
Smart Computer-Assisted Markets

KEVIN A. MCCABE, STEPHEN J. RASSENTI, VERNON L. SMITH

The deregulation movement has motivated the experimental study of auction markets designed for interdependent network industries such as natural gas pipelines or electric power systems. Decentralized agents submit bids to buy commodity and offers to sell transportation and commodity to a computerized dispatch center. Computer algorithms determine prices and allocations that maximize the gains from exchange in the system relative to the submitted bids and offers. The problem is important, because traditionally the scale and coordination economies in such industries were thought to require regulation. Laboratory experiments are used to study feasibility, limitations, incentives, and performance of proposed market designs for deregulation, providing motivation for new theory.

DOMESTICALLY, WE HAVE WITNESSED IN THE LAST DECADE uncommon political and economic forces that have resulted in increased reliance on markets to discipline prices, output, and the entry and exit of firms in industries traditionally regulated by state and federal agencies (1, 2). This has been part of a worldwide move toward privatization in the socialist and command economies of Great Britain, New Zealand, Eastern Europe, and the Soviet Union. In the United States the extent of deregulation has varied among regulated industries and has been less than complete in all of them. Thus, airport landing rights have continued to be allocated by the Federal Aviation Administration (FAA), natural gas pipelines and electric power networks are regulated by some form of federal authority, and local gas and electric utilities are regulated by state authority. Some central control or restraint has been thought necessary because it has not been clear how these industries might be structured so that markets could provide adequate self-regulation. We discuss an experimental research program that explores the use of computerized auctions designed for the complex allocation problems suggested by these examples.

No research program can anticipate all of the problems of each "design for deregulation," nor is this necessary. Institutions must be allowed to evolve in response to the economic and social demands that are placed on them. We do not claim to second-guess these responses. The research program we describe below has more modest objectives. We use the experimental economics laboratory to examine the efficiency, price, and dynamic characteristics of proposed new institutions of exchange. This examination is only the first of several steps that can be taken before such institutional designs become a feature of market practice.

The research we summarize uses smart, computer-assisted auctions for the pricing and allocation of resources in technologically

interdependent environments. The essential idea is to combine the information and incentive advantages of decentralized ownership rights (or responsibility) with the coordination advantages of central processing. The objective is to design person-computer systems in which all optimization data requirements in the form of willingness-to-pay (WTP) demand, willingness-to-accept (WTA) supply, and budget and capacity constraints are provided by decentralized decision-makers as often as prices and allocations require determination. The required input data are available only from dispersed human decision-makers who know best their own circumstances and willingness to exchange and whose interests are served by revealing this information as messages to a dispatch center. The center applies algorithms to the messages to determine the prices and allocations that maximize the gains from exchange implied by the message set. What differs among the various applications is the definition of the rights exchanged and the optimization procedure that is appropriate.

In this research we ask if there are auction mechanisms that can be used to elicit the required information in complicated problems that are not well understood theoretically. An obvious concern is that the center maximizes gains relative to reported characteristics (submitted bids and offers) rather than privately known true characteristics, leaving room for strategic manipulation. However, laboratory evidence suggests that although submitted messages are not truthful, appropriate algorithms can approximately maximize gains relative to true characteristics.

Natural Gas Pipeline Networks

Traditionally, under regulation, gas was purchased from the pipelines that transported it. This arrangement enhanced the monopoly power of pipelines (2). But competition in the natural gas industry has been made possible by the growth and development of a large network of gas pipelines under federal regulation (2). This greater potential for relying on competition to regulate allocations is compromised by the immense complexity in the number of alternative wellhead suppliers and pipeline routes that can serve competing wholesale buyers. Under our proposal, Gas Auction Net (3), all gas contracts are coordinated automatically with their transportation by a computer-assisted market. Wholesale buyers of gas submit location-specific bids defining their demand for gas delivered to their city gates. Wellhead producers submit location-specific offers of gas at pipeline input points. Owners of pipeline capacity rights enter leg-specific offers of pipeline transportation capacity (4). Each of these schedules is a step function—each step stating a bid or offer price per unit and a quantity. The unit prices to buy or sell define coefficients, and the quantities define potential flows entering the objective function of a linear programming (LP) problem that maximizes system surplus (aggregate gains from exchange) for all parties. These quantities, together with the configuration of nodes in the pipeline network and the capacity of each pipeline, determine the constraints on the LP problem.

The authors are with the Economic Science Laboratory, College of Business and Public Administration, University of Arizona, Tucson, AZ 85721.

Table 1. Demand and supply schedules for a two-node experimental environment. Competitive equilibrium: wellhead price, $P_W = 195$; pipeline price, $P_P = 55$; delivery price, $P_D = 250$; quantity, $Q = 21$. All values and costs are in tokens, which convert to cash at 400 tokens per dollar.

Decision-maker	Unit cost or value	Units capacity	Unit cost or value	Units capacity	Competitive profits	Equilibrium units
Wellhead producers						
1	110	2	195	3	170	5
2*	145	2	215	3	100	2
3	110	2	215	3	170	2
4*	145	2	185	3	130	5
5	110	2	210	3	170	2
6*	145	2	175	3	160	5
Pipelines						
1	30	8	50	3	200	8
2	30	8	60	3	200	8
3	45	8	65	3	50	5
Buyers						
1	330	2	240	3	160	2
2	300	3	250	2	150	4
3	320	2	230	3	140	2
4	290	3	260	2	140	5
5	310	3	220	2	180	3
6	280	3	270	2	130	5
Total surplus					2250	

*Owned by pipelines 1 through 3, respectively.

All of these price-quantity messages would be communicated to a central (national or regional) dispatch center (jointly owned and operated by the various interest groups), which computes optimal nondiscriminating prices at every node in the network and the flows between each pair of connected nodes. Consequently, all commodity and transportation alternatives and their terms of availability simultaneously determine market clearing prices and allocations. All buyers at a given node pay the same price, all sellers at a particular node receive the same price, and any two active routes through the network connecting the same two nodes will command the same total transportation rate. The messages are not solicited on a blind, sealed bid basis. Gas Auction Net allows all agents to improve their messages (raise a bid, lower an offer, increase a quantity) for a prescribed countdown period during which they receive real-time feedback of prices and contracts that become binding when the market period closes.

The value and cost environment and the auction procedure are illustrated in the following simple two-node network with 12 decision-makers: six buyers at node 1; three parallel pipelines connecting nodes 1 and 2; six wellhead producers—three independents, and one owned by each of the three pipelines—at node 2 (5). Table 1 and Fig. 1 list each wellhead and pipeline owner's private supply schedule and each buyer's private demand schedule. These experimenter-induced supply and demand schedules, which are unobservable in the economy, serve to motivate experimental subjects as agents are motivated in the economy. These schedules are unknown to the computer and all others except the particular individual and the experimenter. The schedules also enable the experimenter to compute and control for the competitive equilibrium (CE) design. Gas Auction Net can then be evaluated in terms of comparisons between (i) observed and theoretical CE prices and (ii) observed and maximum CE total surplus (profit or gains from exchange). The CE prices, unit allocations, and agent profits are shown in Table 1.

An experiment consists of 30 sequential periods with parameters

such as those in Table 1. Each period begins with each agent submitting a bid (offer) schedule for units of gas (or transportation) to be purchased (sold). Actual bids to buy delivered gas are ordered from highest to lowest price and are plotted (Fig. 1) as the solid step function, d , for period 19 of one experiment. The offers to sell gas and to sell transportation are each ordered from lowest to highest selling price, and the two offer schedules are vertically summed to yield the realized supply of deliverable gas for that period ($s_W + s_P$ in Fig. 1). The observed price to buyers is $p_D = 245$, the wellhead price is $P_W = 196$, and the pipeline price is $p_P = 49$. Efficiency in this period, defined as realized total profits (2245) divided by maximum total surplus (2250) in Table 1, is 99.8%. In this experiment 20 of the 30 periods yielded efficiencies in the 92 to 100% range, and after the first two periods the average was 93.3%.

Figure 2 illustrates a nine-node network connecting six wellhead producers and six wholesale buyers who are served by a pipeline network with three owners. Experiments with this network resulted in prices very near the CE predictions, except that the observed prices at nodes 1 and 2, with predicted prices $P_1 = 155$ and $P_2 = 152$, stabilized at average prices of 152 and 145, respectively (3). The lower wellhead prices were reflected in increased profits for the pipelines serving these nodes, because each wellhead node was served by only one pipeline, and this lack of competition diverted the CE profit of the wellheads to the pipelines.

Downstream, the two pipelines (1 and 2) compete to serve buyers B_1 and B_2 , and the prices at these nodes approximate the CE predictions. Observed total efficiency is about 90%, with the pipeline and buyer shares of the total gains from exchange approximately equal to the CE prediction; producers receive somewhat less than the CE profit prediction (but see below the effect of pipeline cotenancy experiments). Experiments with a less well connected network, achieved by eliminating pipeline segments 3.1 and 3.4, dramatically altered these results: both the wholesale buyers and the wellhead producers lost surplus to the pipelines because of the much-reduced competitiveness of the transportation supply (3).

Any proposal to restructure a nationally networked industry (such as natural gas or electric power) raises many questions. We briefly

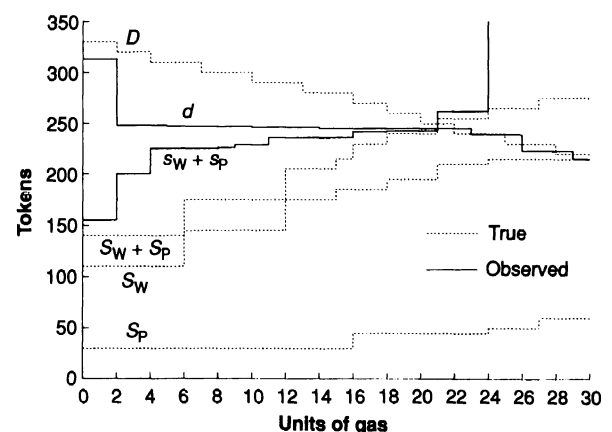


Fig. 1. Comparison of observed and true demand and supply. D , shown dotted, is the true WTP demand schedule; S_W is the true WTA of wellhead suppliers; S_P is the true WTA supply of pipeline transportation. The CE supply of delivered gas is $S_W + S_P$. The true CE price of pipeline transportation, 55 tokens, is the CE delivered price, 250 tokens, minus the CE wellhead price, 195 tokens. Actual bids (reported WTP) for one auction period are shown as the solid step function, d . The vertical sum of actual wellhead offers plus pipeline transportation offers (reported WTA for delivered gas) is the solid step function, $s_W + s_P$. The near-horizontal supply and demand schedules, near the true CE (unknown to all subjects), are typical after the first few periods. Subjects approximate CE outcomes without revealing their true WTP or WTA.

address two that have most frequently arisen in discussions with various government and private concerns. The first deals with the incentives of all agents within the systems to reveal their "true" WTA or WTP schedules and the ability of the system to resist manipulative strategizing. Our experiments with Gas Auction Net show that subjects settle into a behavioral equilibrium in which marginal value (cost) units near the CE are almost fully revealed by the bids (offers) submitted, although interior units tend to be greatly underrevealed. Thus, in Fig. 1, the observed supply and demand realizations are well inside the true supply and demand to the left of the CE, but this situation causes no great loss of efficiency. The nearly flat supply and demand realizations, with many tied bids and offers, illustrate the behavioral mechanism by which each interest group (buyers, producers, and transporters) protects itself from unfavorable price initiatives from their two adversaries: full demand and supply revelation is not an equilibrium behavior.

Dubey (6), Simon (7), and Benassy (8) have shown that in uniform price, sealed bid offer markets (such as Gas Auction Net) the competitive equilibrium can result in a strategic noncooperative Nash equilibrium of the complete information game. Friedman and Ostroy (9) provide a thorough explication of this result. In particular, such Nash equilibrium strategies are not truth revealing. This point is important because generalizing from results in specific experimental environments is especially perilous when no viable theoretical framework exists.

The second issue concerns the treatment of natural monopolies. Cotenancy, or a joint venture property right arrangement, provides a means by which a thinly connected network industry, with natural monopoly segments, might be deregulated (10, 11). Monopoly pipelines, generators, or communications switches could be required to be owned under cotenancy property right rules specifying at least two or three owners who submit independent offers to sell their product or service. Thus, the physical requirement that one and only one capital facility is adequate to satisfy demand does not imply that the market for services must be organized as a natural monopoly. Many examples of such cotenancy ownership already exist, including pipelines, generators, transmission lines, and shopping malls. Our proposal is that the government specify such property right rules as part of the design of deregulated competitive network industries.

We compared the performance of Gas Auction Net with and without cotenants on pipeline segments 1.1, 1.2, 1.3, 2.1, and 3.2 in

Fig. 2 (11); these segments showed the most evidence of monopoly power in the price data without cotenants. In the experiments with cotenants, overall market efficiency increased, prices at buyer nodes decreased, and prices at wellhead nodes increased.

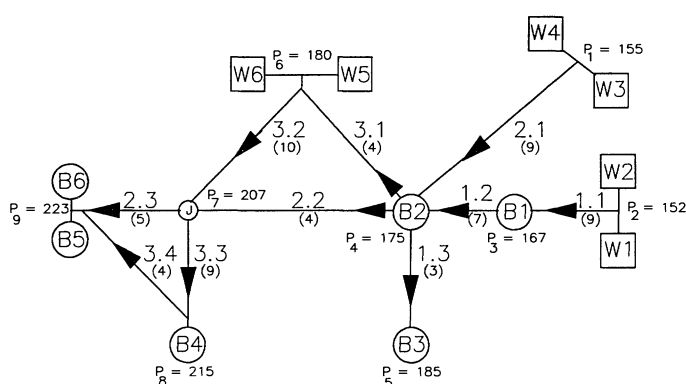
Because of the structure of a gas pipeline network and the nature of the rights exchanged, a linear program suffices to optimize allocation efficiency and compute location-specific prices. If an institution is employed that provides information feedback, the effect of the addition of a new or improved bid or offer is trivially obtained through sensitivity analysis of the incumbent optimal allocation. If an unforeseen curtailment of supply occurs (bad weather, ruptured line, and so on), then there exists a natural hierarchy of bids to abandon and simple rules for the subsequent price adjustment.

Combinatorial Auctions

The simplicity of LP solutions does not extend to many other markets for which smart, computer-assisted auctions have great potential. Thus, unique separating prices above which all bids are accepted and below which all bids are rejected do not exist in markets in which discrete optimal allocations must be made. Suppose we require trading in more than one resource and solicit offers to sell as well as bids to buy for discrete combinations of the different resources. Although there are no unique separating prices, a natural pricing trichotomy can be developed. A vector of resource "buy" prices can provide lower bounds on acceptable bids, whereas a vector of resource "sell" prices can provide upper bounds on acceptable offers. There may be some bids for which acceptability is determined not only by these prices but also because of the combinatorial constraints placed on efficient resource utilization. These bids, corresponding to the core of the mathematical programming problem, generally constitute a small percentage of all bids and are known to decrease in relative number as problem dimensions increase.

Given the optimal allocation derived from the bids and offers submitted by various agents, what prices do buyers actually pay and sellers receive? If successful buyers (sellers) pay what they bid (ask), this provides a strong incentive to underreveal true WTP (WTA). As we saw in Gas Auction Net, we observe good revelation of value (cost) at the margin when we charge the same price for all buyers (sellers) at the same node. In the combinatorial auction this is achieved as follows: all bids above the sum of their component resource demands multiplied by their buy prices pay this sum. All offers below the sum of their component resource supplies multiplied by their sell prices receive this sum. All other accepted bids and offers are transacted at the prices submitted. These rules guarantee nondiscriminatory prices for all bids and offers except for a few marginal packages, which disappear when separating prices exist.

The combinatorial auction was originally motivated by the airport takeoff and landing time-slot problem (12-14). Airlines wish to cycle planes through various airports on any given day according to their preferred schedule, which is constrained by crew and resource availability and the expected demand. Each cycle can be represented by a sequence of flight-compatible takeoff and landing rights in various 15-minute intervals throughout the network. It is these packages of rights across airports that are of value to the airline; individual elements—a takeoff or landing right—are worthless alone. Hence, the combinatorial nature of the entity must be priced in the market. An airline might also wish to impose logical constraints on any slot allocation: it might want, for example, cycle A or cycle B but not both, or cycle B only if it gets cycle A. Such constraints can be mathematically transformed to be indistinguish-



able from other linear resource constraints and are easily honored by smart market algorithms.

Our original study (13) recognized that the slot problem was a subset of the class of set-packing problems in the mathematical programming literature for which we might design a combinatorial auction institution. Our experiments were designed to assess the incentive properties, as measured by allocative efficiency, of our nondiscriminatory pricing of packages. The subjects, students from the campus of the University of Arizona, were introduced through written instructions to an environment in which six agents were able to buy, through an auction process, available quantities of six resources: items A, B, C, D, E, and F. The values of various packages of these items varied among the agents; they could resell the package to the experimenter if they secured it during the auction. For example, agent 1 might place a value of \$8.25 on the combination ACF (13).

Two types of auction institutions were employed to allocate the items. The first used simultaneous, independent sealed bid auctions of uniform price for each item, which had been found to be more efficient than the committee approach (14). The second used our combinatorial auction, in which agents could submit sealed bids for packages of items. Accept and reject prices for each item were then calculated to classify each bid received. In both cases a secondary market was allowed to correct any inefficient allocations of the primary market.

The combinatorial auction almost obviates the need for a secondary market, because the unrealized gains from exchange are very small in the primary auction, especially for experienced subjects (Fig. 3). Both of the institutions tested are subject, theoretically, to strategic manipulation. But strategic behavior is fraught with risk for all agents because they know neither the package values nor the bids of their competitors. Furthermore, in the combinatorial auction, allocations can change dramatically with minor underrevelation of value. In fact, in both institutions, attempts at strategic behavior were uniformly unsuccessful.

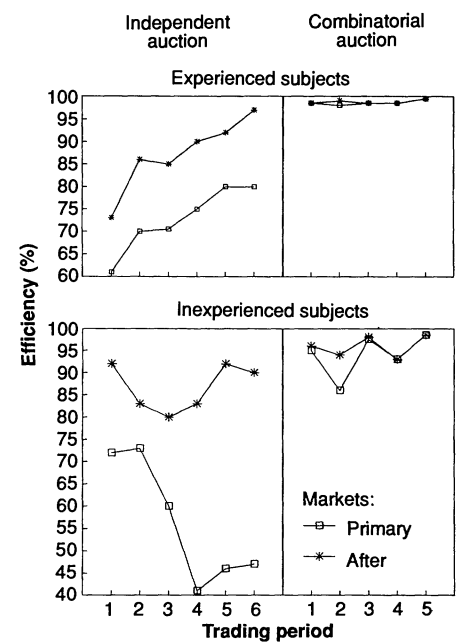
Electric Power Networks

There is a widely held intellectual position that alternating current (AC) transmission networks cannot be made self-regulating by markets. Historically, it was a textbook argument that the market regulation of electricity was infeasible because of scale economies. With the growth of cogeneration and other independent power producers, it was conceded that it was possible to deregulate electric generation in part, but that system stability could not be left up to market forces. These arguments leave out of the equation the fact that every market has elements of uniqueness in the "property" right arrangements (rights to act) that allow a market to do its job.

Our proposal is based on the assumption that deregulation is a researchable hypothesis, worth studying and debating. Individual generators would be privately owned (10). A generator's phase compatibility with the network would be governed by contracts of the kind that already exist. Generator owners would submit a supply schedule (bounded by the generator's minimum and maximum capacities of dispatchable spot power) to the coordination center. Generator owners would also submit a supply price for "spinning," that is, the minimum lump-sum revenue requirement for which the owner is willing to commit his generator at its minimum loaded spinning capacity. Wholesale buyers would submit location-specific bid schedules for power delivered to their local power buses. A regional dispatch center would collect these location-specific bid and offer schedules and supply prices for spinning status.

Given the electrical (resistance/reactance) characteristics of the

Fig. 3. Plot of period to period average efficiency of four experiments in each cell of a 2 by 2 design. Efficiency is measured as the percentage of potentially realizable surplus achieved by all subjects. Regardless of subject experience, the computer-assisted combinatorial auction generally achieved such high efficiency in the primary market that very small gains from exchange were realizable in an aftermarket. Coordination difficulties and speculative bidding require the secondary market to carry a much heavier burden when the primary market uses independent auctions for resource items.



grid and these offer supply and bid demand schedules, the center would compute and continuously update an allocation of the pattern of load to individual dispersed generators so as to maximize system surplus. The first priority of the dispatch center, however, would be to guarantee the integrity and stability of the system if from time to time such considerations were not compatible with maximizing short-run profits.

Optimization by the center requires solving a mixed integer nonlinear program, because transmission losses and power generation costs are approximately quadratic functions of output, and determining which generators to spin or shut down is a discrete optimizing problem. Because this problem must be, and is, solved (if imperfectly) by the large integrated utility using economic dispatch programs, new concepts are not required, but of course the scale is far larger.

There are three important differences with current procedure: (i) the dispatch center treats each generator supply schedule as a marginal cost function, the integral of which enters the center's criterion function—in fact, it is a marginal subjective cost schedule, not necessarily identical to marginal fuel cost. Rather than incur the shutdown and startup costs of a base load generator, its owner might be willing to sell power during low demand periods for less than marginal fuel cost—a decision that the decentralized owner-operator is best able to make. (ii) The lump sum offer prices for spinning provide a mechanism whereby generator owners may recover fixed costs, which are now recovered as part of the regulatory process and are included in the unit charge for power. Under our proposal, the spot price of power would be based on the calculation of "system λ " (the marginal supply price of the most expensive generator required by the optimization program), which is then adjusted for location for incremental transmission loss to determine node prices throughout the network; that is, at node k the price is $\lambda (1 - ITL_k)$, where ITL_k is the increase in transmission loss that would occur if one additional unit of power were injected at node k . (iii) The marginal value of wholesale power to local distribution companies and private commercial-industrial users will be determined by their WTP as expressed in their bid schedules. Thus, buyers with price-sensitive demand for power or who have cogeneration or other sources of power will be motivated to reflect these alternative opportunity costs in their bids and will cycle on and off the system depending on the pattern of optimal prices and

allocations computed by the dispatch center (15).

Transmission prices would be determined by the difference between the spot output and input prices of each line. For a line loaded at less than its capacity, power flowing from node i to node j implies a transmission price $P_{ij} = \lambda (ITL_i - ITL_j)$. When power flow on a line is constrained by a reliability capacity limit, P_{ij} will exceed the marginal value of the power loss on the line. The spot transmission price will reflect the implicit opportunity cost of the constraint (the marginal system value of increasing transmission capacity from i to j). Because of such constraints, a line's price in peak demand periods might be a large multiple of its price at off-peak periods of a day or season. Such market price responsiveness to reliability constraints is desirable because it both signals the need and provides the profit incentive for increased investment. Such investment could take any combination of three forms, depending on the response of decentralized entities: (i) increased transmission capacity from i to j ; (ii) increased generator investment at j ; and (iii) increased investment in energy conservation devices by power consumers at node j .

Other Applications

Many multiple resource allocation problems in the private sector have combinatorial structure. For example, the National Aeronautics and Space Administration (NASA) funded a research project at the Jet Propulsion Laboratory (JPL) (16) to develop an auction process to price resource utilization by commercial users of the space station resources. In this scenario an agent might be interested in placing an experiment aboard the space station that has particular mass, power, and man-hour requirements. NASA, faced with selecting the "best" set of projects to "go," commissioned the development of the California Institute of Technology's Adaptive User Selection Mechanism (AUSM).

AUSM is an auction process allowing agents to express their WTP for a complete package of resources. It is iterative and allows users to improve but not withdraw their bids. Each new or increased bid submitted in real time is tentatively accepted if it requires unused capacity of the various resources. Otherwise, a bid can only be tentatively accepted if it provides more revenue than some set of bids that needs to be displaced to free the required resources. At some prespecified time, the auction ends and all tentatively accepted bids become contracts.

In addition to the original resource constraints, AUSM is concerned with the potential curtailment of these resources under unforeseen circumstances. To deal with this aspect of the allocation problem, it establishes separate priority classes in each of the resources. The user must specify the priority class to which each element of his package applies. Under curtailment, holders of first priority are always serviced before holders of second priority.

A second application, bilateral matching, uses the Gale-Shapley algorithm (17) to construct optimal partnerships in labor market settings. A variant of this algorithm is used in the field (18) to match graduating medical students to hospital internships. Here, field implementation has preceded theoretical and laboratory study. In response, a number of theoretical papers, summarized in (18), show that graduates will misrepresent their true preferences, but equilibrium behavior will lead to efficient outcomes as defined (17). Experiments (19) find that subjects do misrepresent, but not efficiently (in contrast with Gas Auction above).

Another application, the computer scheduling environment, demands microsecond immediacy in the allocation process. In this case

an unconditional optimal allocation would require the impossible—a perfect demand forecast. The market institution must use an easily solvable heuristic to parcel and price processing time. Process Auction, which we are currently developing, would minimize average revenue losses to the center due to congestion by instantaneously adjusting the best order in which to sequence jobs. A four-part "bid" profile accompanies each job submitted to the center: the maximum amount of processing time needed, the bid value, the time due, and the delay cost for lateness. The bid value, minus the delay cost multiplied by the number of time units late, indicates the maximum amount the owner must pay the center for having the job completed. The job owner (or some designated automated process) could update his profile whenever he chooses by raising his bid price or decreasing his delay cost or both. At discrete intervals of time Process Auction would take a snapshot of the currently active set of bids and then heuristically solve an optimal sequencing problem to minimize the opportunity cost of congestion-based tardiness. It would continuously inform each job owner of his job's current status: expected completion time and expected cost of processing, given no new bid profiles.

The list of potential applications for combinatorial auctions might also have included the following: member stations of the Public Broadcasting System selecting a set of commonly funded programs to broadcast, subject to individual budget constraints; scheduling personnel to job tasks or shift assignments using bids solicited from annual rank-based endowments of currency; allocating campus parking spaces to individuals who are willing to pay differing amounts for various time and location combinations; and auctioning stadium tickets, mineral rights leases, or pollution rights certificates where spatial contiguity is important.

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