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Were Thick Galactic Disks Made by Levitation?

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The thick disk of our galaxy displays kinematic and chemical properties that are intermediate between those of the halo and the (thin) disk stellar populations. Not all disk galaxies have a thick disk. A theory of the origins of a thick disk can potentially provide insights into the physical state of our galaxy in its infancy. Levitation, a process that relies on adiabatic capture into resonance of stellar orbits in a growing disk, is presented as a plausible formation mechanism; a 2:2 resonance between vertical and epicyclic oscillations drifts to large vertical energies as the disk grows adiabatically. Resonant stars levitate several kiloparsecs above the plane, forming a thick disk whose spatial distributions, kinematics, and ages leave unique observational signatures on the sky. The same process can also produce the disk globular cluster system.

In addition to the flat disks that give them their names, disk galaxies sometimes have a distribution of stars that forms a thick disk (*1–5*). Near the solar circle, the thick disk is a rapidly rotating population that extends to several kiloparsecs (kpc) above the galactic plane (*6, 7*). The processes that have heated the thin disk (*8*) cannot impart the large vertical velocities (>40 km s^{-1}) needed to support the thick disk (*9*). Several formation mechanisms have been proposed (*10*): some suggest that the thick disk formed during the dissipational collapse of our galaxy, whereas others propose the heating of a preexisting thin disk, for instance, by the accretion of a satellite galaxy. We present a dynamical model, levitation, whose essential ingredients are the slow growth of the disk plus largish initial eccentric motions of orbiting stars.

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Gas falling into the potential well of a dark halo will settle into a disk if it can dissipate energy efficiently (*11, 12*). While stars form, more gas continues to be added to the disk. If the disk is axisymmetric, as we assume it is, the orbital angular momentum L of each star is conserved, which results in an inward drift of the guiding-center radii (R_0) of all orbits. It is likely that the disk grows over times much longer than the orbital periods of stars (that are not too far out in the disk) (*13, 14*). In situations such as these, when the forces vary slowly and the dynamics is nearly integrable, it is useful to consider the actions associated with a stellar orbit; L is one such action. The other two are associated with the epicyclic and vertical oscillations about the inwardly drifting circular motions of the guiding center (*15*). If $X = (R_{max} - R_0)$ is the amplitude of the radial oscillations, Z is the amplitude of the vertical oscillations, and U and W are the corresponding velocity amplitudes, then $I_R = XU$ and $I_z = ZW$ will suffice, for our purposes, as surrogates for the true radial and vertical actions. The normal response of a stellar orbit to the adiabatically growing disk potential will be to conserve I_R and I_z (*15*). Because the frequencies U/X and W/Z may be expected to increase with disk mass, spatial amplitudes decrease while velocities grow. In particular, the vertical thickness of the disk will decrease while the vertical velocity dispersion grows by a corresponding factor. When there are resonances between radial and vertical oscillations, the relative phase might be even more slowly varying than the disk potential, thus breaking adiabatic invariance (*16*). If an orbit happens to be captured into resonance (*17–19*), the star may be dragged across phase space along with the slowly drifting resonance, resulting in a change of order unity in the actions.

Levitation is a process that relies on just such a capture into resonance. We will describe this mechanism after introducing a specific form for the potential of the dark halo:

$$\phi_h = (V_c^2/2)\ln[R_c^2 + R^2 + (z/q)^2] \quad (1)$$

where $R = \sqrt{x^2 + y^2}$ is the radius in the equatorial plane, z is the height above this plane, R_c is the core radius, $1/\sqrt{2} < q < 1$ measures the flatness of the potential, and V_c is the characteristic circular speed due to halo potential. When the disk is still very light, ϕ_h will essentially govern the motions of stars. The ratio of vertical to epicyclic frequency (for small X and Z) is

$$\nu_h/\kappa_h = 1/q\sqrt{2} < 1 \quad (2)$$

The growing disk will add to both frequencies in quadrature; $\nu^2 = \nu_h^2 + \nu_d^2$, and $\kappa^2 = \kappa_h^2 + \kappa_d^2$. Because the disk is geometrically very flat, it will add more to ν than to κ. At the present time, near the solar circle, $\nu/\kappa \approx 3$, so sometime in the remote past the frequencies must have been equal; that is, the star must have passed through resonance. An important property of both ν and κ is that they are decreasing functions of X and Z. Thus, a star that came into resonance when $\nu \approx \kappa$ can remain locked in resonance only by increasing both Z and W. A resonant star will rise far above the plane while its nonresonant fellows are crunched by the growing disk. This implies that the actions are no longer conserved quantities, and indeed they are not. What is conserved is the combination, $(I_R + I_z)$, so that levitation works by exchanging (an initially largish) I_R for (an initially smallish) I_z. The 2:2 resonance gets its name from the $z^2(R - R_0)^2$ terms in the combined potential of the halo and the disk that are the dominant providers of confinement near the drifting resonance [see (*19*) for more details of the dynamics]. The strength of this nonlinearity as well as the slowness of growth of the disk determine how many stars are captured and how far in phase space they are dragged along by the resonance before they escape.

We model the gravitational potential of the disk by a Miyamoto-Nagai potential (*20*):

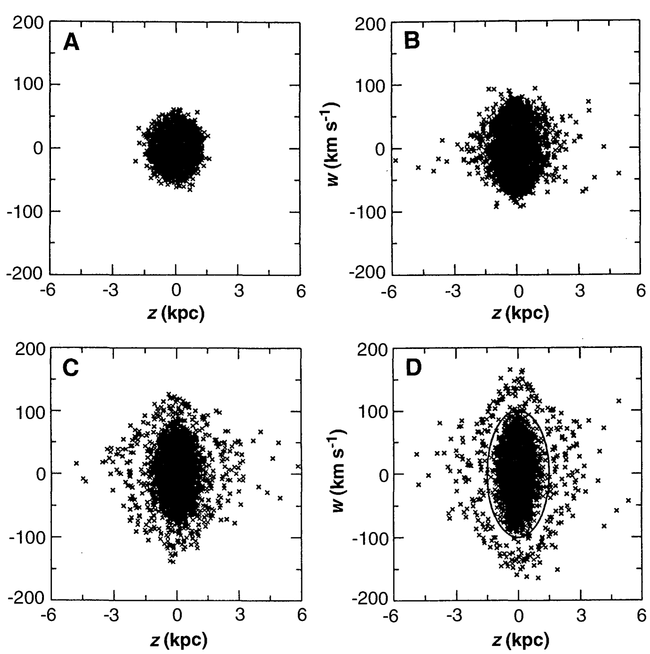
$$\phi_d = -\frac{GM}{\sqrt{R^2 + (a + \sqrt{z^2 + b^2})^2}} \quad (3)$$

where the disk mass M is increased slowly from zero to its final value $M_d = 7.8 \times 10^{10}$ $M_\odot$ (where $M_\odot$ is the mass of the sun), and the scale parameters a and b are held fixed, quite similar to the model in (*21*) (G is the gravitational constant). We then choose a distribution of test particles and integrate their orbits in the combined potential (ϕ_h + ϕ_d). Figure 1 shows the results of a typical integration inside the solar circle. The parameters used in this run are V_c = 190 km s^{-1}, R_c = 500 pc, q = 0.8, a = 5 kpc, and b = 300 pc. Initially M = 0, and all the 12,700 particles were given the same specific z-angular momentum

$$L = V_c R_0^2 / \sqrt{R_c^2 + R_0^2} \quad (4)$$

corresponding to a guiding center radius R_0 = 6 kpc. All four panels in Fig. 1 are projections onto the zw plane of the time evolution of this set of particles. The initial distribution (Fig. 1A) is Gaussian in both z and w with dispersions 430 pc and 17 km s^{-1}, respectively. The dispersions in radial scale and speeds are 1 kpc and 43 km s^{-1}, respectively, but these are not apparent in this projection. Over an initial period of 8 × 10^9 years, M increases linearly from zero to 0.25 M_d. As Fig. 1B shows, particles with large initial radial energies have been captured by the resonance and have already been given large z energies. As the mass increases, now at a greater rate [see figure 4 of (*12*)], the resonance drifts to even larger energies (Fig. 1C). The faster transit allows some of the captured particles to escape at intermediate energies. All the while, those particles that have not been affected by the resonance effectively conserve their I_z and I_R; thus, their z dispersion decreases to 190 pc, while the w dispersion increases to 28 km s^{-1}. Most of these particles are contained within the ellipse in Fig. 1D, when the disk has attained its final mass M_d after 14 × 10^9 years of growth. There are 340 particles (≃2.7%) outside the ellipse, most of which have gotten their large z energies by the capture process. Inside the ellipse, the passage of the resonance results in changes of the order of a couple of percent in the distribution. At this time, the guiding center radius has decreased to 4.8 kpc, as may be seen in Fig. 2A, which gives the spatial distribution. Superimposed on this R–z plot is a near-resonant orbit (*22*). Although the captured stars have a range of energies, the similarity between their distribution and the shape of the orbit is obvious.

We have run orbits with initial guiding center radii ranging from 5 to 10 kpc, in steps of 1 kpc. Both the numbers of stars and the initial velocity variances in each of these six runs were weighted in accordance with an exponential surface density distribution of scale length $h = a/1.3 \approx 3.8$ kpc. A collage of the runs in Fig. 2B is meant to give an impressionistic edge-on view of a thin disk accompanied by what the thick disk might look like were it produced by levitation. The flared shape of the thick disk is a result of the stacking together of many near-resonant orbits such as the one shown in Fig. 2A. For R_0 < 5 kpc, the dynamics might be significantly influenced by the galactic bulge and bar (*23, 24*); we will not discuss this region of the galaxy. From the typical shape (Fig. 2A) of a near-resonant orbit, it is clear that most of the high-|z| stars near the solar circle have come from the inner galaxy and have R_0 as small as 5 kpc. To study the properties of stars near the solar circle, we selected from Fig. 2B all stars with 7.5 kpc < R < 8.5 kpc; the z–w distribution of these solar neighborhood stars (Fig. 2C) shows a clear distinction between thin and thick disk stars. Stars inside the ellipse in Fig. 2C have a z distribution that is excellently fit by a



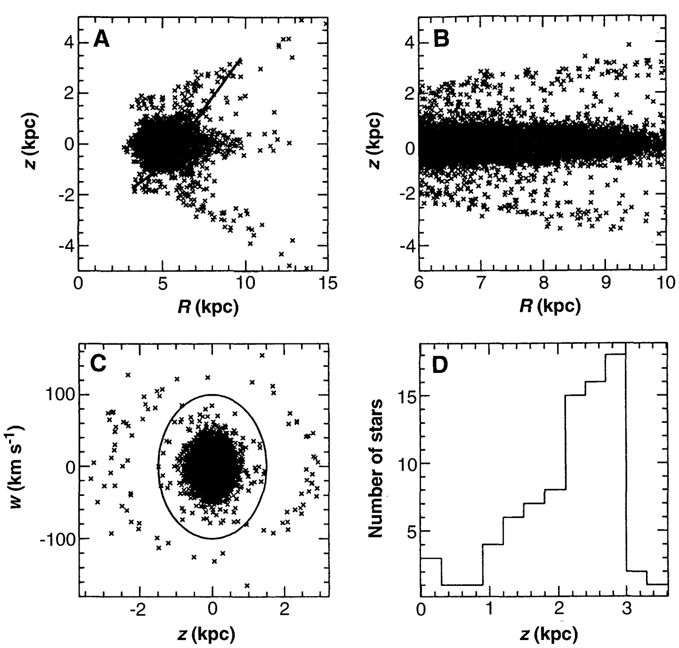


Fig. 1 (left). Projection onto zw space of the evolution of 12,700 test particles ("stars"), all with the same specific angular momentum, corresponding to an initial guiding center radius of 6 kpc. (**A**) t = 0, M = 0; (**B**) t = 8 × 10^9 years, M = 0.25 M_d; (**C**) t = 11 × 10^9 years, M = 0.6 M_d; (**D**) t = 14 × 10^9 years, M = M_d. **Fig. 2 (right).** (**A**) Meridional section of the particle distribution of Fig. 1D. Superimposed on the stars is a near-resonant 2:2 orbit, whose noteworthy features are its asymmetry about the equatorial plane, as well as the large vertical and radial extents. Every orbit has a partner (not shown) that is a reflection about the equatorial plane. (**B**) Meridional section of thin and thick disks. (**C**) Projection onto zw space of stars near the solar circle (7.5 kpc < R < 8.5 kpc) is the best way to distinguish the thin disk (inside the ellipse) from the thick disk (outside the ellipse). The thick disk fraction (≃1.1%) is smaller than that in Fig. 1D because it is somewhat farther out in radius; of course, the stars in (C) have a mix of angular momenta. (**D**) A histogram of the vertical distribution of the thick disk stars selected from (C) shows that most of them are located between 1.5 and 3.3 kpc, with a peak near |z| ≃ 2.5 kpc and a hole at low |z|.

$\mathrm{sech}^2(z/z_0)$ density law with $z_0 \simeq 300$ pc. About 1% of the stars are outside the ellipse, and these should be identified with the thick disk stars. A histogram of their z distribution (Fig. 2D) shows the stars to be distributed mostly between 1.5 and 3.3 kpc, with a peak near $|z| \approx 2.5$ kpc and a hole at low $|z|$.

If such, indeed, were the distribution of thick disk stars, the fitting of exponentials or related density laws to observations would not only miss these features but would overestimate the total number of thick disk stars. Even such an extensive analysis as in (*25*) might miss the high-altitude peak attributable to uncertainties in distance determinations, especially at the faint end. The radial and vertical velocity dispersions averaged over the entire column of the thick disk are 90 and 76 km s^{-1}, respectively. If an observational sample happens to be contaminated with thin disk stars, both velocity dispersions would seem smaller. The high-altitude stars come from the inner galaxy ($R_0 \approx 5$ kpc) and have low angular momenta. Thus, the lag in the rotation velocity V_{lag} rises from 80 km s^{-1} at $|z| = 1$ kpc to 120 km s^{-1} at $|z| = 3$ kpc, giving a vertical gradient in V_{lag} of $\approx$20 km s^{-1} kpc^{-1}, which is of the order that is observed [see (*10*) for a discussion of observations and their analysis]. Similarly, the vertical distribution of metallicities will be a somewhat diluted version of the radial distribution inward of the solar circle, at the early epoch in which these stars were captured into resonance. The observed small gradient (*26*) then implies that the young galactic disk must have had a nearly uniform radial metallicity distribution. Although our orbit computations are admittedly a crude representation of the dynamics, we expect that the two key ingredients necessary for levitation to work are the slow growth of the disk and a largish initial radial velocity dispersion ($\approx$33 km s^{-1} near the solar circle). If the disk grows faster, levitation will make a lighter thick disk, because the more rapid evolution will affect the capture probabilities, allowing more stars to escape from resonance. Another effect that can potentially decrease capture is the scattering of stellar orbits by molecular clouds, which can depopulate the resonant island. This process is most efficient when the resonance has risen to a scale height ($\approx$300 pc near the solar circle). However, a simple estimate (*27*) leads to a time scale of $\gtrsim 14 \times 10^9$ years for this process, so we expect levitation to be relatively unaffected. The large initial eccentricities might well decide which galaxies do grow thick disks, because not all observed disk galaxies have a thick disk (*4*).

Our orbital computations produce a thick disk with the following properties near the solar circle: (i) most of the thick disk stars are distributed between 1.5 and 3.3 kpc, with a hole in the distribution at low and intermediate heights; (ii) the column density is $\gtrsim$1% of the thin disk; and (iii) the high-altitude stars are in near-resonant orbits that carry them well into the inner galaxy (between 1 and 3 kpc, the lag in rotation velocity increases with height at a rate of $\sim$20 km s^{-1} kpc^{-1}). This flared, thick disk in Fig. 2B does not extend much beyond $\sim$10 kpc, because the stars in the outer galaxy have completed too few orbits for capture into resonance to be effective. On the other hand, accretion of a satellite galaxy produces a flare at large radii (*28*). Thus the nondetection of the thick disk toward the galactic anticenter (*29*) probably favors levitation over satellite accretion. Levitation being a collisionless process, it is independent of the mass of the object; thus, the disk globular clusters [see (*30*) for a review] could have been lifted up as effortlessly as individual stars. A prediction of our theory vis-à-vis satellite accretion is that the globular cluster population of the disk should also not extend beyond $R \sim 10$ kpc. If our galaxy is, indeed, nonaxisymmetric (*31*), we expect a larger resonant island [see figure 3-28 of (*15*)], which will lead to a more massive thick disk. Another attractive feature of levitation is that it is a natural by-product of the slow growth of a disk galaxy, and thus it is not necessary to postulate an external agency like satellite accretion. It also produces a population of stars, distinguished by their dynamical history, whose properties are similar to those in the thick disk.

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$$\dot{I}_z \approx 3G^2N_cM_c^2/UW$$
where N_c is the number of clouds per unit area of the disk. Thus, if ΔI_z is the half-width of the resonant island, a captured star will diffuse out of resonance in a time
$$\tau \approx (UW^2Z/3G^2N_cM_c^2)(\Delta I_z/I_z)$$
Near the solar circle, scattering is most effective when $W \sim \sigma_z \approx 13$ km s^{-1} (σ_z is the vertical velocity dispersion), at which time we estimate $(\Delta I_z/I_z) \gtrsim 0.5$. Together with $N_cM_c^2 = 10^6 M_\odot^2$ pc^{-2}, $U = 33$ km s^{-1}, and $Z = 300$ pc at this time, we obtain $\tau \gtrsim 14 \times 10^9$ years, which is much longer than the time a resonant star spends in this phase. At later times, both W and Z grow, so that the star is less effectively scattered by molecular clouds (whose scale height is $\approx$60 pc). However, $\Delta I_z/I_z$ shrinks with time, making the libration period within the island much longer than the fundamental orbital period. Even the slow growth of the disk over a Hubble time is sufficiently nonadiabatic that it promotes escape from resonance. It is amusing to note that, when the disk growth time was increased to the artificially large value of 140×10^9 years, the captured fraction rose to 11% [see (*19*) for a discussion].
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