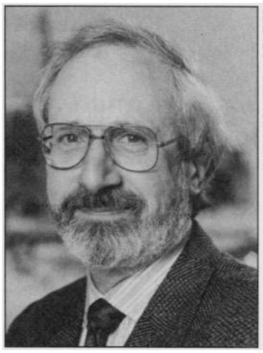
Mathematicians Join the Computer Revolution

Fingertips in sand, brush on papyrus, pencil and paper, chalk and chalkboard—over the centuries mathematicians have used a succession of tools to develop insight and prove theorems, but in each era the world's ever-improving technology has made little difference to the forward march of the field. No matter the century, mathematics has never gotten much fancier than a solitary human mind attacking a problem armed with mere logic and some scratchings in the sand. Until recently, that is.

Now this most conservative of the sciences is facing its first technology-driven revolution. As the telescope did for astronomers and the microscope for biologists, powerful graphic computers offer mathematicians a tool that can open new vistas of knowledge, fundamentally altering the way mathematics is done. And at the forefront of this revolution is the Geometry Center, a National Science Foundation (NSF)-sponsored program at the University of Minnesota.

The center, whose official name is the National Science and Technology Research Center for Computation and Visualization of Geometric Structures, opened in 1991 with annual funding of $2 million, provided by NSF and the Department of Energy. Its goal, says director Albert Marden of the University of Minnesota, is to develop the computer as a tool for mathematicians to use not only in their research, but also to communicate their results to others—both fellow mathematicians and math students. Only months into its young life, the center's research projects are rapidly becoming a paradigm for the mathematics of the future.



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Al Marden, director of the Geometry Center.

"It's a whole new world," says Fred Almgren of Princeton University, one of 19 permanent faculty members of the Geometry Center—all of whom except Marden have full-time jobs elsewhere. Almgren offers his own experience as an example of what the new world can offer: A specialist in the geometric calculus of variations, he works with objects too complicated to be drawn by hand, but they can be pictured by the center's powerful computers and visualization software. "I'm able now to see these minimal surfaces that I've been proving theorems about for years."

Seeing and manipulating these surfaces allows Almgren to develop an intuition about their properties and behavior that is impossible to get any other way. Mathematical proofs are still the final goal of any mathematician's work, he says, "but if you want to get at the heart of problems, you need more"—more than scribbles on a blackboard, he means.

And the center provides more. Its core is a large, open graphics lab with about 50 workstations scattered throughout. The lab is designed to promote interaction and collaboration among the faculty members and visiting researchers as they experiment on the workstations, calculating and visualizing such mathematical objects as fractals, knots, minimal surfaces, and hyperbolic spaces (in which the sum of the three interior angles of a triangle is always less than 180°).

Crystal Growers Compute Together

"Welcome to the future." When Jean Taylor made her opening remarks at the start of a totally new kind of workshop at the University of Minnesota's Geometry Center in Minneapolis in February, she didn't use exactly those words, but she could have. For the next 5 days, the 31 mathematicians, physicists, and materials scientists in attendance worked together in a way that none of them had ever quite experienced before: a free-wheeling, computer-based interaction that just may be the wave of the future in scientific workshops.

Taylor, a mathematician at Rutgers University in New Brunswick, New Jersey, and, simultaneously, a member of the permanent faculty at the Geometry Center (see related article), designed the "Computational Crystal Growers Workshop" to be a "workshop" in the old-fashioned sense of the word: a place where people make things. To that end, she limited the scheduled addresses to "an initial introductory round of 5-minute talks the first day" and then let the attendees loose in the Geometry Center with its 50 computer workstations. It was like setting a bunch of 6-year-olds free in a room filled with Lego building blocks. "It's free-form—everybody is working, trying to figure out problems," said Craig Carter, a materials scientists at Rockwell Science Center on leave from the National Institute of Standards and Technology, as he took a brief break from the action.

Scattered around the graphics lab at the Geometry Center, the scientists and mathematicians were talking in small groups, often centered in front of a computer workstation where one of the group would be testing a new idea or showing a colleague a different approach to a problem. The workshop examined ways to calculate and understand how crystals grow under different physical conditions. Materials scientists are interested in the subject because semiconductors, ceramics, and other important materials are crystalline, and controlling their structures is a key to improving their properties. Physicists are interested because crystal growth is a complex, nonequilibrium physical phenomenon with plenty of challenging, unanswered questions. And mathematicians are interested because many of the problems that arise in traditional mathematical fields such as minimal surfaces have application to crystal growth.

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Computer workstations were at the center of many of the discussions at the University of Minnesota's Geometry Center workshop.

Because analyzing crystal growth can be almost unmanageably complex, the synergy among the disciplines came in handy. To get a sense of the challenges before these communities of researchers, consider that when a ceramic is sintered—heated until its individual crystalline grains start growing together—the grains grow at different speeds depending on such factors as the size and shape of the grains and the temperature of the material. Since the strength and other properties of materials depend sensitively on the sintering process, scientists would like to understand its physics, but even large computers can only roughly approximate what happens when the grains begin to merge. Which is why Taylor arranged for a combined attack: Materials scientists can offer insight drawn from practical experience, physicists can provide models based on physical principles, and mathematicians can ana-

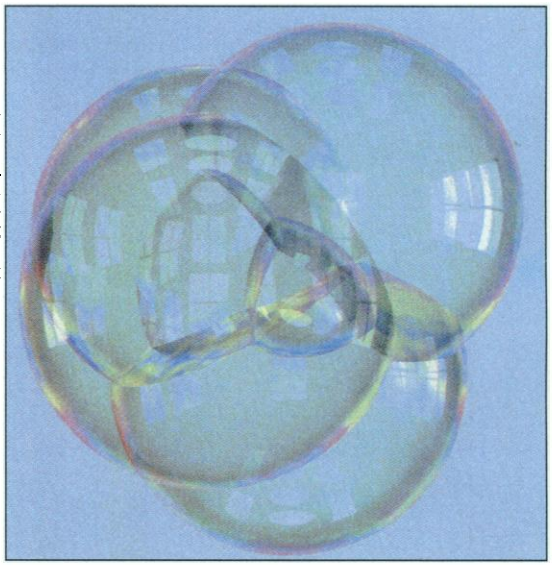
Not long ago, the Minnesota thrust was a narrower one. The center began life in 1987 as the Geometry Supercomputer Project, with the goal of using supercomputers to explore such geometric objects as three-dimensional manifolds (the physical universe is one example of a three-dimensional manifold). But rather quickly, the researchers recognized that increasingly powerful workstations more than made up in convenience what they gave up in speed to the supercomputers. Then, even as the center was broadening its hardware, it broadened its focus, going beyond manifolds to all areas of math—as Marden puts it, "computing/visualization for every aspect of mathematics."

Take Allan Wilks. His specialty lies far from traditional geometry. Wilks, one of the center's faculty, is a statistician at AT&T Bell Laboratories who examines large sets of data in search of correlations and structure. If the data lie in only two dimensions, it's easy enough to plot the points on graph paper and examine the data set for structure—all of the points might lie in one small area, for instance, or close to a single line—but what if the data are in three dimensions, or in four, or even in 10, 50, or 1000? Then, of course, it becomes impossible to visualize the data without ingenious digitized visualization techniques.

One such technique that Wilks describes is "rotating point clouds." Often the best way to visualize a three-dimensional set of points on a computer screen is to instruct the computer to "rotate" the set slowly so that the viewer sees it from a slowly changing angle. This tricks the mind into seeing the set of points as three-dimensional and has an added advantage: Since clusters or structures in the data are often invisible from some angles and obvious from others, the viewer can check the data set from many angles and improve the chance of seeing important details.

The same method works for data sets of more than three dimensions, except that the rotating data set no longer seems to be three-dimensional. "It simply looks like points moving around at varying speeds," Wilks says. Nonetheless, the viewer can still pick out structures from certain angles. "The trick is choosing a 'grand tour' [of different angles] that gives you a good look at the data."

This seemingly simple technique demands great computing power and sophisticated algorithms. "In the Bell Labs statistics department, all 25 members have high-performance graphics stations," Wilks says. "We believe they will eventually be ubiquitous in the data-analysis world." But not very many laboratories can afford the equipment for this kind of work, much less the money and staff needed to research novel approaches, which explains the Minnesota center.



JOHN SULLIVAN, THE GEOMETRY CENTER

Mathematicians can now see objects, such as this soap bubble cluster, that they could only imagine before.

But as useful as the center is in helping researchers visualize complicated problems, its computers may have an even greater calling: actually to create a whole new field of scientific endeavor. "At any time over the centuries, when mathematicians wanted to prove something, what they've often done is to try a bunch of different examples," says David Epstein of the University of Warwick in England and another member of the Geometry Center's permanent faculty. To cite a case in point, centuries before Pythagoras proved his famous theorem, the ancient Babylonians had already deduced it by experimenting with lots of right triangles. So the question arises: What could be deduced by turning computers loose on problems humans can't draw?

Enough, it seems, to found a totally new field that some people are calling experimental mathematics. Indeed, the power of the computer for such experimentation has already created enough intellectual ferment to kick off a brand new journal, *Experimental Mathematics*, which Epstein is editing and whose first issue is due out later this year.

Experimental, or computational, math means different things to different people, Epstein says. To some it can imply using computers in formal mathematical proofs, such as the testing of the thousands of different cases needed for the solution of the four-color mapping problem a decade ago. Others think of testing different strategies for proving a theorem by checking out each step in the proposed attack with computer-generated examples; even if the theorem itself is true, one or more of the steps in the proposed proof can be faulty, so computers permit mathematicians to switch to a different line of proof without wasting a lot of time.

One category of experimental math has gotten the usually placid mathematical community unusually riled. Mathematicians such as IBM's Benoit Mandelbrot, the man who named and popularized fractals in the 1980s, use the computer to explore and experiment with mathematical objects much as a biologist would study a fruit fly. In studying fractals, for instance, they may zoom in for details at higher and higher magnification—a process that can go on without end since fractals are by nature infinitely convoluted—in search of insight into how the fractal is constructed. For them, the formal mathematical proof may sometimes take a back seat to expeditions into computer-generated terra incognita. Other mathematicians, noticeably Steven Krantz at Washington University in St. Louis, argue that this type of exploration isn't really mathematics at all and that it shouldn't be published as such (see *Science*, 27 July 1990, p. 363).

The fracas over experimental math demonstrates better than anything else just how much the use of computers, as epitomized by the work at the Geometry Center, is transforming mathematics. Indeed, Epstein says, it's possible that mathematicians may eventually split into two camps—theorists and experimentalists—and that's a change that no amount of pencil on paper or scratchings in the sand could have ever brought about.

—R.P.

lyze the general features and devise computational methods to model the processes.

And that's just the kind of interaction that developed. Researchers were running their individual computer programs on the workstations, then using the center's video equipment to show each other videotapes they had created—or just talking among themselves.

Carter, for instance, found some mathematicians who could tell him how to handle a particularly tricky part of a model of sintering that he was trying to construct. "Now I see I can do it," he said, adding that he expects "to get a whole new model" in a few months. Nigel Goldenfeld, a theoretical physicist from the University of Illinois at Urbana-Champaign, added, "People have been swapping algorithms and programs." He himself had been handed a "nice graphics program" by some Japanese researchers. Goldenfeld and colleagues came to the workshop with physical questions about crystal growth that they didn't know how to answer and found some mathematicians who had powerful computational methods but weren't aware of all the physics. Using the math wizards' techniques, he reported, "we're now in a position to test the predictions of the [physical] theories, so we've all been sitting around computers to see if it works out."

Sitting around computers—it's not the normal way to hold a workshop, but it just might be a factor in researchers' futures.