A Lower Limit on the Age of the Universe

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A detailed numerical study was designed and conducted to estimate the absolute age and the uncertainty in age (with confidence limits) of the oldest globular clusters in our galaxy, and hence to put a robust lower bound on the age of the universe. Estimates of the uncertainty range and distribution in the input parameters of stellar evolution codes were used to produce 1000 Monte Carlo realizations of stellar isochrones, which were then used to derive ages for the 17 oldest globular clusters. A probability distribution for the mean age of these systems was derived by incorporating the observational uncertainties in the measured color-magnitude diagrams for these systems and the predicted isochrones. The dominant contribution to the width of the distribution (approximately ±5 percent) resulted from the observational uncertainty in RR-Lyrae variable absolute magnitudes. Subdominant contributions came from the choice of the color table used to translate theoretical luminosities and temperatures to observed magnitudes and colors, as well as from theoretical uncertainties in heavy element abundances and mixing length. The one-sided 95 percent confidence limit lower bound for this distribution occurs at an age of 12.07×10^9 years, and the median age for the distribution is 14.56×10^9 years. These age limits, when compared with the Hubble age estimate, put powerful constraints on cosmology.

The apparent dichotomy between the upper bound on the age of the universe obtained from the Hubble constant and the lower bound obtained by dating the oldest globular clusters (GCs) in our galactic halo represents one of the most serious potential conflicts in modern observational cosmology. For Hubble constant $H_0 = 100h$ km sec^{-1} Mpc^{-1}, a flat, matter-dominated universe has an age given by

$$\tau_{\text{Hubble}} = \frac{2}{3} H_0^{-1} = 6.6h^{-1} \times 10^9 \text{ years} \quad (1)$$

For a lower density, open universe, the factor of 2/3 is increased, but even in this case, if the age of the oldest GCs in our galaxy is really 16×10^9 years [as some recent best-fit estimates suggest (*1*)], then the GC age will be inconsistent with the Hubble age if $h > 0.57$. Consistency with the estimate for the flat, matter-dominated universe is impossible if $h > 0.4$. Several recent estimates of the Hubble constant (based on type Ia supernovas and Hubble Space Telescope observations of Cepheid variables in the Virgo cluster) both tend to suggest $h > 0.65$ (*2*); this and other factors have led various groups to argue once again for the need for a cosmological constant (*3*).

Because the Hubble age estimate is unambiguous for a fixed Hubble constant, the crucial uncertainty in this comparison resides in the GC age estimates themselves. Rough arguments have been made that changes in various input parameters in the stellar evolution codes used to derive GC isochrones, or in the RR Lyr distance estimator used to determine absolute magnitudes for GC stars, might result in age estimates that differ by 10 to 20 percent (*4*). However, no systematic study has yet been undertaken to realistically estimate the cumulative effect of all existing observational and theoretical uncertainties in the GC age analysis. Without such a study, the question of whether a real age problem exists in cosmology cannot be properly addressed. This is the primary purpose of the present work.

The age of a GC can be derived from stellar evolution calculations in a number of ways. For the purpose of setting a firm lower limit on the age of the universe, it is necessary to evaluate the various age determination techniques and to select the one that has the smallest possible theoretical uncertainties. Several important considerations are relevant to this evaluation. First, the correct treatment of stellar convection is at present subject to large uncertainties, because our present physical understanding of highly compressible, turbulent motions in a hot plasma is rather limited. Indeed, the treatment of convection is the dominant source of uncertainty in present stellar models. Second, the main sequence phase of evolution is the simplest evolutionary state for a star. As a star evolves off the main sequence, its structure becomes considerably more complex, and hence more difficult to model theoretically. Moreover, helioseismology provides an important test of main sequence stellar models that is not available for other phases of evolution. Inversions of the solar *p*-modes have found that the sound speed and density in solar models agree with the sun to better than 1 percent (*5*), which indicates that our theoretical understanding of main sequence stellar evolution is sound. These factors suggest the use of an age determination technique based on a feature of the main sequence that is nearly independent of convection.

During the main sequence phase of evolution of low-mass stars, only the outer regions of the star are convective. All of the nuclear generation occurs in radiative regions; hence, the luminosity on the main sequence is reasonably independent of the treatment of convection. Because the main sequence turn-off luminosity is quite sensitive to the stellar age, it appears to be the favored age determination technique with respect to the issues raised above when absolute stellar ages are of interest [as has long been known to stellar astrophysicists (*6*)]. Absolute age estimates that are based on other techniques (such as the use of colors or the more advanced stages of evolution) can be performed, but these will be subject to larger theoretical uncertainties. As a result, they will generally lead to a less stringent lower limit to the age of these systems.

Whatever age determination technique is used, one likely reason that an analysis like ours had not previously been carried out is that it is numerically intensive. Each run of a stellar evolution code for a single mass point takes 3 to 5 min on the fastest commercially available workstations. Nine different mass points at three different metallicities must be run to produce each set of isochrones. If, for example, 1000 different isochrone sets were run (requiring the calculation of more than 4 million stellar models) to explore the parameter ranges available, more than 8 weeks of processing time would be required.

Although this is a long time, it is not prohibitive; hence, because of the importance of this issue, we developed the necessary Monte Carlo algorithms. First, we examined the measurements of input parameters in the stellar evolution code to determine their best-fit values, their uncertainties, and the appropriate distributions to use in the Monte Carlo analysis. Next, we rewrote the stellar evolution code (*7*) and isochrone generation code to allow sequential input of parameters chosen from these

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distributions as well as output of the necessary color-magnitude (CM) diagram observables. Finally, we derived a fitting program to compare the predictions to the data. Because the numerically intensive part of this procedure involved the Monte Carlo generation of isochrones, independently incorporating the chief observational CM uncertainty will enable quick refinement of the results as this uncertainty diminishes.

Monte Carlo analysis inputs: general features. The chief input uncertainties in the derivation of stellar evolution isochrones include *pp* and CNO chain nuclear reaction rates, stellar opacity uncertainties, uncertainties in the treatment of convection and diffusion, He abundance uncertainties, and uncertainties in the abundance of the α-capture elements (O, Mg, Si, S, and Ca). Our stellar evolution code was revised to allow batch running with sequential input of these parameters, chosen from underlying probability distributions. Because the equation of state is now well understood in metal-poor main sequence stars, we did not include it among our Monte Carlo variables. The detailed equation of state by Rogers (8) gives GC age estimates that are very similar to those obtained with the Debye-Hückel correction (9), which was used in this study.

The following parameters and distributions were adopted. We included uncertainties for the three most important reactions in the *pp* chain. For $p + p \rightarrow {}^2\mathrm{H} + e^+ + \nu$, we used the analysis of Kamionkowski and Bahcall (*10*), but whereas they took theoretical errors to be Gaussianly distributed, we decided that a uniform distribution better represented the state of our (lack of) knowledge. There are two sources of theoretical error, one from the uncertainty in the particle wave functions and another from meson exchange (*11*). We used a relative modification to this reaction of $1 \pm 0.002\ {}^{+0.0014}_{-0.0009}\ {}^{+0.02}_{-0.012}$, where the second term is 1σ Gaussian and the third and fourth terms are top-hat distributions. For the other two *pp* chain reactions, ${}^3\mathrm{He} + {}^3\mathrm{He} \rightarrow {}^4\mathrm{He} + p + p$ and ${}^3\mathrm{He} + {}^4\mathrm{He} \rightarrow {}^7\mathrm{Be} + \gamma$, the uncertainties included in this study were taken from Bahcall and Pinsonneault [(*12*), table 1]. The CNO nuclear reaction rates and their uncertainties are from Bahcall [(*13*), table 3.4].

The stellar opacities were divided into high-temperature and low-temperature regimes. For the high-temperature regime ($T > 10^4$ K), the opacities of Iglesias and Rogers (*14*) were used. The uncertainties in these opacities were evaluated by a comparison with the Los Alamos Opacity Library opacities (*15*). In the temperature regime relevant for nuclear fusion ($T \gtrsim 6 \times 10^6$ K), differences of 1 percent were typically found, with maximum differences of 3 percent. We therefore multiplied the high-temperature opacities by a Gaussian distribution with a mean of 1 and $\sigma = 0.01$. The Kurucz opacities (*16*) were used for the low-temperature regime. Low-temperature opacities calculated by different researchers can differ by a large amount (*17*), although modern calculations appear to agree to within ~30 percent. In addition, we intercompared Kurucz's calculations for different element mixtures and found maximum differences of 30 percent. For these reasons, we multiplied the Kurucz opacities by a number that was uniformly drawn from the range 0.7 to 1.3.

In stellar astrophysics, the correct treatment of the convective regions of a star has been a long-standing problem that stems from our poor understanding of highly compressible convection and its interaction with radiation in the optically thin outer layers. Most stellar evolution codes use the mixing length approximation, a simple model of convection in which blobs of matter are supposed to rise or fall adiabatically over some distance and then instantaneously release their heat (*18*). The uncertainties in this treatment of convection are parameterized into a single nondimensional quantity called the mixing length, which is usually taken to be fixed during a star's evolution. Its value is typically chosen by requiring that a solar model have the correct radius and luminosity at the solar age, or by comparing theoretical isochrones to observed CM diagrams. A mixing length of ~1.8 appears to provide a reasonable match to the observations, but its exact value depends on the input physics (opacities, model atmospheres, and so forth) used to construct the stellar models. Modern solar models typically use mixing lengths between 1.7 and 2.1 (*19, 20*). To further explore this issue, we conducted tests in which isochrones were constructed from models with different mixing lengths and were then compared with a metal-poor GC CM diagram. We found that if all other input parameters were held constant, changes of 0.3 in the mixing length could be ruled out because the isochrones no longer fit the data. To allow for possible variation among GCs and for the fact that a broader range in mixing length might fit the data if other parameters are allowed to vary at the same time, we used a rather broad Gaussian distribution with a mean of 1.85 and $\sigma = 0.25$.

Whether to include the effects of element diffusion (whereby helium tends to sink to the center of the star while hydrogen rises to the surface) is a difficult question. Physical models of compressible plasmas suggest that diffusion should be occurring in stars, and they predict diffusion coefficients with a claimed accuracy of ~30 percent (*21*). However, these models assume that no other mixing process occurs within the radiative regions of a star, which may not be true. Helioseismology appears to suggest that diffusion is occurring in the sun (*22*), but the evidence is not compelling (*19*). Models of halo stars that incorporate diffusion predict curvature in the Li-effective temperature plane that is not observed (*23*), which suggests that some process is inhibiting diffusion in halo stars. Given these uncertainties, we elected to multiply the diffusion coefficients given by Michaud and Proffitt (*21*) by a number uniformly drawn from the interval 0.3 to 1.2. The use of a flat distribution with a rather large range reflected our conviction that the incorporation of diffusion within stellar models is subject to large uncertainties.

The primordial He abundance, which is relevant to old halo stars and is an important input in our stellar codes, has taken on renewed interest as a result of recent calculations of Big Bang nucleosynthesis (BBN) light element production (*24*). For BBN estimates to agree with inferred primordial abundances, potentially large systematic uncertainties must be taken into account. It has become clear that such uncertainties are the dominant feature of the comparison between theory and observation. We therefore used a flat distribution for the primordial He mass fraction between 0.22 and 0.25, which encompasses the range of recent estimates.

The abundance of elements heavier than He (denoted by Z) must be specified before a stellar model can be calculated. The abundance of Fe in globular cluster stars is relatively easy to determine because of its numerous spectral lines. Unfortunately, the abundances of the other heavy elements are more difficult to determine, and it has been common to assume that the other heavy elements are present in the same proportion as they are in the sun. However, from both theoretical arguments and observational evidence, it is clear that the elements that are produced by means of α-capture are enhanced in abundance relative to their solar value. It is relatively easy to incorporate the effects of the enhancement of the α-capture elements on the stellar models by redefining the relation between the model Z and the iron abundance (*25*). Oxygen is by far the most important of the α-capture elements; it accounts for roughly half of all the heavy elements (by number) present in the sun. For this reason, we took observations of oxygen to be representative of the α-capture elements. The determination of oxygen abundances in stars is extremely difficult and is subject to a number of systematic uncertainties [for example, (*26*)]. Recently, high-quality [O/Fe] abundances (*27*) were obtained for a number of halo stars (*28*); the mean abundance was found to be [O/Fe] =

0.55 ± 0.05, where the error is simply the standard deviations of the measurements. In addition to this error, possible systematic errors (*26, 29*) must be considered. An analysis of published data led us to conclude that possible systematic errors in the determination of [O/Fe] may be as large as ±0.2 log units. Thus, the abundance of the α-capture elements was taken to be [α/Fe] = 0.55 ± 0.05 (Gaussian) ± 0.2 (top-hat).

Finally, a color table must be used to convert our theoretical luminosities and temperatures to observed magnitudes and colors. The construction of an accurate color table requires the use of theoretical model atmospheres, which are still subject to large uncertainties. We accounted for this uncertainty by randomly choosing one of two totally independent color tables (*30*) with equal probability in constructing each isochrone set in the Monte Carlo analysis. These two tables reasonably span the present range used to transform theoretical temperatures and luminosities to observed colors and magnitudes.

The fitting procedure and the probability distribution for GC ages. After a set of isochrones (which consisted of isochrones of three different metallicities for a set of 15 different ages between 8×10^9 and 22×10^9 years) was derived, the comparison with the observed parameters of a specific set of GCs required a fitting procedure. To minimize the large uncertainties in the effective temperatures of the models (*6*), we used the difference between the main sequence turn-off magnitude and the horizontal branch (HB) magnitude as our age diagnostic. This age determination technique is commonly referred to as ΔV_{HB}^{TO} and has been extensively used in the astronomical literature [for example, (*31*)]. Our isochrone sets provided values for the main sequence turn-off luminosity as a function of age and metallicity. Because of the importance of convection in the nuclear burning regions of HB stars, theoretical HB luminosities are subject to large uncertainties, and thus we combined our theoretical main sequence turn-off luminosities with an observed relation for the luminosity of the HB (see below). The result was a grid of predicted ΔV_{HB}^{TO} values as a function of age and [Fe/H], which was then fit to an equation of the form

$$t_9 = \beta_0 + \beta_1 \Delta V + \beta_2 \Delta V^2 + \beta_3[\mathrm{Fe/H}] + \beta_4[\mathrm{Fe/H}]^2 + \beta_5 \Delta V[\mathrm{Fe/H}] \quad (2)$$

where t_9 is the age (in billions of years). The observed values of ΔV_{HB}^{TO} and [Fe/H], along with their corresponding errors, were input in Eq. 2 to determine the age and its error for each GC in our sample.

Because there is abundant evidence for a large age range within different GC systems (*32, 33*), it was important to select a sample that only includes old GCs. Observational errors in the determination of the turn-off and HB magnitudes can lead to an error of ~10 to 20 percent in the derived age of any single cluster. Thus, to minimize the observational uncertainties, we chose to determine the mean age of a number of GCs. In light of the strong evidence for an age-metallicity relation (in which metal-poor clusters are the oldest), only metal-poor clusters were selected ([Fe/H] ≤ −1.6). From this list of metal-poor clusters, any cluster that had been shown to be young [as judged by the difference in color between the giant-branch and main sequence turn-off (*33*)] or that was suspected of being young [because its HB was unusually red for its metallicity (*34*)] was discarded. From the sample of 43 GCs (*32*) for which high-quality observations were available, 27 survived the metallicity cut; of these, 10 were discarded because they were young, leaving a total of 17 GCs. Our final sample contained the following clusters: NGC 1904, 2298, 5024, 5053, 5466, 5897, 6101, 6205, 6254, 6341, 6397, 6535, 6809, 7078, 7099, 7492, and Terzan 8.

We checked that the inferred dispersion in the age of the 17 GCs was not larger than expected on the basis of the observational uncertainties. Using the uncertainties in the individual ages determined from uncertainties in the observed turn-off magnitude and metallicity, we examined the dispersion about the mean age for the 17 clusters by means of a χ^2 test. We found a reduced χ^2 of 0.55 per degree of freedom, which indicated that there was no evidence for any intrinsic dispersion in age for our sample and that the quoted observational uncertainties for each cluster may be too generous. In any case, given the quoted accuracy, it is certainly consistent to assign a single mean age for the sample.

One chief observational uncertainty common to all GCs is retained until the end of the analysis: the determination of the absolute magnitude M_v for RR Lyr variables. To determine ΔV_{HB}^{TO} as a function of age and metallicity, we combined our theoretical turn-off magnitude with an observationally based estimate for the absolute magnitude of the HB as determined from the RR Lyr variable stars [M_v(RR)]. A number of independent, observationally based techniques can be used to derive M_v(RR). In general, it has been found that the absolute magnitude of the RR Lyr stars can be represented by an equation of the form

$$M_v(\mathrm{RR}) = \mu[\mathrm{Fe/H}] + \gamma \quad (3)$$

where μ is the slope with metallicity and γ is the zero point. Note that M_v(RR) is independent of age (at least, with systems greater than 8×10^9 years old). Recent estimates for μ vary from 0.15 to 0.30 (*31, 32, 34*). Fortunately, because we sought to determine the mean age of 17 GCs in the restricted metallicity range −2.41 ≤ [Fe/H] ≤ −1.60, the uncertainties in the slope had only a small effect on our age estimate. We

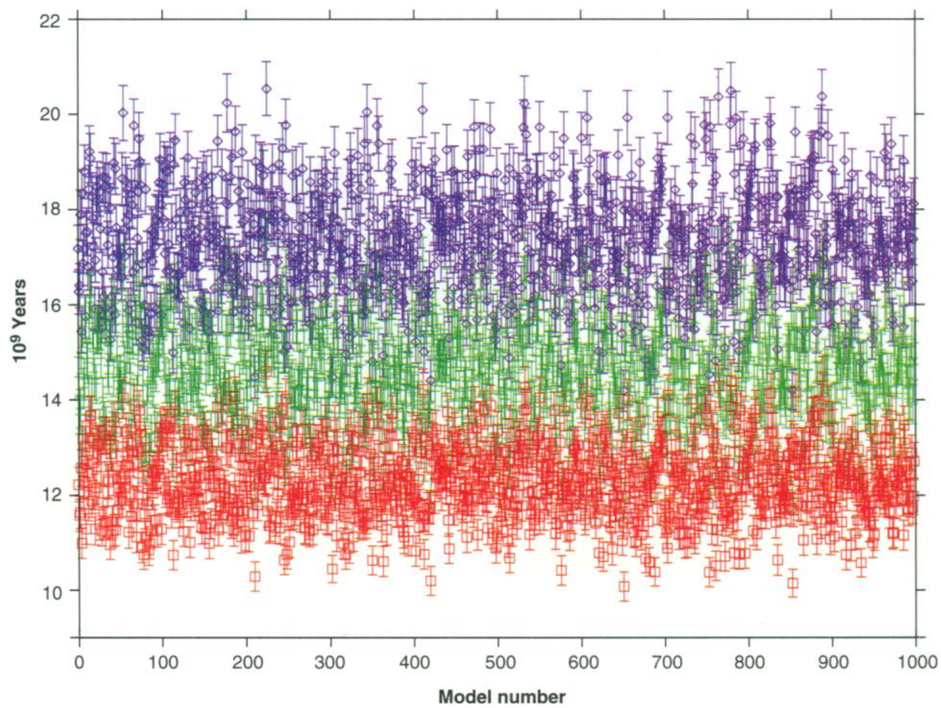

Fig. 1. The suite of models generated by the Monte Carlo procedure. For each model, the 1σ age range is plotted for three values of M_v(RR). The red, green, and blue data points show the age variation for M_v(RR) = 0.44, 0.60, and 0.76, respectively.

conducted tests that indicated that the maximum difference in our mean age was only 0.5 percent when the slope was varied between 0.15 and 0.30. Recent work suggests that $\mu = 0.20$ is likely to be correct (*35*), and this is the value we used. Uncertainties in γ have a large impact on our derived age estimates; for this reason, we conducted a thorough review of recent observational estimates of γ. These estimates for γ are usually given as a value of M_v(RR) at a specific metallicity. Because the GCs in our sample have a median metallicity of [Fe/H] = –1.82 and a mean metallicity of [Fe/H] = –1.93, we used a slope of $\mu = 0.20 \pm 0.04$ to transform the various γ estimates to [Fe/H] = –1.90.

Layden *et al.* (*36*) recently used the statistical parallax technique to determine M_v(RR) = 0.68 ± 0.12 in field halo RR Lyr stars. Walker (*37*) measured the apparent magnitude of the Large Magellanic Cloud (LMC) RR Lyr stars and assumed an LMC distance modulus of 18.5 magnitudes to infer M_v(RR) = 0.44 ± 0.10. This choice for the LMC distance modulus was based on the Cepheid distance, main sequence fitting, and the SN1987A ring distance to the LMC. This last method is a purely geometrical method and should be the most reliable. However, the SN1987A distance to the LMC was recently revised to 18.37 magnitudes (*38*), implying M_v(RR) = 0.57 ± 0.10. Main sequence fitting of GC CM diagrams to local halo stars with well-determined parallaxes can be used to determine the distance to GCs, and hence M_v(RR). Unfortunately, there is only one relatively metal-rich subdwarf that has a well-determined parallax. Application of this technique to the GC M5 yields M_v(RR) = 0.76 ± 0.12 (*31*). The only direct determination of M_v(RR) in a metal-poor GC was done by Storm *et al.* (*39*), who used a Baade-Wesselink infrared flux analysis to determine M_v(RR) = 0.52 ± 0.26. Although the error is large because of possible systematic uncertainties, it does suggest that the RR Lyr stars in metal-poor GCs are somewhat brighter than those found in the field (*36*) or in metal-rich GCs (*31*).

In light of the above estimates, we elected to use M_v(RR) = 0.60 ± 0.08 (corresponding to $\gamma = 0.98 \pm 0.08$). This central value was chosen by a straight average of the four published M_v(RR) estimates referenced above. The error bar was chosen to ensure that the 1σ range would include the central value obtained for M_v(RR) in a metal-poor GC, and to ensure that the 2σ range (0.44 to 0.76) would encompass all of the estimates cited above. Our choice is further supported by a very recent study that compared kinematic, RR Lyr, Cepheid, and type II supernova distance estimators for consistency (*40*) and found a range for M_v(RR) similar to the one we chose.

The ensemble of age estimates from our Monte Carlo analysis for different values of M_v(RR) is shown in Fig. 1. For each of the sets of isochrones, we determined a mean age and 1σ uncertainty in the mean for three values of M_v(RR): the mean, 0.60, and the end points of our 95 percent confidence limit range, 0.44 and 0.76. To obtain the final histograms (Fig. 2, A and B), we followed an analogous procedure, but in this case we allowed M_v(RR) to be a random variable and sampled the sets of isochrones with replacement 12,000 times. For each sample, rather than the mean age, we recorded a random age drawn from a Gaussian distribution, with the mean age and variance appropriate for the sampled isochrone set at the sampled M_v(RR). The data were then sorted and binned to produce the full distribution for the assumed Gaussian spread in M_v(RR) of ±0.08 (Fig. 2A) and the distribution for a fixed value of M_v(RR) of 0.6 (Fig. 2B). In this way, the effect of the uncertainty in M_v(RR) could be explicitly examined.

Conclusions. Taken at face value, our results indicate that at the one-sided 95 percent confidence level (set by requiring 95 percent of the determined ages to fall above this value), a lower limit of $\sim 12.1 \times 10^9$ years can be placed on the mean value of these 17 GCs. (The symmetric 95 percent range of ages about the mean value of 14.56×10^9 years is 11.6×10^9 to 18.1×10^9 years.) The distribution deviates somewhat from Gaussian, as might be expected. In particular, at the lower age limits the rise is steeper than Gaussian, reflecting the fact that essentially all models give an age in excess of 10×10^9 years, whereas the tail for greater ages is larger than Gaussian. The explicit effect of the largest single common observational uncertainty, that in M_v(RR), increases the net width of the distribution by $\sim \pm 0.6 \times 10^9$ years (that is, ~±5 percent). The width introduced by the systematic ambiguity of our choice of color table is $\sim \pm 0.2 \times 10^9$ years. On the other hand, simply varying M_v(RR) over its full 2σ range (keeping all other parameters fixed) would produce a change of ±16 percent in GC age estimates. For comparison, the next most important input parameter uncertainties in this same sense are [α/Fe] (±7 percent effect), mixing length (±5 percent effect), and diffusion, ^{14}Np reaction rate, and primordial He abundance, each of which would affect age estimates at the ±3 percent level if allowed to vary over its entire range, keeping all other parameters fixed.

Several caveats are in order. First, in our analysis, although we have attempted to identify and vary all of the parameters that could affect the determination of the main sequence luminosity, we did not account for all types of variations in all parameters dur-

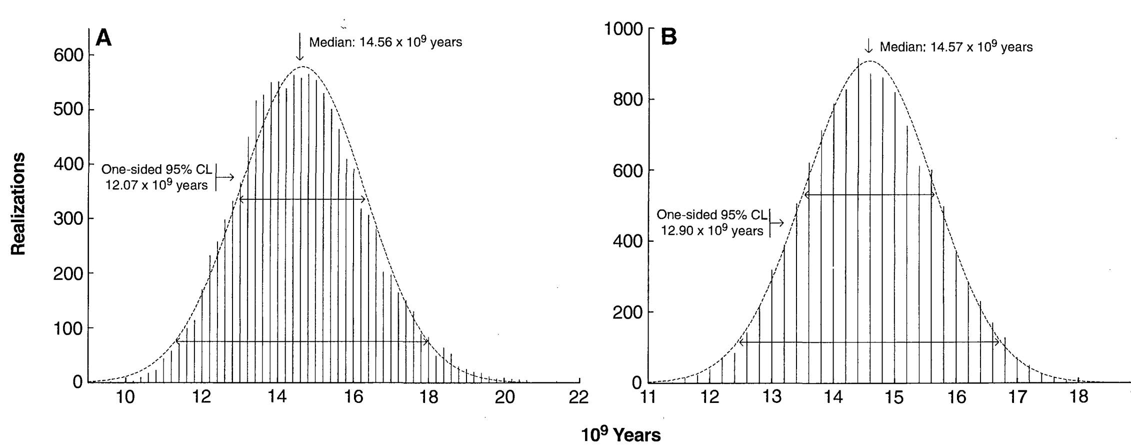

Fig. 2. Histograms showing the relative numbers of realizations of mean GC ages drawn randomly from the Monte Carlo data set (with uncertainties on individual age estimates taken to be Gaussian). The value for M_v(RR) was chosen from one of two different distributions: (**A**) a Gaussian, M_v(RR) = 0.6 ± 0.08, and (**B**) a delta function, M_v(RR) = 0.6. The dashed line is a Gaussian approximation to the actual distribution. The means and standard deviations of the Gaussians are $14.6 \pm 1.7 \times 10^9$ years in (A) and $14.6 \pm 1.1 \times 10^9$ years in (B). The horizontal lines show the 1σ and 2σ ranges in age, based on the Gaussian approximation. The one-sided 95 percent confidence limit for a lower bound on the age of the universe (arrow extending to the right from a vertical bar) was calculated directly from the generated distribution.

ing the evolutionary process. In particular, the mixing length parameter we chose was fixed for each star throughout each evolutionary run, although there is no a priori reason to expect that this is actually the case. Allowing for a possible variable mixing length during stellar evolution would change the shape of our isochrones. However, the variation in the fixed mixing length we allowed is broad enough so as to exceed any actual variation that might occur during a single star's evolution on the main sequence. As a result, the actual main sequence turn-off luminosities should be spanned by our Monte Carlo set.

Next, note that the detailed shape of the age distribution we derived depends on the manner in which systematic uncertainties in various input parameters were taken into account, and thus it would vary slightly if different distributions were chosen to model these systematic uncertainties. However, the distribution is affected by many different variables (most of them statistically distributed), so that unless the overall scale of the systematic uncertainties has been underestimated, we believe the general features of our derived distribution provide a robust representation of the actual age distribution, and hence of the uncertainty in age.

It could also be argued that there is other age-sensitive information in the CM diagrams, which, if used to complement the main sequence turn-off data, could perhaps lead to yet tighter 95 percent confidence limits. For example, isochrones that produce the youngest ages tend to have rather red turn-off colors. However, as we have only crudely approximated the span in predicted colors, we chose to be somewhat conservative and used only the main sequence luminosity as an age indicator. In addition, until our knowledge of convection and other factors that affect stellar colors (such as model atmospheres) is more secure, we feel it is premature to use other points on the CM diagram to constrain GC ages.

Finally, some people have questioned whether a more conservative number—namely, the absolute lowest age that can be obtained by simultaneously varying all parameters to their estimated extreme limits—is not a better lower bound for the age. Although this value ($\sim 10 \times 10^9$ years, touching on the lower limit obtained from white dwarf cooling and nucleocosmochronology) is certainly more conservative, if our analysis is at all correct, it is also vanishingly probable. As such, it seems unrealistic to use this value, rather than our one-sided 95 percent confidence limit or a similarly chosen limit, as a constraint.

Within the context of the above remarks, we believe our results can be used with some confidence to compare GC ages with cosmological age estimates based on the Hubble expansion (*41*). Of course, in addition to the age determined here, it is necessary to add some estimate of the length of time required for the formation of our galactic stellar halo from the initial density perturbations present during the big-bang expansion. Conservative estimates for this formation time vary from 0.1×10^9 to 2×10^9 years. If the lower value is chosen, we find that with a high degree of certainty, the age of GCs in our galaxy is inconsistent with a flat, matter-dominated universe unless $h < 0.54$, or with an open ($\Omega \approx 0.1$), matter-dominated universe unless $h < 0.75$. If the value of h is determined to be greater than either of these values, some modification, such as the addition of a cosmological constant, would seem to be required.

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41. For further details of the analysis and our results for covariance matrices, correlation functions, dependence on individual parameters, and observational uncertainties, see B. Chaboyer, P. Demarque, P. J. Kernan, L. M. Krauss, in preparation.

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