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In What Sense Is Turbulence an Unsolved Problem?

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Turbulence can be narrowly defined as a property of incompressible fluid flow at very high Reynolds number, and thus an attempt can be made to specify what is and what is not understood about it. The applicability of the Navier-Stokes equations of hydrodynamics to real turbulent flows and the successes and limitations of direct numerical simulation are considered. A discussion is presented of universality, and mention is made of the remarkable success of Kolmogorov's 1941 scaling ideas despite uncertainties about basic underlying assumptions such as local isotropy. Extensions of this scaling to the multifractal picture of dissipation fluctuations are discussed, but this picture remains phenomenological. Turbulence as defined above remains "unsolved" in the sense that a clear physical understanding of the observed phenomena does not exist.

IT IS FREQUENTLY STATED THAT TURBULENCE IS ONE OF THE great unsolved problems of classical physics. I agree, but what is turbulence, and what do we mean by an unsolved problem in classical physics? When a fluid flows rapidly, its flow pattern typically exhibits a subtle mixture of order and chaos, and it is this structured chaotic fluid motion that we refer to as turbulence.

Turbulent fluid flows are ubiquitous in the atmosphere, the oceans, and the stars. They also occur in a wide variety of engineering applications. Most studies of turbulence have an applied objective, whether this application be to engineering, to geophysics, to astrophysics, or to weather prediction. But is there a basic problem in physics common to all of these applications, and in what sense is this problem unsolved?

Navier-Stokes Equations and Statistically Universal Behavior

To define such a problem, I consider a restricted class of flows. I neglect density variations, whether these be due to buoyant convection or to the dynamic effects of pressure as in flows with high Mach number. I neglect thermal effects and all effects of electric and magnetic fields. I assume that the fluid satisfies Newton's law of viscosity. We are left with incompressible fluid flow governed by the Navier-Stokes equations of hydrodynamics (*1*). These are partial differential equations for a velocity field $\mathbf{v}(\mathbf{r}, t)$. Because the density is assumed to be constant, this field has zero divergence

$$\nabla \cdot \mathbf{v}(\mathbf{r},t) = 0 \qquad (1)$$

where $\mathbf{r}$ is the position vector and t is time. The momentum balance of a moving fluid element is described by the Navier-Stokes equations

$$\partial \mathbf{v}/\partial t + (\mathbf{v} \cdot \nabla)\mathbf{v} = -(1/\rho)\nabla p + \nu \nabla^2 \mathbf{v} \qquad (2)$$

where $p(\mathbf{r},t)$ is the dynamic pressure field, ρ is the constant density, and ν is the kinematic viscosity of the fluid. When supplemented by the boundary condition that the fluid in contact with any bounding solid surface does not move with respect to that surface, Eqs. 1 and 2 define the mathematical problem that I wish to study (*2*). These equations should accurately apply whenever the density changes are small and the flow is slowly varying on a molecular space and time scale. These conditions are comfortably met in many observed air and water flows on a laboratory or geophysical scale.

I emphasize one feature of these equations. The kinematic viscosity ν is the only molecular property of the fluid that enters the equations. This has the value 0.15 $cm^2\ s^{-1}$ for air and 0.01 $cm^2\ s^{-1}$ for water. If the density is constant, this is the only way to distinguish fluid flows in air from fluid flows in water. This relation can be expressed in a well-known, but still remarkable, scaling property of the fluid equations. Suppose we have a flow where a cylinder of diameter L is placed in a wind or water tunnel with a uniform upstream speed U. When put in an appropriate dimensionless form, the Navier-Stokes equations contain only one dimensionless parameter, the Reynolds number, Re, defined by

$$Re = U L/\nu \qquad (3)$$

All incompressible flows with the same Reynolds number and the same flow geometry should have the same flow properties when measured in the appropriate units. This Re scaling is of great engineering importance. It is the basis of subsonic wind tunnels for aircraft design or water tunnels for submarine design. There is also a more basic physical point. If the Navier-Stokes equations apply to the physical problem at hand, then Re scaling applies whether the flow is laminar and well understood or turbulent and less well understood. In fact, Re scaling does work well where it should. Thus we have confidence that we are starting from the right equations. The only problem is our inability to solve them. In most problems in physics, we have a subtle mixture of uncertainty about the validity of the equations we study, and uncertainty about how to solve them.

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Here we have only the latter.

Qualitatively, turbulence is not a mystery. The Navier-Stokes equations are a dissipative dynamical system with many degrees of freedom, and the effective number of degrees of freedom increases strongly with increasing *Re*. For low *Re*, we observe and can compute smooth laminar solutions with little spatial structure. As *Re* increases, the generic sequence of events is a transition to a steady spatially structured flow, then to a spatially structured and time-periodic flow, and then to a flow that is chaotic in time (*3*). This behavior is qualitatively similar for different flow geometries but is quantitatively far from universal. In this article, I will consider only fully developed turbulence, which occurs for *Re*'s very large compared to the critical value Re_c where chaos first occurs. My hope is to find statistically universal behavior for large *Re* independent of the flow geometry. In this article "universal" means that the statistical properties of turbulence are independent of the external geometry of the flow, being the same, when properly scaled, for jets, for wakes, and for boundary layers.

Isotropic Homogeneous Turbulence

High *Re* turbulence is observed to contain turbulent eddies of widely varying size. The large eddies, whose size is of the order of the externally imposed length scale *L*, are not universal. There is some indication, however, that the small eddies with size $r << L$ have statistically universal properties and are locally isotropic (*4, 5*). Local isotropy of the small eddies is essential to the possibility of a universal statistical theory, but it is not proven to be true. For now I assume it, which allows me to consider a simpler but somewhat artificial problem, isotropic homogeneous turbulence (*6*). The mean flow is absent, and all of the interactions are among the turbulent fluctuations. Whether such a flow exists in nature, it can surely be simulated on a computer. If the small-scale fluctuations are locally isotropic, then this idealized problem also becomes relevant to experiment. In restricting attention to this idealization, I have discarded most of the problems of practical interest, which relate to the interaction of the mean flow with the turbulent fluctuations. In return, I have the possibility of a universal problem in statistical physics. I will argue that even this restricted problem is "unsolved" in the sense that a firmly based qualitative physical understanding is still lacking.

Direct numerical solutions of the Navier-Stokes equations are limited to modest *Re*'s. Suppose the smallest eddies in a turbulent flow are of size η. The number of coupled ordinary nonlinear differential equations that must be solved is of order $(L/\eta)^3$, where *L* is a geometrically determined external length scale in the flow (such as the diameter of a cylinder in a wind tunnel). Experiment and phenomenological theory (see below) indicates that (L/η) scales as the 3/4 power of *Re*. Thus there are about 10^6 degrees of freedom in a turbulent flow with $Re = 1000$, which is about the limit for solution on today's supercomputers. But the observed *Re*'s in laboratory turbulent flows are of order 10^4 to 10^5, and in the atmosphere of order 10^6 to 10^7. This is too big a problem to compute directly. Direct numerical simulation plays an increasingly important role (7), particularly in comparison with relatively low *Re* laboratory flows, but it is not a substitute for a physical understanding of the universal statistical properties of the flow.

Kolmogorov Theory and Universality

The one striking success in turbulence theory is the phenomenological picture introduced by Kolmogorov in 1941 (*4*). The essential qualitative idea is a cascade of energy from large scales to small scales with an eventual dissipation of the energy by viscosity at the smallest scales. At each cascade step, the eddy size changes by some finite factor *b*. I take $b = 2$ for convenience, but any other value that is not very large will do equally well. As the energy flows toward smaller scales because of the nonlinear terms in the Navier-Stokes equations, time scales decrease so that the result is an essentially steady state. In this steady state one dynamical quantity plays a key role. This is $\langle\epsilon\rangle$, the average rate of energy dissipation per unit mass. (I use the averaging symbol $\langle\rangle$ to represent a time average in a statistically steady state. In a laboratory flow this presents no problems. In the atmosphere this introduces subtle questions about relevant time scales, which I ignore.) The quantity $\langle\epsilon\rangle$ plays multiple roles. It is the rate at which energy is fed into the turbulent fluctuations, it is the rate at which energy is transferred from large to small scales by the nonlinear terms in the Navier-Stokes equations, and it is also the rate at which energy is dissipated at the smallest scales by the action of molecular viscosity.

The 1941 Kolmogorov theory assumes that $\langle\epsilon\rangle$ is the only dynamical property of the flow that is relevant. When combined with ν, the kinematic viscosity of the fluid, the smallest length scale in the flow can be calculated by dimensional analysis to be

$$\eta = (\nu^3/\langle\epsilon\rangle)^{1/4} \sim L\,Re^{-3/4} \quad (4)$$

It should be emphasized that this is a prediction in terms of quantities that can be directly measured. In its principal dynamical role as an energy transfer rate, $\langle\epsilon\rangle$ can neither be calculated from first principles nor directly measured. But in its role as a rate of energy dissipation, $\langle\epsilon\rangle$ can be directly measured if we assume that the turbulence is statistically isotropic at the smallest scales. Statistical isotropy implies (*4*)

$$\langle\epsilon\rangle = 15\nu < (\partial u/\partial x)^2 > \quad (5)$$

where the velocity vector $\mathbf{v} = (u,v,w)$, and the position vector $\mathbf{r} = (x,y,z)$. In Eq. 5 the right side is, to a good first approximation, a directly measurable quantity. If Eqs. 4 and 5 are combined, the smallest scale in all incompressible turbulent flows is estimated to a good approximation. This is an important achievement because there is no a priori theoretical basis to know what this scale should be.

The Kolmogorov theory can be made more quantitative. I introduce the velocity structure function

$$D(r) = \langle[u(x + r) - u(x)]^2\rangle \quad (6)$$

which is a measure of the kinetic energy per unit mass in eddies of size *r*. By straightforward Fourier transform arguments, this can be related to $E(k)\,dk$, the kinetic energy per unit mass between wave numbers *k* and $k + dk$. If the Kolmogorov arguments are correct, the energy spectrum $E(k)$ is given by

$$E(k) = \langle\epsilon\rangle^{2/3}k^{-5/3}f(k\eta) \quad (7)$$

where the scaling function $f(x)$ is universal. Because Eq. 7 is expressed entirely in terms of measurable quantities, it allows a large amount of data to be collapsed onto a putatively universal single curve. The measured energy spectrum is divided by $\langle\epsilon\rangle^{2/3}k^{-5/3}$ and is plotted as a function of $k\eta$, where η is given by Eq. 4. An example of applying this procedure is shown in figure 2 of (*1*). There is good experimental evidence for approximate universality.

If I further assume that viscosity plays no role except at the smallest scales, then there is a range of scales $\eta << r << L$, where the turbulence is universal, isotropic, and independent of viscosity. In this "inertial subrange," the Kolmogorov theory gives, by dimensional analysis, that

$$E(k) = C\langle\epsilon\rangle^{2/3}k^{-5/3} \quad (8)$$

which is the famous Kolmogorov five-thirds law. This is equivalent to assuming that the scaling function $f(x)$ in Eq. 7 has a finite value at $x = 0$. The five-thirds law has abundant experimental support, and the Kolmogorov constant C has an experimental value of about 1.6. It is not at all clear whether Eqs. 7 and 8 are exactly correct or are only a good first approximation. I discuss this point later in this article.

One hope for turbulence theory is to develop a statistical theory of turbulence. Typically this gives an approximate nonlinear integral equation for the energy spectrum $E(k)$. It is well understood (*8*) what features such a theory must have in order to reproduce the 1941 Kolmogorov scaling. When the theory has these features, it allows a calculation of C and $f(x)$. These calculations are usually in good agreement with experiment. Because such theories work with only the limited statistical information contained in $E(k)$, they give an incomplete description of the complex structure of the velocity field. Despite this, they can be very useful for applications because they allow the essential features of the small scales to be described analytically. Combining an analytic description of scales smaller than some convenient cutoff with a direct numerical simulation of the larger scales ("large eddy simulation") is an active and promising approach for practical turbulence calculations.

Up to this point, this article is not very controversial. Most physicists and engineers agree among themselves and can get useful results. It is not really understood why the 1941 Kolmogorov scaling works so well, but this scaling can be built into approximate theories. But all is not well, even at the level of the most elementary statistical properties.

I have been considering the equal time spatial correlation function $\langle u(x,t)\, u(x + r,t)\rangle$, and there is general agreement as to its behavior. Suppose instead I consider the one-point time correlation $\langle u(x,t_0)\, u(x,t_0 + t)\rangle$ and its Fourier transform, the frequency spectrum $E(\omega)$. This describes the distribution of kinetic energy in frequency instead of wave number. If there is a mean flow U whose magnitude is large compared to a typical turbulent velocity u_0, then I can use Taylor's frozen turbulence assumption (*4*), which states that the turbulence is swept by our measuring probe at speed U with negligible distortion so that frequency and wave number are simply related through $\omega = kU$. In the universal range, this implies that

$$E(\omega) = \langle\epsilon\rangle^{2/3}(U/\omega)^{5/3}f(\omega\eta/U) \qquad (9)$$

In fact, most measurements of $E(k)$ are made with a probe at a single spatial point and are based on this assumption. Suppose instead that I consider the frequency spectrum measured following a fluid particle (in a Lagrangian instead of an Eulerian description of the fluid). This spectrum is not subject to direct experimental measurement, but it can be studied by direct numerical simulation. Dimensional analysis of the Kolmogorov type can be applied to this quantity to give

$$E_L(\omega) = \langle\epsilon\rangle\omega^{-2}g(\omega/\omega_0) \qquad (10)$$

where the characteristic frequency $\omega_0 = (\langle\epsilon\rangle/\nu)^{1/2}$. Equation 10 is not controversial, but consider instead a slightly different problem. What is the frequency spectrum $E(\omega)$ measured at a fixed spatial point in the fluid in the absence of a mean flow? The most commonly accepted phenomenological picture is that the Taylor hypothesis applies, but in modified form. The small scales are still swept, but now they are swept by the large-scale turbulent fluctuations that have a typical speed u_0. I thus obtain Eq. 9 but with U replaced by u_0. This result was given explicitly by Tennekes (*9*) and is commonly assumed to be correct.

More recently, a statistical renormalization group theory of turbulence has been applied to this problem (*10*). In this theory there is no sweeping of the small scales by the large scales, and the single point spectrum is given by Eq. 10. Nelkin and Tabor suggested (*11*) that experiment favors Tennekes, but at a theoretical level this fundamental and rather deep question remains open. It is not known for certain if the large eddies sweep the small eddies past a fixed probe without distorting the internal dynamics of the small eddies. Here there is a disagreement about a qualitative physical point, not just about its sophisticated theoretical justification. The problem of turbulence, even in our restricted definition, is beginning to look somewhat more "unsolved."

A similar lack of understanding occurs for the question of local isotropy. The Kolmogorov theory assumes that turbulence becomes more nearly statistically isotropic as scale size decreases, but this problem is seldom addressed in a quantitative way. For example, I could consider the cospectrum $E_{12}(k)$, which is the Fourier transform of the cross-correlation function $\langle u(x)\, v(x+r)\rangle$. For isotropic turbulence this would be identically zero. The conventional view is that the mean shear in the flow generates anisotropy and that the local cascade processes tend to restore it. This suggests Lumley's result (*12*) that $E_{12}(k)/E(k)$ should be proportional to the ratio of two time scales. The first is the eddy turnover time for an eddy of wave number k, which in the Kolmogorov theory is $(\langle\epsilon\rangle k^2)^{-1/3}$. The second is the large eddy turnover time, (L/u_0). This ratio of time scales goes as $k^{-2/3}$. There is some experimental support for this result (*13*), but no recent experiments have been performed. Nelkin and Nakano suggested (*14*), from a crude nonlinear dynamical model, that anisotropy could relax no faster than $k^{-2/3}$ but that a slower relaxation is possible. Recently, Yeung and Brasseur argued (*15*) that the Kolmogorov idea of local isotropy is wrong and supported their argument with direct numerical simulation. They suggested that the energetics of the cascade are in agreement with Kolmogorov but that a residual anisotropy remains at the smallest scales even in the limit of infinite Re. Again there is basic disagreement about a qualitative physical point, and our problem looks even more "unsolved."

Are there other statistical properties to which the simple Kolmogorov scaling applies? For example, consider pressure fluctuations. Simple dimensional analysis with the same assumptions as before give

$$p(k) = \text{const.}\ \rho\langle\epsilon\rangle^{4/3}k^{-7/3} \qquad (11)$$

for the spectrum of pressure fluctuations in the inertial subrange. This quantity is difficult to measure, and a spectrum as steep as $k^{-7/3}$ is difficult to observe reliably in direct numerical simulation. Thus, there is no direct evidence for the validity of Eq. 11. Without a more explicit dynamical theory, there is little reason to believe that Eq. 11 is even qualitatively correct. I return to this point at the end of this article.

As a second example, consider a passive scalar θ that might be temperature or the concentration of a dye marker. The Kolmogorov theory was extended by A. M. Obukhov and by S. Corrsin to this case [see (*4*)] and gives

$$E_\theta(k) = \text{const.}\ \chi\langle\epsilon\rangle^{-1/3}k^{-5/3} \qquad (12)$$

where χ is the dissipation rate for the passive scalar. There is an extensive literature suggesting that Eq. 12 fits reasonably well to experiment, but there have always been experimental anomalies. A recent analysis by Sreenivasan (*16*) suggests problems with Eq. 12 and indicates that there are serious doubts about small-scale universality for passive scalars. This result naturally leads to some skepticism about small-scale universality of the velocity field, but there is less reason to expect universality for the linear problem of the passive scalar than for the nonlinear problem of the velocity fluctuations.

Multifractal Picture of Dissipation Fluctuations

It has been clear since 1962 that the 1941 Kolmogorov theory is at best an incomplete description of the small-scale statistics of the velocity field. The experiments that confirmed the five-thirds law for $E(k)$ showed also that higher order statistical properties behave in an unusual way. Historically, most experiments have studied the statistical properties of a one-dimensional surrogate for the local dissipation

$$\epsilon(x) = \nu(\partial u/\partial x)^2 \tag{13}$$

and its average over a linear interval of length r,

$$\epsilon_r = (1/r) \int_0^r \epsilon(x)dx \tag{14}$$

In Eq. 14, the integral performs a spatial average over an interval but no statistical averaging has been performed. The quantity ϵ_r is still a random variable whose statistics are to be studied. The qualitative behavior of ϵ_r shows increasing intermittency as r is decreased corresponding to an increasing intermittency of $\epsilon(x)$ as Re is increased. By intermittency I mean the statistical tendency for very large and very small values of the (nonnegative) random variable to occur with unusually high probability compared to typical values. The probability distribution of ϵ_r is far from Gaussian with high moments being much larger than their Gaussian counterparts.

In order to make this discussion quantitative, I consider the moments of ϵ_r and how they scale with r when r is an inertial range distance. I assume for now that this scaling is universal and is described by

$$\langle(\epsilon_r)^q\rangle \sim (r/L)^{A(q)} \tag{15}$$

For notational convenience (*17*), I define a "generalized dimension" $D(q)$ by

$$A(q) = (q - 1)[D(q) - 1] \tag{16}$$

In 1962, Kolmogorov (*18*) suggested that ϵ_r was a lognormal random variable. If its moments go as powers of r, and the behavior is universal, then

$$D(q) = 1 - (\mu/2)q \tag{17}$$

where μ is a universal constant. In modern terms, it is expected that the dissipation field should define a universal function $D(q)$ rather than a single constant μ. This description, encapsulated in Eqs. 15 and 16, is equivalent to stating that the local dissipation field is a multifractal (*19*). Recent experiments suggest that Eq. 17 is a good approximation for $0 < q < 2$. The observed value of μ is approximately 0.25 and is approximately universal. For larger values of q, Eq. 17 is known to be theoretically inconsistent (*20*). It is likely that $D(q)$ goes to a finite value in the limit of large q, but the experiments for large q are extremely difficult because of the need to sample the rare intense events that dominate high moments.

The above multifractal description gives a good summary of the observed statistical properties of the dissipation field. This phenomenological picture raises, however, as many physical questions as it answers. Starting from the Navier-Stokes equations, why should the dissipation be universal or multifractal? What is the relation between the statistics of the dissipation and the statistics of the velocity field? In particular, do dissipation fluctuations modify the five-thirds law for the energy spectrum? Kolmogorov in 1962 predicted a correction of $(\mu/9)$ to the five-thirds exponent (*18*). This correction is about 0.03 and is probably too small to measure reliably. The phenomenological picture that leads to $(\mu/9)$ has no basic theoretical justification, however, so that the question of corrections to five-thirds is open both theoretically and experimentally. If I go beyond moments of the dissipation and take the multifractal description literally, then the dissipation length scale η should fluctuate strongly from point to point in the fluid (*21*). This has many observable consequences both for local quantities and for a subtle revision of the Kolmogorov scaling of Eq. 7. Some of these consequences can be tested experimentally, and these tests are of great interest (*22*).

Conclusions

Throughout this article, my discussion has emphasized universal statistical properties, and I have not connected these to the actual geometry of the small-scale structures in turbulence or to dynamical mechanisms. These connections are only beginning to be developed, for example, in (*7*). As a final example, however, I discuss one problem for which these two points of view collide in a striking manner. If one takes the divergence of the Navier-Stokes equation, one obtains

$$(1/\rho)\,\nabla^2 p = \omega^2 - \sigma^2 \tag{18}$$

Equation 18 states that the local pressure is the solution of a Poisson equation whose source is the difference between the squared vorticity ω^2 and the squared rate of strain σ^2. Both of the latter are highly intermittent in space, and it is therefore reasonable that their difference and consequently the pressure are also highly intermittent. In a recent article, Douady (*23*) presented a technique for direct visualization of the high- and low-pressure regions of a turbulent flow and interpreted the results in light of Eq. 18. The results suggest that pressure fluctuations are dominated by intermittency and that Eq. 11 may very well be qualitatively incorrect.

I have restricted the discussion to a particular turbulence problem, the putatively universal statistical properties of the small-scale velocity fluctuations in incompressible flows with high Re. I have found that phenomenological theory gives a compact description of a large amount of data. There is an adequate theoretical starting point in the Navier-Stokes equations. There is a useful phenomenological picture that suggests at least approximate universality. It is not known if this universality is exact, and I cannot say from first principles whether it should be (*24*). Even in the restricted sense that I have defined turbulence, it remains a fascinating unsolved problem. In the next several years I expect that new carefully controlled experiments, improvements in direct numerical simulation, and better mathematical understanding of the underlying Navier-Stokes equations will combine to increase our basic understanding. I also hope that basic progress in statistical field theory will add essential contributions from theoretical physics, but nothing so far has given much substance to this hope.

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Research Article

Lunar Impact Basins and Crustal Heterogeneity: New Western Limb and Far Side Data from Galileo

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Multispectral images of the lunar western limb and far side obtained from Galileo reveal the compositional nature of several prominent lunar features and provide new information on lunar evolution. The data reveal that the ejecta from the Orientale impact basin (900 kilometers in diameter) lying outside the Cordillera Mountains was excavated from the crust, not the mantle, and covers pre-Orientale terrain that consisted of both highland materials and relatively large expanses of ancient mare basalts. The inside of the far side South Pole–Aitken basin (>2000 kilometers in diameter) has low albedo, red color, and a relatively high abundance of iron- and magnesium-rich materials. These features suggest that the impact may have penetrated into the deep crust or lunar mantle or that the basin contains ancient mare basalts that were later covered by highlands ejecta.

The Galileo spacecraft encountered the Earth-moon system in December 1990 in the first of two flybys that are part of a sequence of planetary gravity assists that will deliver the spacecraft to Jupiter. The geometry of the flyby was such that the western limb of the moon was illuminated (*1*), in contrast to the Apollo missions, during which the eastern limb was illuminated to ensure safe descent and landing. The geometry of the Galileo flyby provided the opportunity to obtain multispectral images of the western part of the lunar near side, the western limb (including the Orientale basin), and parts of the lunar far side, all relatively unexplored regions of the moon. These data provide important information on crustal heterogeneity and the nature of impact basin formation. In this article we describe these data and their implications for knowledge of lunar composition and evolution.

The lunar crust is composed of two components. The globe-encircling highland crust formed in earliest lunar history and was highly modified during the period of heavy bombardment; abundant impact craters and large impact basins such as Imbrium and Orientale remain from this period. Subsequent interior melting produced volcanic surface deposits. These surface flows form the second component of the crust, the maria, which accumulated primarily in the old impact basins and which cover less than about 20 percent of the lunar surface. Impact events have fragmented the surface at all scales so that it is covered with a surface layer of crustal material called regolith. Larger impacts excavate material from greater depths, perhaps as deep as the lower crust or underlying mantle. Although these general relations are understood, many questions remain regarding the composition and time of onset of mare volcanism and the lateral and vertical compositional heterogeneity of the highland crust, particularly on the far side of the moon, where little compositional data are available.

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