same "value" each time they enter a triangle. The pattern, then, is flat-foldable only if the 3-SAT expression is satisfiable.

The proof means that if origami experts could find an efficient way to answer their flat-folding question, the result would apply across the board to a vast range of computational problems, including those whose difficulty provides the security for modern cryptographic systems. But origami enthusiasts are no more likely to solve the flat-folding problem than computer scientists are to solve such NP-hard chestnuts as the Traveling Salesman problem. Says Hayes: "This has no practical applications to [either] origami or computer science; I will be really surprised if one turns up."

Still, Hull is confident that origami will soon be producing something more than beautiful models for problems in other fields. "There are lots of places where you see things being folded up in nature," from insect wings and flower buds to the crumpled aftermath of a car crash, he says. Using paper to study what nature does with a range of materials, Hull thinks, "might be the next revolution in origami."

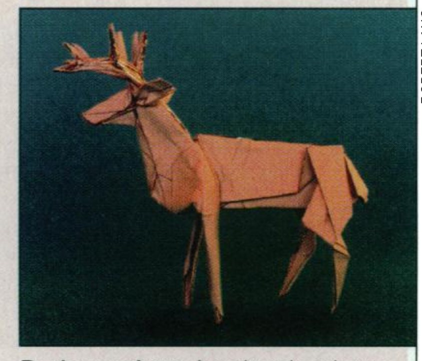
–Barry Cipra



How to fold a proof. In a folding pattern that mirrors a logic problem called not-all-true 3-SAT, the lines indicate strips formed by an upward and a downward fold; the up-fold is always on the right in the direction of the arrow. The strips converge in two "triangle twists" that correspond to clauses in which only two of the three variables carry the same true or false value.

Algorigami, Anyone?

Predicting whether any given origami design can be flat-folded—pressed flat between the pages of a book without damage—is about as hard as a problem can get (see main text). It's easier to start with the flat-foldable criterion and create designs that meet it, as Robert Lang, a physicist at SDL Inc. in San Jose, California, and a renowned origami expert, has shown. At the mathematics meetings, he described a way to create arbitrarily complex patterns that are easy to fold flat. More important—at least to origamists—the method also provides a way to sculpt in paper essentially any object whose general shape can be suggested by a stick figure.



ROBERT LANG

By the numbers. An origami sculpture folded with the help of a new algorithm.

The stick figure specifies the numbers and lengths of various body parts—six legs for an insect, say, plus mandibles if it's a beetle. These in turn constrain how close together various vertices can be in the crease pattern. Within those constraints, Lang's algorithm works out a crease pattern that puts flesh—the flat-foldable kind—on this framework. The basic shape can then be decorated using familiar origami techniques to produce details such as feet and joints.

The algorithm also aims at producing the largest possible sculpture from a given square of paper. Lang notes that this challenge is closely related to what mathematicians call the circle-packing problem: determining how large a square you need to enclose *N* nonoverlapping circles of radius 1. In the origami context, it's only necessary to enclose the centers of the circles, which correspond to the tips of the stick figure's extremities and thus can lie at the edge of the paper. Lang's algorithm can tell you, for example, that a three-pointed star with 1-centimeter spokes can be folded from a square with sides approximately 1.93 centimeters in length.

But for more complex shapes, there's no guarantee that the algorithm will make the most efficient use of paper. The circle-packing problem, like other mathematics related to origami, is a longtime puzzler. The state of the art, Lang notes, is such that no one knows how small a square can be used for stars with more than 20 points. **–B.C.**