Stability of the Astronomical Frequencies Over the Earth's History for Paleoclimate Studies

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The expected changes over the past 500 million years in the principal astronomical frequencies influencing the Earth's climate may be strong enough to be detectable in the geological records, and such effects have been inferred in several cases. Calculations suggest that the shortening of the Earth-moon distance and of the length of the day back in time induced a shortening of the fundamental periods for the obliquity and climatic precession, from 54 to 35, 41 to 29, 23 to 19, and 19 to 16 thousand years over the last half-billion years. At the same time, the precessional constant increased from 50 to 61 arc seconds per year. The changes in the frequencies of the planetary system due to its chaotic motion are much smaller; their influence on the changes of the periods of climatic precession, obliquity, and eccentricity of the Earth's orbit around the sun can be neglected. Eccentricity periods used for Quaternary climate studies may therefore be considered to have been more or less constant for pre-Quaternary times.

The Earth's climate has always been changing, and the magnitude of these changes has varied from place to place and from time to time. The yearly fluctuations and longer term changes have been the result of natural internal processes and external influences at work on the complex system that determines the Earth's climate, a system that includes atmosphere, oceans, land ice masses, and biosphere. These subsystems interact with one another, producing feedback effects that result either in an amplification or a damping of the processes involved. Understanding such a complex system requires the gathering of a considerable and varied mass of data along with global interactive modeling of the different subsystems.

These models are used to simulate both the history and present state of the Earth system and then to aid in predicting the evolution of the system in response to selected changes in input variables and to man-made or natural forcings. Understanding and modeling change on the Earth requires that we go beyond the traditional approaches, with the ultimate aim of modeling the Earth system over all time scales. Quantitative information on global changes of the past can be used to put observed trends in contemporary data in a broader context, to evaluate Earth system models, and to identify unknown and often more important interconnections between physical, chemical, and biological processes. During the past years, it has become more and more evident that global change studies should focus also on the natural evolution of the Earth system (*1*), in particular at the thousand-years time scale, usually referred to as the long-term astronomical time scale in climate change studies (*2*). At this time scale, the best documented geological period is the Quaternary, which covers roughly the last 2 million years (Myr). This period is characterized by the succession of ice ages that are best explained by the astronomical theory of paleoclimates.

The aim of this astronomical theory, a particular version of which comes from Milankovitch (*3, 4*), is to study the relation between the radiation the Earth received from the sun at the top of the atmosphere and the climate at the global scale. This insolation for each latitude changes over geological time scales because the energy output from the sun changes with time, but also because the characteristics of the Earth's orbit and rotation change according to the laws of celestial mechanics (*5*). Because the total radiation emitted by the sun affects the whole Earth homogeneously, it can be taken as a scale factor in the computation of the energy available for the Earth. Therefore, the astronomical theory comprises mainly four different parts: the theoretical computation of the long-term variations of the Earth's orbital parameters and related geometrical insolations, the design of climatic models to transform these insolations into climate, the collection of geological data and their interpretation in terms of climate, and the comparison of these proxy data to the simulated climatic variables (*2*). In this article, we briefly review geologic records suggesting that variations in solar insolation have had significant effects on Earth's climate not just during the Quaternary but for at least most of the Phanerozoic (past 600 Myr). We then consider the effect of the changes of the Earth's spin due to tidal friction and of the weakly chaotic nature of the orbits of the inner planets on the frequencies of the principal orbital parameters.

General Astronomical Theory and Quaternary Climates

The energy available at a given latitude ϕ on the Earth, on the assumption of a perfectly transparent atmosphere and of a constant solar output, depends on the semimajor axis of the ecliptic (a), its eccentricity (e), its obliquity (ϵ) (tilt of the axis of rotation), and the longitude of the perihelion measured from the moving vernal equinox (ϖ), which combines with e to define the climatic precession parameter $e \sin \varpi$ (*6*).

The semimajor axis and the eccentricity specify the size and shape of the Earth's orbit around the sun. In order to measure the angles used to locate the orbit in space and the Earth on the orbit, a reference frame is fixed with a reference plane, usually the ecliptic (orbital plane) at a particular fixed date of reference, called the ecliptic of epoch (Ec_0), and a point in that plane, the vernal equinox (γ_0) or First Point of Aries, that indicates the position of the sun when it crosses the celestial equator from the austral to the boreal hemisphere. The orientation of the orbital plane in space with regard to the reference plane is specified by two angles (Fig. 1): the

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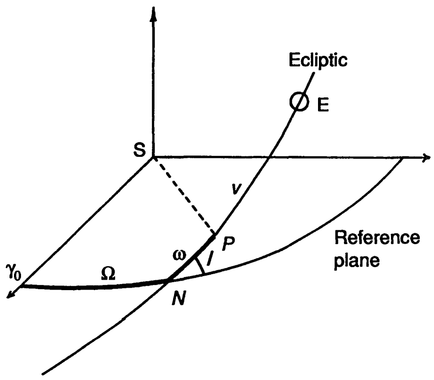

Fig. 1. Position of the Earth (E) around the sun (S). In astronomy, it is usual to define the orbit and the position of a body with six quantities called elements. Three elements define the orientation of the orbit with respect to a set of axes, two define the size and the shape of the orbit (a and e, respectively), and the sixth (with the time) defines the position of the body within the orbit at that time. In the case of a planet moving in an elliptic orbit about the sun, it is convenient to take a set of rectangular axes in and perpendicular to the plane of reference, with the origin at the center of the sun. The point γ_0 is the reference from which the angles are measured. Because the reference plane is usually chosen to be the ecliptic at a particular fixed date of reference [named the epoch of reference in celestial mechanics (*74*)], γ_0 is, in such a case, the vernal equinox at that fixed date (the vernal equinox is also referred to as the First Point of Aries). P is the perihelion; Ω is the longitude of the ascending node; ω is the argument of the perihelion; π ($= \Omega + \omega$) is the longitude of the perihelion; I is the inclination; ν is the true anomaly; $\lambda = \pi + \nu$ (the longitude of the Earth in its orbit).

longitude (Ω) of the ascending node (N), measured on the ecliptic of epoch between γ_0 and the intersection between the ecliptics of date (Ec) and of epoch (Ec_0); and the inclination (I), which is the angle between these two planes. The position of the perihelion is measured by the longitude of the perihelion (π), which is the sum of two angles lying in different planes: Ω and ω, the angular distance in the orbital plane from N to the perihelion.

Because of the attraction of the moon and of the sun, the equatorial plane is not fixed in space, nor is the vernal equinox. Their positions are given, respectively, by the obliquity, which is the angle between Ec and the equator (Eq), and by the general precession (ψ) (Fig. 2). This last angle provides the longitude of the moving perihelion ϖ through $\varpi = \pi + \psi$.

Spectral analysis of Quaternary paleoclimatic records has provided substantial evidence (*7–9*) that, at least near the obliquity and precession frequencies, a considerable amount of the climatic variance is driven in some way by insolation changes brought about by changes in the Earth's orbit (*10, 11*). Marine foraminiferal oxygen isotope ratios can be used as monitors of global ice volume, and the enormous sets of data now collected and analyzed by time-series methods leave no doubts that the Quaternary glacial regimes were forced by the orbital periodicities. Just how the variations in insolation patterns influence climate is still an active topic of research, but the general consensus is that in glacial times the

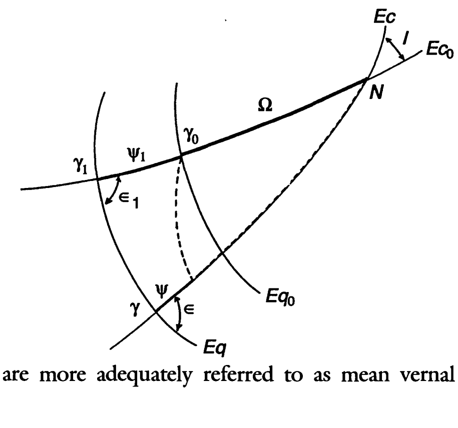

Fig. 2. Precession and obliquity. The symbol γ is the vernal equinox of date; γ_0 is the vernal equinox of reference; $\gamma_0\gamma_1 = \psi_1$, the luni-solar precession in longitude; ϵ_1 is the inclination of the equator of date on the ecliptic of reference (*75*). Because we are interested in the long-term variations of the astronomical elements, their short-term variations are removed, and γ and γ_0 are more adequately referred to as mean vernal equinoxes.

climatic effects of insolation patterns were strongly amplified and modified by the albedo of the expanding ice sheets, by the isostatic rebound of the underlying lithospere, and by other feedback mechanisms (*12, 13*), including those related to the greenhouse gases in the atmosphere (*14*).

This evidence, both in the frequency and in the time domains that orbital influences are felt by the climate system, implies that the astronomical theory might provide a clock with which to date old sediments with a precision several times greater than that now possible. In addition, significant advances made in climate theory, combined with advances in paleoclimatology, indicate that there is now an opportunity to use the geological record as a criterion against which to judge the performance of physically motivated models of climate and thus to identify mechanisms by which different parts of the climate system respond to changes in radiative boundary conditions. The importance of this opportunity is that both the temporal and the spatial pattern of these changes can be specified exactly. Except for the daily and annual cycles, we know of no other part of the climatic spectrum of which this can be said convincingly (*15*). But did orbital variations drive climatic oscillations in nonglacial times, as in the Cretaceous, which had no amplification from ice sheets?

Astronomical Periods in Pre-Quaternary Times

Gilbert (*16*) suggested that fluctuations of climate and the record of fluctuations in sedimentary successions might be due to the influence of astronomical parameters, mainly precession. On such a basis, he gave an estimate of the time span of an Upper Cretaceous pelagic rock sequence in Colorado that fits remarkably well with the results of modern radiometric datings. His concept was based on the work of astronomers such as Newton (1643–1727) and d'Alembert (1717–1783), who had discovered the precessional cycle, and Le Verrier (1811–1877), who was the first to compute the long-term variations of the Earth's orbital elements, works on which Adhémar (*17*) and Croll (*18*) based their theories of the Quaternary ice ages (*19, 2*). Despite Gilbert's great challenge, it is only since about 20 years ago that geologists under the leadership of Schwarzacher and Fischer [see their early works (*20*) and (*21*), respectively] started to recognize the effects of orbital cycles on climates and sediments for pre-Quaternary times as well (*22–25*).

Rhythmicity is indeed a pronounced characteristic of nearly all Cretaceous and Cenozoic pelagic carbonate rock sequences, whether deposited in shallow- or deep-water environments. Intercalations of carbonate-rich and carbonate-poor beds, on the order of tens of centimeters thick, are common in environments where there is also an abundant supply of clay. In anoxic depositional environments, intervals of oxygen depletion are periodic as well, and enrichment of organic carbon occurs in the relatively clay-rich intervals. In highly compacted pure chalk, the rhythmicity is reflected by decimeter-thick limestone beds separated by thin stylolitic seams (*26*). The estimated average periodicities of all of the most obvious types of cycles in Cretaceous-Cenozoic pelagic strata are usually either about 20 thousand years (kyr) or 40 kyr; groups of beds (bundles) commonly form less obvious cycles with periods of 100 kyr. However, studies of cyclicity in the more ancient record are hampered by several significant problems. Diagenesis has altered the primary chemistry and stratigraphic relations across cycles; the absolute time scales used for estimating the duration of cycles become more imprecise with increasing age; the periodicities of orbital variations are changing with time; and suitable mechanisms for coupling variations in temporal and latitudinal receipt of insolation to sedimentation changes are difficult to find (*27*).

In order to bypass this difficulty, three approaches have been used to measure rhythm periods in pre-Quaternary geological records: varve counts, direct radiometric determinations, and calculation of mean sedimentation rates with the timing assigned to stages or other units by means of radiometric scales. Only varve counts have so far provided sufficient accuracy to answer the question of whether the periodicities of ancient cycles are those of the orbital variations. Radiometric dates and extrapolated periodicities are limited by uncertainties, but other factors work to one's advantage (*28*). The character of the precession index is indeed such that precessional cycles must wax and wane in strength with the eccentricity. Furthermore, if the full range of precession cycles is recorded, the number of precessional events in an eccentricity group must lie between 3 and 7, averaging a fraction under 5, on the basis of the length of the precession cycles, which extend from 13.9 to 31.3 kyr with a mean of 21.74 kyr, as computed by Berger (*29*) for the last 5 Myr. On the other hand, the obliquity signal is more steady and can be expected to yield simple, regular sedimentary oscillations. Thus, the distinctive patterns come to be useful criteria for the identification of driving cycles. However, we must also consider the possibilities of other Milankovitch cycles, of their variations in time, of additional cycles induced by the response of the Earth (not only its climate but also its geological environment) to Milankovitch forcing, and of cycles unrelated to the orbital parameters.

The Umbrian section exposed in the Apennines of central Italy contains a remarkable sequence of pelagic sedimentary rocks that extend from the mid-Jurassic to the mid-Tertiary; these strata are suitable for testing the astronomical hypothesis from the Late Triassic until the Miocene (*30*). Rhythmicity has been investigated in the Cretaceous part of this sequence. From a time-series analysis of the variations in bedding thickness in the Maiolica and Cenomanian Scaglia Bianca (basal Upper Cretaceous) limestones, Fischer and Schwarzacher (*31*) suggested that the couplets showed a precession cycle and that bundles of beds showed a 100-kyr periodicity. Studying a typical black shale formation, the upper Fucoid Marls located at Piobbico, Italy (Upper Albian and Cenomanian part of this Umbria-Marche sedimentary rock sequence), de Boer (*32, 33*) and de Boer and Wonders (*34*) found the precession cycle for the marl-limestone bedding couplets; bundles of approximately five couplets record the 100-kyr eccentricity cycle, and superbundles of four bundles record the 400-kyr cycle. Carbonate production in these pelagic Aptian-Albian (mid-Cretaceous) sediments, quantified by calcium carbonate and optical densitometry time series with the assumption of a constant sedimentation rate of 5 mm/kyr, reflects the orbital eccentricity and precessional cycles (*35*). Planktonic foram abundance and the color of the rocks allowed recognition of periods close to those of the obliquity and precession cycles (*36*). Spectral analysis of two other series (percentage carbonate content and photodensitometer record of light-dark variation) from this sequence (after the curves were tuned to the bundle peak found from independent time control) showed the presence of two major components that match in spacing the 98- and 126-kyr components of the eccentricity curve (*37*). The identification of the bundle with the 100-kyr eccentricity cycle has thereby received independent confirmation, which in turn strengthens the identification of other cycles. As in Pleistocene marine sequences, the 100-kyr term thus dominates the sedimentary variance in the cycles in the Umbrian strata, but anoxic episodes occur in phase with a limestone-marl repetition and reflect minima of the precessional cycle (*38*).

In summary, it appears that the precession index has been widely recorded in marine sediments and is fingerprinted by the grouping of individual precession events into sets that correspond to the 100-kyr eccentricity cycle also found in these Cretaceous nonglacial times. But in contrast to the Cenomanian rocks of the Umbria sequence, Cenomanian rocks of the maritime Alps show a fairly regular oscillation in the 40- to 50-kyr bracket, suggesting that the obliquity cycle was the driver for what appears to have been a dilution rhythm (*39*).

Weedon (*40*) has studied Jurassic cycles in the Blue Lias Formation using Walsh power-spectral analysis. Two cycles with durations less than 90 kyr were found, which may record changes in orbital precession and obliquity; the sedimentary cycles were tentatively attributed to changes in the volume of runoff. The same technique was also applied to three Lower Jurassic limestone-shale sequences from Breggia Gorge, Switzerland (*41*). The regularity of the cycles observed, combined with crude direct dating and wavelength ratios, implies that the 21-kyr precession cycle is present in all three cases, the 100-kyr modulation of this cycle is present in two cases, and the 41-kyr obliquity cycle is present in one case.

The occurrence of rhythmic bedding in platform limestones of the Alpine Triassic System was recognized in late 1930s, and Schwarzacher (*20*) first noted the now well-established ratio of five couplets to a bundle. Studies by Goldhammer *et al.* (*42*) of the emergence of the Middle Triassic Latemar marine carbonate platforms of the Dolomites (Italy) also showed that a cycle ratio of 5:1 was evident, suggestive of glacial control forced by precession (20 kyr) and eccentricity (100 kyr). The stacked meter-scale cycles, each made of a subtidal unit with a cap rock, must be the result of relative sea-level oscillations that occurred with an average frequency in the 10-kyr range (*43*). The high variability of this carbonate platform sequence has been examined with a prolate spheroidal expansion of the couplet thickness spectrum (*44*). The results indicate that multiple signals are present with characteristics similar to those predicted from the astronomical theory. These results confirm that orbital cycles are significantly imprinted in pre-Cretaceous sedimentary records.

Van Houten (*45*) first recognized the cyclical nature of Newark Supergroup sediments in eastern North America, which he associated with the orbital variations. More recently, Fourier analysis of the thicknesses of beds in sections of the Upper Triassic Lockatong and Passaic formations of the Newark basin showed periods of 25, 44, 100, 133, and 400 kyr, which correspond roughly to the astronomical periodicities (*46*), as judged by radiometric time scales and varve-calibrated sedimentation rates. Analysis of the alternation of black shale deposited in deep water with drab-to-red playa deposits yields a period of 21 kyr by extrapolation of varve thickness; by the same method, alternations of detritus-rich and chemical sequences show a 100-kyr periodicity, and red sequences occur at 400-kyr intervals.

All of these interesting results encourage research on the time variation of the orbital parameters in order to determine, for example, the extent to which the changing Earth-moon distance [see, for instance, (*47*)] influenced the length of the main astronomical periods and to test the validity of the astronomical theory of paleoclimate over a broader time range.

Orbital Elements for Climate Research

We have thus analyzed the variation over the whole Earth's history of the orbital elements needed for the computation of the insolation at the top of the atmosphere for use in climate models (that is, the eccentricity e, the obliquity ϵ, and the climatic precession $e \sin \varpi$). As was shown by Milankovitch (*4*), the expansion of these astro-climatic parameters can be obtained from the trigonometrical expansion of (e, π) and (I, Ω) given by

$$e \sin \pi = \sum_{j=1}^{m} M_j \sin(g_j t + \beta_j) \quad (1)$$

$$\sin(I/2)\sin\Omega = \sum_{i=1}^{n} N_i \sin(s_i t + \delta_i) \quad (2)$$

where M_j, g_j, β_j, and N_i, s_i, δ_i are the fundamental amplitudes, frequencies, and phases.

The astro-climatic parameters can be expressed in the general form

$$e = e_0 + \sum E_i \cos(\lambda_i t + \phi_i) \quad (3)$$

$$\epsilon = \epsilon^* + \sum A_i \cos(\tilde{f}_i t + \tilde{\delta}_i) \quad (4)$$

$$e \sin\varpi = \sum P_i \sin(\alpha_i t + \zeta_i) \quad (5)$$

where the frequencies λ_i are linear combinations of g_j only, and $\tilde{f}_i$ and α_i are of the form

$$s_i + k,\ 2(s_i + k),\ s_i + s_j + 2k,\ s_i + g_j + 2k,$$
$$g_i + k,\ s_i - s_j,\ s_i - g_j \quad (6)$$

and where k is the precessional constant. Numerical values of the parameters in Eqs. 1 and 2 are given in Bretagnon (*48*) for the Quaternary and in Laskar (*49*) for longer time intervals. The corresponding values for Eqs. 3 through 5 are given in Berger (*6*) and in Berger and Loutre (*50*).

Stability of Periods for Precession and Obliquity

According to Eq. 6, changes of the orbital frequencies can result from changes in the precessional constant k and the characteristic frequencies, g and s, of the planetary point masses. Thus, these changes have two sources: (i) changes in the Earth's spin due to tidal friction and (ii) changes in the planetary orbits due to the weakly chaotic nature of the orbits of the inner planets.

The precessional frequency, k, arises from the solution of the Poisson equations describing the Earth-moon system. Its analytical expression (*51*) is given by

$$k = \frac{3}{2}\frac{\eta^2}{\omega_e}\frac{C-A}{C}\left[(1-e^2)^{-1.5} + \frac{m_{☾} a^3}{m_{\odot} a_{☾}^3}(1-e_{☾}^2)^{-1.5}\left(1-\frac{3}{2}\sin^2 I_{☾}\right)\right]\cos h \quad (7)$$

where η is the mean motion of the sun in a geocentric reference frame, ω_e is the rotational angular velocity of the Earth, A and C are the Earth's moments of inertia around the equatorial and polar principal axes of inertia (they combine to define the dynamical ellipticity of the Earth: $H = (C - A)/C$), $a_{☾}$ and $e_{☾}$ are the semimajor axis and the eccentricity of the moon's orbit around the Earth, $I_{☾}$ is the inclination of the lunar orbit on the ecliptic, $m_{☾}$ and $m_{\odot}$ are the masses of the moon and the sun, respectively, and h is a constant with a value of 23.40109°. Thus, a change in any one of these parameters influences the value of k.

To understand how k varies in time, we have calculated the sensitivity of k to both fast-varying and slow-varying parameters of the Earth-moon system. The eccentricity of the Earth's orbit and the eccentricity and inclination of the lunar orbit (the fast-varying parameters) contribute very little to the change of k, so their influence on the periods of obliquity and climatic precession can be neglected (*52*). But this is not the case for the slow-varying parameters ω_e, A, C, and $a_{☾}$ (*52, 53*).

This last parameter, $a_{☾}$, is of particular importance as it is related to the other two (*54–56*). The values used for our calculations over the last 450 Myr are deduced from the present-day Earth-moon distance and an average rate of lunar recession, $\dot{a}_{☾}$, equal to 10^{-9} m/s (*53*). This value has been chosen from published ones deduced from both indirect proxy data and recent observations. Indeed, evidence for the rate of lunar recession is available on several different time scales (*57*). Growth rhythms in corals, bivalves, and stromatolite fossils have been interpreted in terms of variations in their environmental conditions, which are related to astronomical cycles. These observations on fossils from the Ordovician to the present day (*58*) provide lower bounds for the number of solar days per year and per synodic month in the geological past. Though rough, sparse, and containing scatter, these data lead to estimated variations, respectively, of the Earth rotational angular velocity ($\dot{\omega}_e$), of the lunar mean motion ($\dot{\eta}_{☾}$), and consequently of $\dot{a}_{☾}$ given by $\dot{a}_{☾} = -2/3a_{☾}(\dot{\eta}_{☾}/\eta_{☾})$. Comparisons between some observed and computed astronomical phenomena from the last millennia have allowed more direct derivations of $\dot{\eta}_{☾}$ and $\dot{\omega}_e$; this is the case for the earlier Babylonian, Hellenic, Chinese, and Islamic records (*59*) and for telescope observations over the last three centuries. More recently, the lunar orbit and the rotation of the Earth have been monitored by lunar laser ranging (*60, 61*).

Using this value of $\dot{a}_{☾}$ and related values of ω_e, A, and C, one can show that over the last 450 Myr, k increased from its present value of ~50 arc sec per year to 61 arc sec per year. This estimate decreases the 40-kyr period and the 54-kyr period of obliquity by as much as 25% and 30%, respectively, at 450 million years ago (Ma). Over the same time interval, the 23-kyr and 19-kyr periods of precession became 19.2 and 16.3 kyr (*62*). These calculations rely on the assumption of a constant rate of lunar recession over the last 450 Myr, but this rate could have varied with time as a consequence of tidal friction, which is strongly related to ocean basin geometries and consequently to plate tectonic motion. However, because these constraints are fairly sparse and scattered and seem to have remained similar for ancient times (*63, 64*), the time values listed in Table 1, for the assumption of a constant lunar recession rate, can be regarded as a reasonable guess.

To counteract the impact of these variations of k on the astro-climatic periods, the planetary frequencies, g and s, should have changed by an equivalent amount (~10 arc sec per year) over that time interval. Because these frequencies are of the order of 20 arc sec per year at the maximum (*48, 49, 65*), their relative change should therefore have amounted to as much as 50% or more. This assumption can now be tested with the calculations made by Laskar (*65*) for the last 200 Myr.

Table 1. Estimated past values of the main astronomical periods, derived taking into account the slow variations of the Earth-moon and planetary systems.

Age (Ma)	Periods (years)			
0	19,000	23,000	41,000	54,000
50	18,800	22,600	39,900	52,100
100	18,500	22,300	38,800	50,200
150	18,200	21,900	37,700	48,500
200	18,000	21,500	36,600	46,700
250	17,700	21,200	35,600	45,000
300	17,400	20,700	34,200	42,900
350	17,000	20,200	32,900	40,700
400	16,700	19,700	31,600	38,700
450	16,300	19,200	30,300	36,800
500	16,000	18,700	29,000	35,000

Consider the main periods used for the Quaternary paleoclimate studies, that is, around 19 and 23 kyr for the climatic precession and around 41 and 54 kyr for the obliquity. The corresponding frequencies are related to the fundamental frequencies, g and s, of the planetary system according to the relations (66)

$$\frac{1}{18964} \text{ comes from } k + g_4 = (k + 17.922491) \quad (8)$$

$$\frac{1}{23708} \text{ comes from } k + g_5 = (k + 4.248976) \quad (9)$$

$$\frac{1}{41055} \text{ comes from } k + s_3 = (k - 18.850133) \quad (10)$$

$$\frac{1}{43804} \text{ comes from } k + s_6 = (k - 26.330042) \quad (11)$$

The fundamental frequencies g_4, g_5, s_3, and s_6 given in Eqs. 8 to 11 in arc seconds per year are those computed by Laskar (49) for the secular evolution of the solar system over 10 million years. The value of k can be estimated as explained above. If the fast-varying parameters in k are those calculated from the Laskar (49) solution, the present-day value of k is estimated to be 50.417262 arc sec per year (50, 67).

Over the last 200 Myr, Laskar (68) found that the motion of the solar system, and especially the motion of the inner planets, is chaotic; such chaotic motion has two consequences. First, it is impossible to compute the exact motion of the solar system over more than about 100 Myr, and the solution given over 200 Myr can be considered just a qualitative possibility before 100 Ma. Second, the fundamental frequencies of the planetary system are not fixed quantities, which would be the case if the motion were not chaotic, but slowly vary with time (Fig. 3). Although this solution should not represent the actual solution before 100 Ma, it seems reasonable to consider that this slow diffusion of the frequencies was qualitatively the same over the whole time interval, which is the assumption we make. This assumption leads to a maximum deviation from present-day values of 0.20 arc sec per year for g_4, 2×10^{-5} arc sec per year for g_5, 0.06 arc sec per year for s_3, and 8×10^{-5} arc sec per year for s_6 (65). Over the same interval of time, the maximal deviation of k due to the effect of the changing lunar orbit is 5 arc sec per year (52). Consequently, the impact of the changes in g's and s's on the obliquity and climatic precession periods is much less important than that of the variation of k (Fig. 4). Similarly, the effect of the variations of k on the amplitudes is much larger than the chaotic effect of the planetary system, and as a consequence the amplitudes in the ϵ expansion all decrease (in absolute value) by 25% over the last 500 Myr (although their relative importance remains the same) (69).

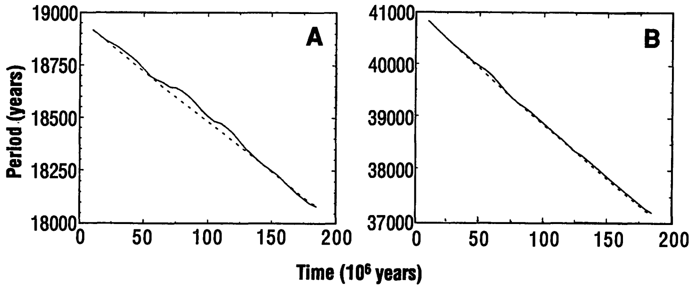

Fig. 4. Evolution of the main periods of (**A**) the climatic precession (19,000 years) and (**B**) the obliquity (41,000 years), over time backwards to 200 Ma, derived taking into account the variation of the precessional constant k only (dashed line) and the variations of both k and the main frequencies, g and s, of the planetary system (solid line).

Stability of Periods for Eccentricity

Finally, it is interesting to see whether, and to what extent, the eccentricity periods, λ_i in Eq. 3, are affected by the changes in g's. As shown in Berger (6) and Berger and Loutre (66), the origin of the period of the five largest amplitude terms in the development of eccentricity is as follows: the 400-kyr period is associated to g_2 and g_5, the 95-kyr period to g_4 and g_5, the 123-kyr period to g_4 and g_2, the 100-kyr period to g_3 and g_5, and the 131-kyr period to g_3 and g_2. Because, according to Laskar (65), the largest deviations for g_2 to g_5 are, respectively, 0.013, 0.17, 0.20, and 0.00002 arc sec per year, the corresponding periods can change only by roughly 1.5% at the maximum. This confirms recent results obtained by Bond *et al.* (70) from the analysis of proxy paleoclimatic data for the early Paleozoic, Triassic, and Jurassic, according to which the 100-kyr eccentricity cycle would remain constant over these entire remote geological times.

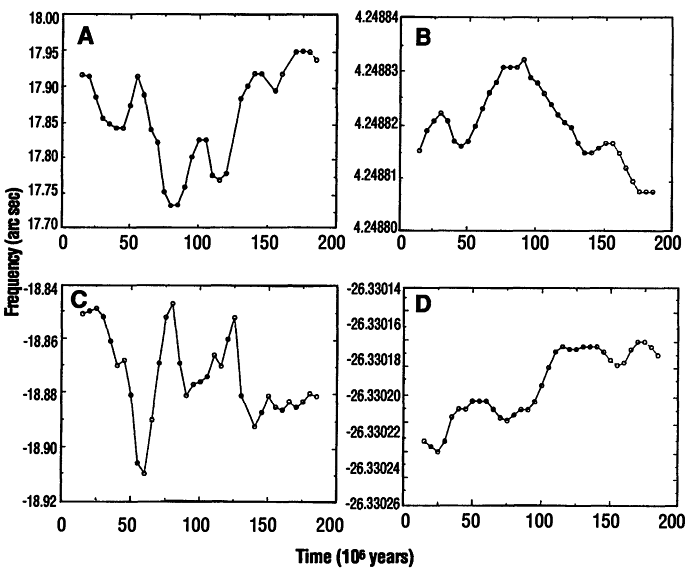

Fig. 3. Evolution of the secular frequencies (**A**) g_4, (**B**) g_5, (**C**) s_3, and (**D**) s_6 over time backwards to 200 Ma. [Adapted from (65)].

Conclusions

In conclusion, with the assumptions that the general solution of the planetary system keeps its general form and that the lunar recession rate is constant, the shortening of the obliquity and climatic precession periods over the last half billion years [roughly from 54 to 35, 41 to 29, 23 to 19, and 19 to 16 kyr (Table 1 and Fig. 4)] is therefore mostly driven by the increase of k; the planetary effect (changes in g's and s's) has a much smaller influence that is unable to counteract the effect of k.

However, most of the periods found in pre-Quaternary geological data seem to be approximately 100, 41, and 21 kyr. This apparent difference between our calculated values and those deduced from proxy data is most probably related to the difficulty of obtaining a precise, absolute time scale for Cretaceous, Jurassic, and Triassic. The reality of the changes in periodicities of the sedimentary cycles is indeed questionable even during the Neogene (71) because of these uncertainties in the time scale. The use of the ratio between the main periods of eccentricity and precession can enlighten the problem differently: recorded rhythms seem to be grouped into bundles of 3 to 6 couplets, just as precessional events are grouped by the 100-kyr cycle of eccentricity. These bundles in turn seem to be grouped into large sets of 3.5 to 5 bundles, which would reflect the 400-kyr eccentricity cycle. Unfortunately, this aspect of the computation is not precise enough to allow one to distinguish between one present-day ratio (for example, 19,000/123,000) and a past one computed, for example, at 440 Ma, when the period corresponding theoretically to the present-day 23-kyr period was 19,300 years, a value very close to the present-day 19-kyr period.

In spite of this, a first promising result has been obtained by Park and Herbert (*37*) studying a mid-Cretaceous pelagic core from central Italy. Using the eccentricity periods as a timekeeper, they estimated the inferred obliquity oscillation to have a period of 39.2 kyr with uncertainty 1.1 kyr. This period is roughly 4.4% shorter than the modern obliquity period, and this result is of the same order of magnitude as the result presented in this paper (Fig. 4B). Nevertheless, additional work is still needed to refine the time scale and to improve the astronomical solution, mainly for the Earth-moon system.

The periods of the eccentricity are not affected by the precessional parameter k. Therefore, if we consider the small diffusion of the frequencies of the planetary point masses system given in Laskar (*65*), their changes are negligible, amounting to 1.5% at the most over the last 200 Myr.

On the assumption that for times before 200 Ma the impact of the chaotic behavior of the solar system on the astro-climatic periods was still negligible, these periods can be estimated for older times (*69*). These computations are similar to that made by Walker and Zahnle (*72*) to estimate the Milankovitch periodicities that might be expected in the Weeli Wolli Formation dating from ~2450 Ma.

Our computation deals only with the past variations of the obliquity and climatic precession. However, if the development of these orbital elements is studied for the far future, the fact that the moon will tidally drive the Earth into resonance, as discussed by Ward (*73*), will have to be taken into account.

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In What Sense Is Turbulence an Unsolved Problem?

Mark Nelkin

Turbulence can be narrowly defined as a property of incompressible fluid flow at very high Reynolds number, and thus an attempt can be made to specify what is and what is not understood about it. The applicability of the Navier-Stokes equations of hydrodynamics to real turbulent flows and the successes and limitations of direct numerical simulation are considered. A discussion is presented of universality, and mention is made of the remarkable success of Kolmogorov's 1941 scaling ideas despite uncertainties about basic underlying assumptions such as local isotropy. Extensions of this scaling to the multifractal picture of dissipation fluctuations are discussed, but this picture remains phenomenological. Turbulence as defined above remains "unsolved" in the sense that a clear physical understanding of the observed phenomena does not exist.

It is frequently stated that turbulence is one of the great unsolved problems of classical physics. I agree, but what is turbulence, and what do we mean by an unsolved problem in classical physics? When a fluid flows rapidly, its flow pattern typically exhibits a subtle mixture of order and chaos, and it is this structured chaotic fluid motion that we refer to as turbulence.

Turbulent fluid flows are ubiquitous in the atmosphere, the oceans, and the stars. They also occur in a wide variety of engineering applications. Most studies of turbulence have an applied objective, whether this application be to engineering, to geophysics, to astrophysics, or to weather prediction. But is there a basic problem in physics common to all of these applications, and in what sense is this problem unsolved?

Navier-Stokes Equations and Statistically Universal Behavior

To define such a problem, I consider a restricted class of flows. I neglect density variations, whether these be due to buoyant convection or to the dynamic effects of pressure as in flows with high Mach number. I neglect thermal effects and all effects of electric and magnetic fields. I assume that the fluid satisfies Newton's law of viscosity. We are left with incompressible fluid flow governed by the Navier-Stokes equations of hydrodynamics (*1*). These are partial differential equations for a velocity field $\mathbf{v}(\mathbf{r}, t)$. Because the density is assumed to be constant, this field has zero divergence

$$\nabla \cdot \mathbf{v}(\mathbf{r},t) = 0 \tag{1}$$

where $\mathbf{r}$ is the position vector and t is time. The momentum balance of a moving fluid element is described by the Navier-Stokes equations

$$\partial \mathbf{v}/\partial t + (\mathbf{v} \cdot \nabla)\mathbf{v} = -(1/\rho)\nabla p + \nu \nabla^2 \mathbf{v} \tag{2}$$

where $p(\mathbf{r},t)$ is the dynamic pressure field, ρ is the constant density, and ν is the kinematic viscosity of the fluid. When supplemented by the boundary condition that the fluid in contact with any bounding solid surface does not move with respect to that surface, Eqs. 1 and 2 define the mathematical problem that I wish to study (*2*). These equations should accurately apply whenever the density changes are small and the flow is slowly varying on a molecular space and time scale. These conditions are comfortably met in many observed air and water flows on a laboratory or geophysical scale.

I emphasize one feature of these equations. The kinematic viscosity ν is the only molecular property of the fluid that enters the equations. This has the value 0.15 $cm^2\ s^{-1}$ for air and 0.01 $cm^2\ s^{-1}$ for water. If the density is constant, this is the only way to distinguish fluid flows in air from fluid flows in water. This relation can be expressed in a well-known, but still remarkable, scaling property of the fluid equations. Suppose we have a flow where a cylinder of diameter L is placed in a wind or water tunnel with a uniform upstream speed U. When put in an appropriate dimensionless form, the Navier-Stokes equations contain only one dimensionless parameter, the Reynolds number, *Re*, defined by

$$Re = U L/\nu \tag{3}$$

All incompressible flows with the same Reynolds number and the same flow geometry should have the same flow properties when measured in the appropriate units. This *Re* scaling is of great engineering importance. It is the basis of subsonic wind tunnels for aircraft design or water tunnels for submarine design. There is also a more basic physical point. If the Navier-Stokes equations apply to the physical problem at hand, then *Re* scaling applies whether the flow is laminar and well understood or turbulent and less well understood. In fact, *Re* scaling does work well where it should. Thus we have confidence that we are starting from the right equations. The only problem is our inability to solve them. In most problems in physics, we have a subtle mixture of uncertainty about the validity of the equations we study, and uncertainty about how to solve them.

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