Keplerian Complexity: Numerical Simulations of Accretion Disk Transport

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Supercomputer simulations have been used in conjunction with analytic studies to investigate the central issue of astrophysical accretion-disk dynamics: the nature of the angular momentum transport. Simulations provide the means to investigate and experiment with candidate mechanisms, including global hydrodynamic instabilities, spiral shock waves, and local magnetohydrodynamic (MHD) instabilities. Simulations have demonstrated that accretion disks are generally MHD turbulent. These results suggest that the fundamental physical mechanism for angular momentum transport in accretion disks has now been identified.

Some of the most energetic photons astronomers observe originate not within stars but outside them, in orbiting disks of gas. These accretion disks are powered by the release of gravitational potential energy as gas spirals down onto a compact star or black hole. Half of the energy is retained by the gas in its slowly decaying orbit, but the other half must be dissipated. It is likely that this occurs through shock waves and turbulence; as a result of this dissipation the gas becomes hot and radiates. At such high temperatures the gas is a fully ionized plasma. Indeed, at the inner edge of the disk, near a central black hole or neutron star, the gas can be hot enough to emit hard x-rays and produce electron-positron pairs.

Accretion disks power several types of high-energy astrophysical systems, ranging from stellar mass binaries up to the most powerful active galactic nuclei (AGN) and quasars. Accretion disk signatures, such as double-peaked emission line profiles, have been observed in binary systems, and in some eclipsing binary systems the size and shape of the disk can be mapped out (*1*). In the case of quasars and AGN, the evidence is more circumstantial, inferred mainly from the high luminosity and rapid variability emerging from a compact region. The available gravitational binding energy in accreted matter far exceeds the potential luminosity available from nuclear reactions, so that even in the absence of a detailed model it seems very likely that AGN are driven by gravity power (*2*). (A third type of accretion disk is the protostellar disk, a flattened, rotating, self-gravitating system out of which stars and any orbiting planets form; I will not specifically consider the unique aspects of this type of disk in this review.)

Ever since the existence of accretion disks was recognized, astronomers have worked to understand their structure and evolution. The simplest analytic accretion disk models are one-dimensional and time steady, two quite restrictive assumptions. As such restrictions are lifted, it becomes increasingly difficult to solve the equations by analytic techniques. Numerical simulations are needed to investigate the complex time-dependent physics of accretion disks. In the last decade, the widespread availability of supercomputers has made such disk simulations possible.

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Angular Momentum Transport and Energy Generation

A brief review of the history of stellar evolution theory is instructive, as it illustrates what one might hope to achieve in accretion disk theory. Astronomers now have a fairly complete description of stellar structure and evolution. The first step was the realization that, to good approximation, stars are spheres of gas in hydrostatic equilibrium, with the inward force of their self-gravity balanced by an outward-directed pressure. In 1926, the distinguished astronomer Sir Arthur Eddington completed his classic work *The Internal Constitution of the Stars*, in which he laid out the general principles of stellar structure. Energy released in the core of the star provides the high temperature required for pressure support, and the gradual leakage of that energy through the stellar surface gives the star its luminosity and determines its evolution. What was unknown at the time was the precise source of the star's energy, although Eddington knew that the answer must lie with the release of "subatomic energy." The rapidly developing sciences of quantum mechanics and nuclear physics eventually provided the specifics required for detailed models.

The spherical symmetry and hydrostatic equilibrium of stars allow them to be described through a series of ordinary differential equations that, although complicated, are today readily solvable by numerical methods. Stellar models have steadily improved, along with our understanding of the underlying physics: nuclear reaction rates, radiative opacities, and turbulent mixing are all included in modern stellar models. As computational power has increased, it has been possible to relax even the restrictive assumption of spherical symmetry [see, for example, (*3*)].

The impressive accomplishments in a field as complex as stellar structure and evolution have inspired astrophysicists working on accretion disks to hope that their models would have similar successes. But progress has been slow because, despite the absence of nuclear physics, disks are considerably more complicated than stars. First, they are not spherically symmetric; many critical aspects of disks are inherently three-dimensional. Second, accretion disks are much more dynamic and variable than most stars. Finally, the basic physics of accretion disks has proven more complicated than that of stars. The most obstinate difficulty lies with the energy generation mechanism. Although accretion disk theory recognized from its inception that disk luminosity originated with the release of gravitational energy as gas spirals down onto a star or black hole, the physical process by which angular momentum is transferred outward, thus allowing for inspiral, was unknown. In an accretion disk, molecular viscosity is many orders of magnitude too small to be important.

In order to progress, disk modelers must assume the existence of an anomalous viscosity or stress tensor to account for angular momentum transfer. Typically, this is parameterized in some particularly simple, dimensionally consistent manner by, for example, setting the appropriate component of the stress tensor equal to the disk pressure times a constant of proportionality, designated α (*4*). Such models are generally referred to as "alpha models," after this free parameter. This procedure is somewhat akin to modeling stellar cores without the benefit of detailed nuclear reaction rates. By assuming a steady state, it is possible to construct a set of height-integrated, one-dimensional (1D) differential equations, in which the angular momentum transport equation is represented by the unknown α.

Although the α-viscosity approach has proven reasonably successful in describing mean disk properties, it can be justified only when considering the largest length scales and the longest time scales. Time-dependent models that rely on adjustable viscosity parameters are not predictive; the parameters and functional forms for the viscosity can be modified as required to fit observations. Thus, while observations provide constraints on the size of this unknown α parameter, the lack of knowledge about its underlying physical properties is a serious limitation for such issues as dynamical stability, detailed structure, and time-dependent evolution. As an alternative to including a parameterized viscosity, numerical simulations can study the known dynamical equations from first principles. In this way, simulations can search for the source of the anomalous viscosity, rather than assume its existence.

Although this review will focus on possible mechanisms for disk angular momentum transport, that is only one example of how simulations can contribute to our understanding of accretion disks. As they do for stellar evolution, simulations will ultimately play a fundamental role in accretion disk theory as our knowledge of the basic physics improves. Except for a few large protostellar disks, accretion disk systems are too small for direct imaging. Recently, indirect imaging of some disks has become possible, with the use of the techniques of eclipse mapping and Doppler tomography (*1*, *5*), but detailed data on the internal structure of disks are still hard to come by. The scarcity of observations means that theoretical speculations are only loosely constrained. Numerical experiments represent an important technique for turning such speculation into science.

This important role that simulations have and the possibilities for new breakthroughs from supercomputer experiments were outlined by Smarr in 1985 (*6*). Smarr emphasized the mutually supporting roles of analytic and numerical techniques for solving equations that describe the physical world, particularly for systems such as black hole accretion disks, where direct observations are impossible. With the advent of supercomputing centers, high-speed supercomputers capable of performing multidimensional simulations have become widely available to astrophysicists. Smarr's expectations have been largely borne out by progress in accretion disk theory, where simulations in concert with careful analysis have led to advances in our understanding of both dynamical flows and disk stability. Here I will focus on the role of simulations in the specific issue of the source of disk transport and anomalous viscosity.

Disk Hydrodynamic Stability

A plausible source for the anomalous viscosity lies with turbulent stresses that might be present in a disk. Without an instability, however, there is no assurance that gas orbital motions should be anything but laminar. It has long been appreciated that accretion disks satisfy the Rayleigh criterion for hydrodynamic local stability; that is, angular momentum increases with radius, so local hydrodynamic instabilities appear to be ruled out. Instabilities could arise from the violation of other stability criteria, however. One possibility is the development of vertical convective instabilities as a result of unstable temperature gradients from the disk midplane to the disk surface. It has been suggested that the resulting predominantly vertical, thermally driven motions set up a general turbulence favorable to the outward transport of angular momentum. Although simple arguments suggest that angular momentum transport is possible (*7*), Ryu and Goodman (*8*) carried out a detailed analysis of the linear stage of a convective instability and found that the angular momentum transport is inward, opposite the desired direction. Numerical simulations should be able to address this question directly, but it has been only recently that the required, highly resolved, multidimensional simulations have been possible. The first such nonlinear simulations (*9*) appear to indicate that inward transport is favored, consistent with and generalizing Ryu and Goodman's analysis. Follow-up simulations should finally resolve this issue in the near future.

In the absence of local instabilities, global instabilities are still possible, and one important application of numerical simulations has been to studies of the global stability of disks and the generation and propagation of global spiral waves. These waves can carry angular momentum, and if they are dissipative, they are a possible angular momentum transport mechanism.

Papaloizou and Pringle (*10*) uncovered one such global instability in so-called thick disks, or accretion tori. In the standard accretion disk model, gas is mainly confined to a narrow region near the equatorial plane. In a thick accretion disk, large internal pressure distends the disk into a toroidal shape, something like a bagel. For a time, thick disks gained favor because they can have a narrow, evacuated vortex along their spin axis, and this empty funnel might collimate winds driven by radiative pressure into the jets that emerge from AGN. However, the instability brought into question the existence of such tori. The instability arises from global nonaxisymmetric waves that have pattern speeds equal to the orbital speed at a corotation radius within the torus. Waves on either side of the corotation radius exchange energy and angular momentum with each other and grow in amplitude as they tap into the free energy of the rotating disk. Provided a feedback mechanism exists, such as a reflecting disk boundary, the waves can build to a large amplitude and exert considerable torque. What was unclear, however, was whether the instability would (i) destroy the disk, (ii) saturate less catastrophically, but with sufficient amplitude to act as a small, but important, source of anomalous viscosity, or (iii) experience so little amplification as to produce no effect.

The answer to this question depends on the instability's nonlinear evolution, and investigating nonlinear dynamics requires numerical simulations. The first step was to isolate the essential physics of the instability by further simplifying the model to the slender torus, an isolated orbiting ring of gas resembling an orbiting bicycle inner tube. For certain types of slender tori, the unstable modes can be obtained analytically (*11*). In addition, because these slender tori are quite unstable to the Papaloizou-Pringle instability, and because their fastest growing modes have almost no vertical structure, the problem is immediately adaptable to 2D time-dependent numerical simulations. Linear analysis provided eigenmodes and growth rates for comparison with, and testing of, the numerical simulations, which could then be evolved into the nonlinear regime. Figure 1 shows the result of the evolution of a typical slender torus. The initial exponential growth continues up to the point at which the perturbations in the fluid variables are as large as the equilibrium amplitudes. When the unstable mode becomes nonlinear, an unanticipated structure emerges: an elliptical fluid blob in which Coriolis and pressure forces are in balance and the sense of internal rotation is opposite to that of the overall orbit. These counterrotating vortex structures, discovered numerically, were dubbed "planets" (*12*).

The first computer-generated image of a slender torus breaking up into blobs led Goldreich, Goodman, and Narayan (*13*) to derive a closed-form, analytic solution for these counterrotating coherent structures. They further showed that the planet solution was itself linearly unstable as an equilibrium solution, although simulations suggested that it is more robust in fully dynamic evolutions. Thus, an unanticipated coherent structure was discovered numerically, which led to a new analytic solution, precisely the sort of mutuality between numerical and analytic work described by Smarr.

The coherent planet solution arises only for particular unstable modes and under limited circumstances, primarily for the slender torus. The next step was to extend the understanding of the nonlinear Papaloizou-Pringle instability into more general regimes, first through 2D simulations of radially extended thick accretion disks (*14*), and then with full 3D simulations of tori orbiting around black holes (*15*). These simulations showed that in radially wide tori, the instability saturates in a strong spiral pressure wave. They also confirmed the analysis of Blaes (*16*), who suggested that accretion flows through the torus could reduce and even halt the growth of the global instability. The conclusion of this work was that tori with constant angular momentum distributions, those with the narrowest funnels, are ruled out, but the instability is unlikely to be important in disks with Keplerian angular momentum distribution. Because most disks should be Keplerian, we must look elsewhere for the anomalous viscosity.

The Papaloizou-Pringle instability is not the only source of global spiral waves. External forcing by tidal gravitational fields can lead to strong nonaxisymmetric (spiral) waves in accretion disks in binary systems where gas is transferred from one star to its compact companion. The effects of tidal forces have been successfully simulated in two dimensions (radius and angle) by a number of researchers using the inviscid equations of hydrodynamics [see, for example, (*17*)]. Again, although the idea of spiral shock waves in disks has been around for many years, numerical simulations have greatly improved our understanding of spiral wave formation and propagation. The results of numerical simulations by Sawada *et al.* (*18*) led to the development of self-similar stationary shock solutions (*19*) that have proven important in interpreting and generalizing the numerical simulations. To date it appears that these shocks may be significant for properties such as disk structure and variability, but their efficiency as an angular momentum transport mechanism appears to be rather low.

The efficiency of spiral waves may be even lower than simulations suggest. By confining the waves to a plane, 2D simulations exaggerate their importance by not allowing vertical propagation out of the disk. Three-dimensional simulations are required, but to date only preliminary steps have been taken with finite difference techniques. The rapid increase in computer speed and memory holds promise that detailed global 3D accretion finite difference models will soon be carried out. However, global 3D simulations are particularly appropriate to particle methods. Particles representing fluid elements can travel wherever needed, whereas a finite difference grid must contain many zones if it is to cover all possible areas for fluid motion. The drawback to particle methods is that they are not as well suited as finite difference schemes for evolving hydrodynamic effects, such as pressure forces, sound waves, and shocks. As with all numerical techniques, however, given the appropriate problem, they produce useful insights.

An example is the simulation of disk formation in binary systems and the effects of tidal torques on disk structure. Although it is simple to visualize the idea of gas transfer from a star toward a compact companion, the details are impossible to derive without simulations. Several groups have simulated tidally induced eccentricities (*20*) using particle methods, in both two and three dimensions. The introduction of an accretion stream, transferring matter from one star to the other, leads to the formation of a disk. In certain cases, tidal forces produce a distorted disk. This eccentric disk is a promising model for observed superoutbursts in certain types of cataclysmic variable binary systems.

Another interesting tidal effect was uncovered analytically by Goodman (*21*) [see also (*22*)], who found that a tidal potential can produce a parametric disk instability. This instability is another excellent candidate for numerical study. Finite difference simulations by Ryu and Goodman (*23*) of a local section of disk show that substantial angular momentum transport can be produced by this instability in regions of the disk where the tidal effects are strong. The general efficiency of such tidal effects and the ultimate fate of the angular momentum transferred outward by them are interesting issues that await more complete multidimensional simulations of the full binary system.

Accretion Disk MHD and Turbulence

Global instabilities, spiral shocks, and tidal instabilities may well be important for some disks, but these mechanisms lack the desired generality. At best they produce transport only under special conditions: the Papaloizou-Pringle instability only for tori with extreme constant angular momentum; spiral shocks where there are strong tidal distortions; the tidally driven instability primarily in the outer regions of a disk in a binary system where the tides are very strong. Because angular momentum transport is a generic feature of accretion disks, its fundamental physical mechanism should be similarly generic. A good candidate is the direct transport of angular momentum through magnetic stresses. Because the disks under discussion here are fully ionized, they are highly conducting, and such plasmas are quite capable of supporting magnetic fields. Historically, the perceived problem with such a mechanism was the expectation that the field must be amplified to an energy comparable to the disk's thermal energy before it would become important. It was thought that magnetic fields might be amplified by preexisting turbulence in the disk, but such turbulence was the phenomenon that was to be explained in the first place. The standard view of a decade ago is stated clearly by Zel'dovich, Ruzmaikin, and Sokoloff in their text *Magnetic Fields in Astrophysics* (*24*):

> The transport of angular momentum (i.e. accretion) in the disk is therefore possible only through turbulence and/or magnetic fields. The presence of magnetic fields in the matter flowing out from the visible component is hardly in doubt (every star has a magnetic field), but the presence of turbulence is more questionable. It is

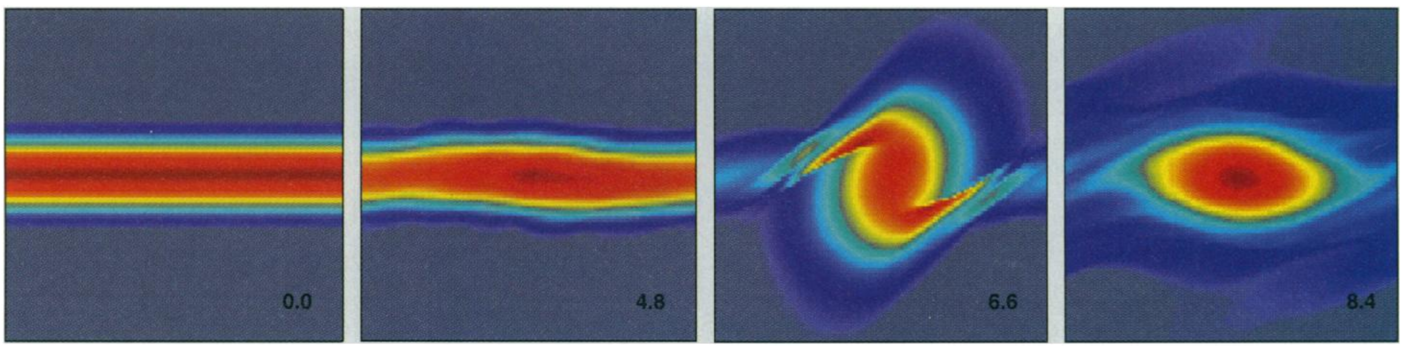

Fig. 1. The evolution of a section of slender torus into an elliptical planet caused by a global hydrodynamic instability. Colors represent density, red corresponding to high density, blue to low. The vertical direction is the radial coordinate; the horizontal, the angular. The images are labeled by time in orbits.

known that a medium in nonuniform rotation (Keplerian rotation in our case) in which the angular momentum density increases outward is stable against all small perturbations.

Note the tacit assumption that the purely hydrodynamic angular momentum stability criterion applies even in the presence of a magnetic field. For all the attention given to hydrodynamic stability, it has emerged that the critical issue is magnetohydrodynamic (MHD) stability. Recent developments suggest that a general mechanism for the anomalous viscosity lies with a linear, local MHD accretion disk instability.

The discovery and analysis of this MHD instability provides another illustration of the mutually beneficial interaction between large-scale numerical simulations and analytic work. In the case of the coherent planet solution discussed above, the discovery can be credited to the simulation. In this case, analytic theory provided the crucial breakthrough. Balbus and Hawley, working on the extension of the earlier disk simulations to include magnetic fields, began a local disk study to analyze the propagation of waves in a magnetized disk, planning to demonstrate that such waves can be reproduced by a numerical code. However, a surprising result emerged from the analysis. Instead of a wave, one of the modes found was an exponentially growing instability (*25*) present when the magnetic field is weak. Remarkably, the growth rate of the instability was found to be independent of the field strength. This was a clear and simple result, but in an area as well studied as accretion disk stability, it was completely unexpected. This type of instability had been investigated before in a quite different context. Velikhov (*26*), over 30 years ago, performed a global linear analysis of a rotating vertically magnetized Couette flow and found it to be unstable. What was for Couette flow a seemingly isolated result, relevant to a specific laboratory setup, is for accretion disks both general and unavoidable.

In contrast to the hydrodynamic stability criterion, which requires angular momentum to increase outward, MHD stability is assured only when angular velocity increases outward. This is a very important distinction; the angular momentum distribution in an accretion disk easily satisfies the hydrodynamic requirement, because a Keplerian angular momentum distribution in a disk of radius R goes like $L \sim R^{1/2}$, but always violates the MHD requirement, because the angular velocity is $\Omega = L/R^2 \sim R^{-3/2}$.

Once the criterion for instability was understood from the linear analysis, the next step was to reproduce those results numerically (*27*). The reproduction of linear modes and growth rates is more than a simple test of a numerical code; an advantage a numerical simulation has over a linear perturbation analysis is that the simulation includes all of the higher order nonlinear terms. Thus, the agreement between numerical simulation and linear theory simultaneously validates both the code and the assumptions that went into the analytic derivation.

Ongoing MHD simulations, combined with detailed analysis, have provided a clear picture of the nature of the instability and its properties. The underlying physical cause of the instability stems from the elastic nature of the magnetic field and from nonintuitive properties of orbital dynamics. Fluid elements in orbits with larger radii have larger angular momentum and smaller angular velocities than fluid elements in orbits with smaller radii. A magnetic field couples fluid elements at different radii but allows joined elements some freedom to evolve independently. Consider, for example, a fluid element displaced radially outward but connected by a magnetic field to another fluid element near its original location. The displaced fluid element will be centrifugally accelerated by the pull from the more rapidly rotating fluid element at the smaller radius to which it remains magnetically tethered. Angular momentum is lost from the fluid element at the smaller radius, which already has lower angular momentum, and is transferred to the fluid element at larger radius. The transfer of angular momentum causes the outwardly displaced fluid elements to continue outward, while the fluid element that has lost angular momentum falls inward to smaller radius. This transfer ceases only when angular velocity is constant or increasing outward. Very generally, angular velocity gradients, rather than angular momentum gradients, are the stability discriminants in rotating magnetized plasmas, even when pressure support is important (*28*).

One of the most important aspects of this instability is that the effects of the magnetic field on small perturbations enter the equations only in the combination $\mathbf{k}\cdot\mathbf{v}_a$, where $\mathbf{k}$ is the wave number vector and $\mathbf{v}_a$ is the magnetic Alfvén velocity ($\mathbf{v}_a = \mathbf{B}/\sqrt{4\pi\rho}$, where $\mathbf{B}$ is the magnetic field vector and ρ is the fluid density). This dependence shows that all orientations and strengths of the magnetic field can be unstable: the magnetic field strength simply determines the wave number at which magnetic tension becomes important. At a given wave number, the instability works when the field is weak and ceases if the magnetic tension becomes strong enough to overcome the destabilizing excess centrifugal force that drives a displaced fluid element. In the strong field case, the result is ordinary Alfvén wave propagation. In an accretion disk, a "weak field" corresponds to magnetic pressures that are less than the thermal gas pressure, precisely the sort magnetic field strength expected within disks.

As it turned out, this instability had been at work in other accretion disk simulations, although it was not recognized as such. Uchida and Shibata (*29*) investigated the creation of MHD jets through large-scale axisymmetric numerical simulations by threading a disk of gas with fairly strong vertical magnetic fields and allowing the disk to fall inward (mimicking the accretion process), by giving it too little angular momentum to remain in a circular orbit. The infall produced radial fields that were then wrapped up by differential rotation into strong toroidal fields that drove dynamic outflows along the vertical field lines. Although the jets were a transient phenomenon resulting from a special choice of initial conditions, the simulations provided the first time-dependent demonstration of the efficacy of magnetic fields for jet acceleration and collimation.

Stone and Norman (*30*) reproduced and extended Uchida and Shibata's work. As a

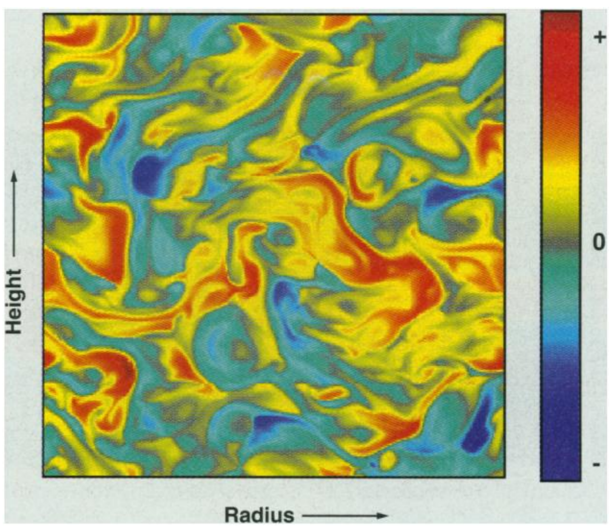

Fig. 2. Simulation of a height versus radius cross section of an accretion disk showing MHD turbulence. Colors indicate angular momentum perturbation: red indicates gas with excess angular momentum (moving outward), blue corresponds to gas with too little angular momentum (moving to smaller radius).

first step, they placed a stable Keplerian accretion disk on a finite difference grid and applied a weak vertical magnetic field. Expecting nothing more than stable magnetosonic waves, they instead observed a rapid collapse and infall of the accretion disk. Suspecting numerical problems, they set about reworking the MHD algorithms. Although this led to important developments in numerical technique (*31*), the apparent difficulties with the disk model remained.

The timely appearance of the accretion disk magnetorotational instability explained these apparently anomalous results. Disks containing weak fields are unstable. The fields used in Stone and Norman's simulations rapidly transferred angular momentum outward, and as a result, the disk promptly fell inward. Stone and Norman's global disk simulations demonstrated that both strong and weak vertical fields cause disk inflow on orbital time scales (*30*), albeit for different reasons. Their results suggest the need to rethink many popular accretion disk paradigms that involve disks embedded in magnetic fields. Although numerical simulations did not make the crucial breakthrough in this specific case, this history suggests that the discovery of important basic physical processes is inevitable once numerical simulations allow the full dynamical equations to speak for themselves.

As yet another example of the complementary roles of analysis and simulation, the first local MHD simulations of disks (*32*) showed an interesting exponentially growing, streaming behavior that remained coherent well into the nonlinear regime. This feature led Goodman and Xu (*33*) to examine the simulated problem analytically. They found an exact, incompressible, exponentially growing, axisymmetric, nonlinear coherent solution consisting of channels of fluid streaming radially inward and outward. Indeed, the linear solution and the nonlinear solution turned out to be identical. Moreover, the analysis by Goodman and Xu further predicted that the nonlinear solution would be unstable in three dimensions, a prediction subsequently verified by simulations (*34*).

The rapid transfer of angular momentum seen in all of the simulations highlights the central role of the magnetorotational instability in the long-standing problem of anomalous viscosity. In general, the outcome of a linear instability in an initially laminar shear flow is turbulence. The puzzle had always been that disks are hydrodynamically stable, as discussed above, and it was assumed that weak magnetic fields would not qualitatively alter this conclusion. But disks are magnetohydrodynamically unstable in the presence of even weak fields, and strong fields vigorously transport angular momentum directly. Because outward transport of angular momentum is a cause, not merely a consequence, of the instability, it is particularly adept as an anomalous viscosity. Both 2D and 3D numerical simulations have demonstrated this explicitly (*27, 30, 34, 35*). Figure 2 shows an image of the turbulence that develops after three orbits in an initially laminar disk flow containing a weak magnetic field. Fluid elements colored red have excess angular momentum and move to a larger radius. Blue fluid elements have sub-Keplerian angular momentum and move to a smaller radius. There is a structure on all scales, consistent with the presence of turbulence.

Figure 3 is an image from the most complete numerical simulations that have been done to date, which has about one million grid zones. Although it is still not possible to model the full accretion disk, this compressible MHD simulation is a 3D local section of a disk that extends for two pressure-scale heights of vertical thickness above and below the equatorial plane. A periodic boundary condition in the radial and angular directions allows this domain to represent a more extended disk. Into this disk are placed a number of weak initial magnetic fields, with various strengths and orientations. These fields are unstable and are quickly amplified to larger energies as turbulence is produced and sustained within the disk. To date, simulations such as these give every indication that the turbulence is sustained by the magnetic instability and that it can produce angular momentum transport at levels consistent with observations of accretion disks (*35*).

Future Prospects

Accretion disks are too complex and cover too broad a dynamic range for a complete simulation from first principles, at least for the present. This is not to say that progress is impossible. Quite the contrary: the same complexity that makes modeling disks difficult means that there is a rich collection of interesting properties to discover. Indeed, the lack of detailed astronomical observations suggests that numerical experimentation, interpreted with analytic theory, will play a vital role.

There are two approaches to disk simulation. One is the simulation of large-scale global models that investigate the overall structure of disks and the gas flows that

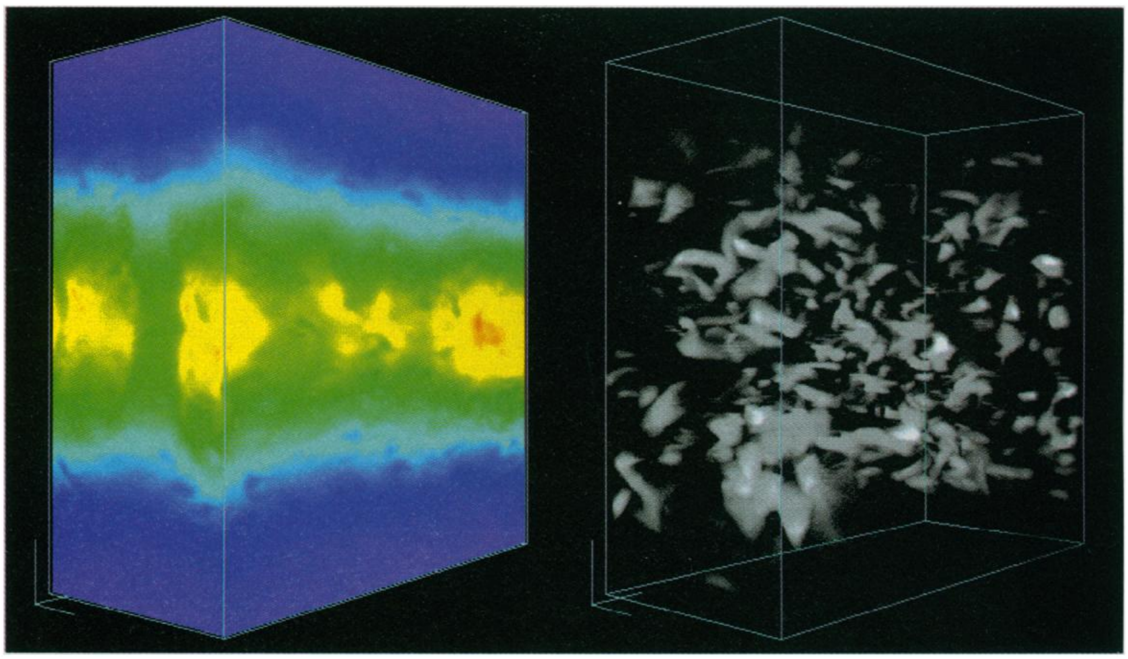

Fig. 3. Three-dimensional simulation of a section of vertically stratified accretion disk. The left image shows the density: high density is red, low density blue. The right image is a volumetric rendering of magnetic pressure: high-pressure regions are illuminated. The magnetic field is highly tangled, without large-scale organization, and still weak on average, with a total pressure of a few percent of the total gas pressure.

form them. Global 3D models of binary systems and detailed global MHD disk simulations are two problems of immediate interest. The other approach is a more detailed simulation of a small local region within a disk that includes more complex physics. Examples include radiation transport, partial ionization, buoyancy, reconnection, and improved equations of state. Both of these tasks require significant computer resources. Both kinds of simulation have led, and will lead, to significant progress in furthering our understanding of accretion disks. Larger simulations with greater resolution can take advantage of the latest advances in computer speed, but including more physics in the model will depend as much on new algorithms and insightful analysis as on new hardware.

A crucial recent development has been the discovery that a local MHD instability produces precisely the type of turbulence necessary to transport angular momentum outward within the disk. We are now in the position to address specific questions of accretion disk dynamics from first principles. For example, we can study precisely what sort of transport levels are to be found in disks (or in the usual parlance, the value of α) and whether the instability can act as a dynamo and amplify magnetic fields. Magnetic buoyancy and reconnection are two important physical processes limiting magnetic field strength that require further attention. Of course, the relevant equations are by no means simple; understanding accretion disks has become, in large part, the task of understanding 3D, nonisotropic, radiative, inhomogeneous MHD turbulence. Considering the effort that has gone into investigating hydrodynamic turbulence in much simpler contexts, the problems are daunting. But prospects are far from bleak. The recent strides in understanding turbulence made possible by numerical simulations (*36*) hold forth the promise of similar advances in accretion disk modeling.

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Turbulent Dynamics in the Solar Convection Zone

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Observations of the sun reveal highly complex flows and magnetic structures that must result from turbulent convection in the solar envelope. A remarkable degree of large-scale coherence emerges from the small-scale turbulent dynamics, as seen in the cycles of magnetic activity and in the differential rotation profile of this star. High-performance computing now permits numerical simulations of compressible turbulence and magnetohydrodynamics with sufficient resolution to show that compact structures of vorticity and magnetic fields can coexist with larger scales. Such structured turbulence is yielding transport properties for heat and angular momentum at considerable variance with earlier models. These simulations are elucidating the coupling of turbulent fluid motions with rotation and magnetic fields, which must control the interlinked differential rotation and magnetic dynamo action.

The outer layers of the sun are observed to be in continuous agitated motion because of vigorous turbulent convection. Theoretical models of stellar structure and evolution indicate that in the present sun some manner of thermal convection must extend from the surface well into the interior, forming a zone occupying about the outer 30% by radius in which convective motions transport nearly all of the energy that emerges from the radiative interior. Observations suggest that the actual dynamics within this zone are extremely intricate. The velocities and magnetic fields are complex, exhibiting large-scale structure (Fig. 1A) and ordered behavior amidst rapidly varying and intense small-scale turbulence. High-resolution observations of the solar surface show that convection involves multiple and somewhat discrete scales of roughly cellular motion. The flows range from the fast and short-lived solar granules (Fig. 1C) with typical horizontal scales of 1000 km, through mesogranules with scales of about 5000 km, to fairly persistent supergranules (Fig. 1B) about 30,000 to 50,000 km across (*1*). It is so far unclear from theory as to how these different scales arise. Each of the convective flows tends to sweep and con-

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