CHAOS

Putting the Pedal to the Metal In a Controlled Chaotic Laser

Imagine trying to drive a race car down a rutted, gravel road. At low speeds, no problem. But as you continue to accelerate, the steering gets trickier, until finally the only thing that keeps you on the road is luck.

Something very similar happens with lasers. Certain solid-state lasers produce a steady, stable beam only at low power settings. When pushed to higher power levels, their output intensity fluctuates wildly, showing all the characteristics of chaos. For applications in areas such as optical information processing, chaos is not exactly what you want to see in a laser beam. But with the help of some new mathematical techniques for controlling chaos, developed earlier this year by mathematicians at the Naval Research Lab (NRL) in Washington, D.C., Rajarshi Roy and his co-workers at the Georgia Institute of Technology have succeeded in maintaining a stable beam while boosting a laser's output power by a factor of approximately 15. That's like accelerating from 5 to 75 miles per hour along a bumpy road.

The feat isn't the first time investigators have succeeded in stabilizing a chaotic system. But earlier successes were achieved in systems that were cruising at a constant speed, with their parameters—such as the power of the laser—kept fixed. In this case, says Roy, "we took a laser that emits unstable light, and we were able to stabilize it and then keep pumping harder and harder and maintain that stability over a huge range of parameter change." Moreover, the 15-fold improvement is just a demonstration. "We don't think by any means that that's a limit," Roy adds. "We could probably go much higher in light power."

That added measure of control bodes well not only for lasers, but also for other systems where chaos arises, according to Neal Abraham, a physicist at Bryn Mawr College in Pennsylvania. Thanks to such successes, he says, "one can envision a whole range of applications, which before were just speculative." The possibilities run the gamut from medicine to profiteering on Wall Street.

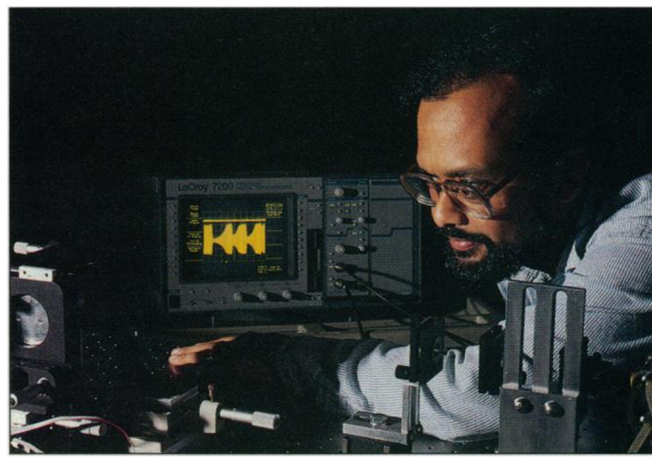
At first glance, the entire field of chaos control seems founded on a contradiction. A chaotic system's extreme sensitivity to tiny disturbances would seem to preclude any hope of controlling its behavior. Not so. In recent years, researchers have found that the hair-trigger response of chaotic systems can actually be exploited, ju-jitsu style, to control them. In 1990, for example, Edward Ott, Celso Grebogi, and James Yorke of the University of Maryland showed in a theoretical paper in *Physical Review Letters (PRL)* that it's possible to stabilize a chaotic dynamical system by making small, time-dependent perturbations to a parameter of the system—in effect, giving the system tiny kicks from time to time. In a subsequent *PRL* paper with Troy Shinbrot, also of Maryland, they showed it should be possible to steer a chaotic system toward a desired state by the same kind of small adjustments.

That theory was put to the test by physicists at the Naval Surface Warfare Center in Silver Spring, Maryland, who controlled the chaotic dance of an elastic metal ribbon to the music of an oscillating magnetic field by making tiny adjustments in the field (*Science*, 10 May 1991, p. 776, and 29 May 1992, p. 1283). Recently, researchers at the University of California, Los Angeles, have taken a similar tack in stabilizing cardiac arrhythmias in rabbit hearts (*Science*, 28 August, p. 1230).

Last year Roy's laser group developed a variant on Ott, Grebogi, and Yorke's control technique that enabled them to stabilize the output of a ticklish laser by adding tiny fluctuations to the input power—which is sort of like keeping a car on course by tapping on the gas pedal. The device they worked with is a neodymium-doped yttrium aluminum garnet (Nd:YAG) laser equipped with a potassium titanyl phosphate crystal that changes a fraction of the Nd:YAG's infrared radiation into green light. Chaos arises in part because the crystal causes the Nd:YAG to lase simultaneously at up to 10 closely spaced frequencies, and the energy wanders among the different modes according to a system of coupled nonlinear differential equations—the mathematical breeding ground for chaos. But even though the researchers' technique allowed them to steady the laser at a given power level, they still lacked a way to "step on the gas" without losing that control. In effect, each time they increased the power, the laser would revert to chaotic behavior, and they would have to steer it back to stability.

An answer came from NRL mathematicians Ira Schwartz and Ioana Triandaf, who had developed a mathematical method that keeps a dynamical system "on track" even while one of the parameters of a system is varied. Their tracking technique uses the same kind of tiny kicks as earlier methods, but the mathematics guides the application of the kicks to stabilize the system even in parts of the parameter range where it is unstable but nonchaotic—a feat that eluded earlier techniques, which paradoxically could maintain stability only in realms of chaos.

Roy and his colleagues were also drawn to the tracking method's simplicity. "We don't work with the equations of a model," says Triandaf; "we work directly with an experimentally obtained series [of measurements]." That makes the control-of-chaos method very appealing to experimentalists. "Anybody can implement it," Schwartz says of the tracking method. Roy agrees: "I was surprised, because it's a very simple algorithm, and it works quite well in our laser system."



MARGARET BARRETT/GEORGIA TECH

A steadying influence. Rajarshi Roy and an oscilloscope display of the chaotic laser's output.

Schwartz and Triandaf think it should work well in other systems, too. Triandaf is currently gearing up to work on biological applications, including systems of coupled beta insulin cells, whose chemical response to each other and their environment can be chaotic. She and Schwartz have refined their method so that it can track and steer a stabilized state of a dynamical system anywhere on the system's range of potential behavior. And they are currently working to extend the technique to work with systems that are described by partial (as opposed to ordinary) differential equations, such as the reaction-diffusion equations that crop up in chemistry and mathematical biology.

As for the combination of chaos and control in general, "a lot of people don't believe it," says Schwartz. "Our hope is to convince the nonbelievers by applying these mathematical techniques to real experiments." The recent laser results, he hopes, may help people see the light.

–**Barry Cipra**