averaging 40 years of age, however. The premature mortality expected to result from ambient air pollution falls predominantly on the elderly. Thus fewer life years are likely to be saved per case of premature mortality avoided by air pollution control than through programs that reduce occupational risks, for instance. For this reason, we choose a value for reduced mortality risk that falls toward the lower end of the observed range.
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Back-Action Evasion as an Alternative to Impedance Matching

B. YURKE

Back-action evasion is a measurement technique originally devised to overcome certain limits imposed by quantum mechanics on the sensitivity of gravitational radiation detectors. The technique is, however, more generally applicable and can be used to improve the sensitivity of instrumentation with noise floors much greater than the quantum noise floor. The principle of back-action evasion is described here by means of a simple example. A comparison of back-action evasion with impedance matching is made to clarify when back-action evasion may be useful. Back-action evasion allows one to achieve a sensitivity comparable to that achieved by impedance matching.

IT HAS BEEN A LONG-STANDING GOAL OF A NUMBER OF experimental groups (*1*) to detect the gravitational radiation that should be emitted during violent astrophysical events such as supernova explosions, as predicted by general relativity. One method of detection, pioneered by Weber (*2*), utilizes a massive cylindrical bar of aluminum (Weber bar), typically weighing about 1 ton, suspended so that it is isolated from ambient acoustic and seismic noise. Tidal forces exerted on the bar by passing gravitational radiation cause the length of the bar to oscillate. If the length of the bar could be monitored with sufficient sensitivity for detection of this vibration, one would have a gravitational wave detector. Because the tidal forces exerted on a Weber bar are very weak, there was considerable interest in determining the ultimate sensitivity of this type of detector (*3, 4*). Of particular concern was whether quantum mechanics, through the energy-time uncertainty principle or the position-momentum uncertainty principle, placed limitations on the sensitivity of Weber bar detectors. Quantum nondemolition detection schemes (*5*) were devised which showed that the "standard quantum limits" obtained by naive application of the uncertainty principles could be overcome by sufficient cleverness in instrumentation.

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Back-action evasion (*6–8*) is one type of quantum nondemolition measurement scheme. Although the scheme was originally devised with Weber bar detectors in mind, the technique can be applied more generally. The method is not limited to quantum noise. It can also be used to overcome classical noise and thermal noise and, thus, may be useful even in instrumentation in which noise is much larger than that of the standard quantum limit. So that the usefulness of back-action evasion can be evaluated, this technique should be compared to the standard technique for optimizing sensitivity, namely impedance matching. It will be shown here that with back-action evasion one can achieve a performance level comparable to the level one would achieve if the detector were impedance-matched to the source, provided one is content with looking at the information carried by only one phase of the signal (*9*). Back-action evasion thus provides an alternative to impedance matching. It may be particularly useful in cases where it is difficult or undesirable to match impedances. Voltage measurement provides an example where impedance matching is undesirable, because the volt meter should not significantly load the circuit on which the measurement is being made. It is also inconvenient to match impedances if the source impedance of a signal source changes with time.

To keep the discussion simple, I will consider back-action evasion methods applied to purely electrical systems rather than to the electromechanical systems that constitute Weber bar detectors. Also, for simplicity, I will only consider circuits in which all the impedances are real (resistive).

Equivalent Circuits for a Source

Consider a black box with one port, that is, two terminals, across which one can put a meter. With a volt meter one could measure the open circuit voltage V across the two terminals. I will take this voltage to consist of two parts: a voltage V_S, which is the signal of interest, and V_N, which is a fluctuating noise voltage uncorrelated

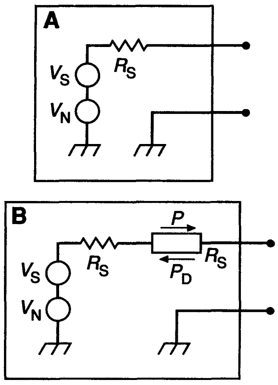

Fig. 1. A black box containing a signal that one would like to measure by placing an instrument across the two output terminals. (**A**) An equivalent circuit for the black box, consisting of a source resistor R_S, a signal voltage generator V_S, and a noise generator V_N. (**B**) An equivalent circuit for the black box with a transmission line having an impedance R_S included to aid the visualization of power flow. The two equivalent circuits are indistinguishable in terms of measurements that can be made across the output terminals.

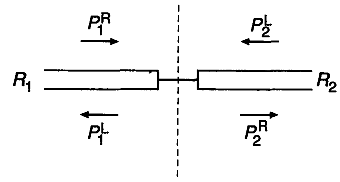

Fig. 2. An impedance discontinuity (dashed line) formed by two transmission lines having different impedances. The line on the left has an impedance R_1. The line on the right has an impedance R_2. The arrows indicate the direction of power flow.

with the signal. With a current meter placed across the terminals, one could measure the short-circuit current I. The source impedance (which I take to be purely resistive) of the black box is then given by $R_S = V/I$. The equivalent circuit for the black box is thus a series circuit with a resistor R_S and two voltage sources V_S and V_N (Fig. 1A). A more complicated equivalent circuit, in which the series combination of the resistor and the two voltage sources feed a transmission line of impedance R_S, is shown in Fig. 1B. The two equivalent circuits are indistinguishable if one is restricted to measurements that can be made across the output terminals of the black box. This more complicated equivalent circuit allows one to visualize the propagation of signals. In particular, the power P propagating to the right along the transmission line consists of the sum of the available source power $P_S = \langle V_S^2 \rangle / 4R_S$ and the available noise power $P_N = \langle V_N^2 \rangle / 4R_S$, where triangular brackets denote time averaging. I will consider the case where the signal occupies a band about the center frequency, ν, and the bandwidth B is small compared to ν. The specific source of the fluctuating voltage, V_N, is not important. It could be thermal noise, quantum noise, or some nonequilibrium noise such as flicker noise. The minimum available noise from a resistive element is, however, given by $P_N = h\nu B/2$, where h is Planck's constant. This noise is due to quantum fluctuations and is often called vacuum fluctuation noise. For electronic circuits operated at room temperature, the available noise from a resistive element will generally be much larger than this quantum lower bound.

The power P_D propagating to the left along the transmission line is dumped into the source resistor R_S and is never seen again; that is, none of this power is reflected off of the left end of the transmission line to propagate back toward the output terminals. The signal-to-noise ratio (SNR) for the signal propagating to the output terminals of the black box is thus

$$SNR = P_S/P_N \tag{1}$$

Because the noise is internal to the black box, this is the optimum SNR that can be achieved by a measuring instrument attached across the output terminals of the black box. Most measuring instruments will achieve an SNR that is worse than that given in Eq. 1.

Reflections Off of an Impedance Mismatch

There is an impedance discontinuity or mismatch where two transmission lines, one of impedance R_1 and the other of impedance R_2, are joined (Fig. 2). I will consider the case when the signals (which could be noise) propagating towards the impedance mismatch are uncorrelated so that interference effects average to 0. Let P_1^L and P_1^R denote the power propagating to the left and to the right, respectively, on transmission line 1, and let P_2^L and P_2^R denote the power propagating to the left and to the right, respectively, on transmission line 2. Then the power propagating away from the impedance discontinuity is expressed in terms of the power propagating towards the discontinuity as follows:

$$P_1^L = RP_1^R + TP_2^L$$
$$P_2^R = TP_1^R + RP_2^L \tag{2}$$

In the first equation, the power propagating away from the impedance mismatch on transmission line 1 consists of (i) the power that propagates in on side 1 and is reflected off of the impedance mismatch with a reflection coefficient R and (ii) the power that propagates in from side 2 and is transmitted through the impedance mismatch with a transmission coefficient T. The second equation has a similar interpretation. T is given by

$$T = \frac{4R_1R_2}{(R_1 + R_2)^2} \tag{3}$$

The reflection coefficient is

$$R = \left[\frac{R_1 - R_2}{R_1 + R_2}\right]^2 \tag{4}$$

Note that $T + R = 1$, as required by the conservation of energy; that is, the power leaving the impedance discontinuity must be equal to the power incident on the discontinuity. When one is impedance matched, that is, when $R_1 = R_2$, there is no reflected power, and the transmission coefficient is 1.

Back-Action Noise and Impedance Matching

An amplifier emits noise out of both its input port and its output port. For simplicity, the case will be considered where the noise leaving the output and input ports are uncorrelated. The noise

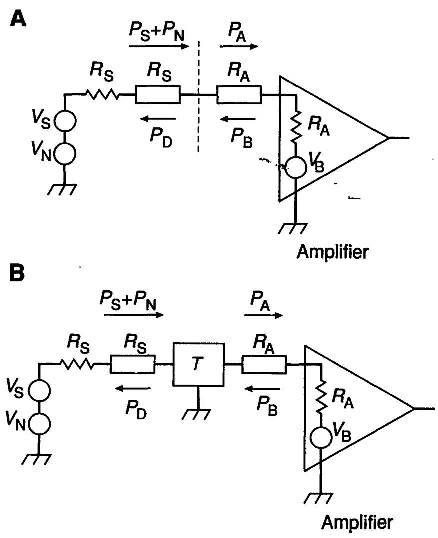

Fig. 3. An amplifier attached to a signal source. (**A**) The impedance mismatch between the source and the amplifier is located at the junction of the R_S and R_A transmission lines (vertical dashed line). Back-action noise P_B coming from the amplifier is reflected back into the amplifier by the impedance mismatch, thus degrading the SNR. (**B**) Impedance matching is restored by means of the impedance transformer T.

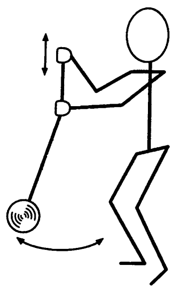

Fig. 4. A mechanical parametric amplifier. The stick figure holds one hand at a fixed height to provide a fulcrum for the pendulum, which consists of a weight attached to a length of string. By raising and lowering the other hand, the stick figure can vary the length of the pendulum. If the right hand is raised and lowered at twice the natural frequency of the pendulum, the stick figure can amplify the initial swinging motion of the pendulum.

leaving the input port is called the back-action noise. The noise leaving the output port will be called the amplifier noise. The amplifier has an input impedance R_A. For simplicity, I will consider the case where the noise impedance is equal to the input impedance. The equivalent circuit for the input port of the amplifier then consists of a resistor R_A with a noise voltage generator V_B that generates the back-action noise (Fig. 3A). Transmission lines of impedance R_S and R_A have been inserted into the equivalent circuit. It is immediately apparent how the back-action noise degrades the SNR. The power P_A propagating towards the amplifier consists of the signal source power, TP, transmitted through the impedance discontinuity and the back-action noise, RP_B, reflected off of the impedance discontinuity. The power propagating toward the amplifier is

$$P_A = TP_S + TP_N + RP_B \tag{5}$$

The SNR for the signal delivered to the input port of the amplifier is thus

$$SNR = \frac{TP_S}{TP_N + RP_B} \tag{6}$$

This is the best SNR that could be hoped for at the output of the amplifier. Generally, the SNR at the output of the amplifier will be worse because of the noise the amplifier emits out the output port. The SNR, Eq. 6, is worse than the optimum SNR, Eq. 1, as long as the source impedance R_S and the amplifier's input impedance R_A are not equal. When the impedances are matched, that is, when the source impedance and the amplifier's input impedance are equal, Eq. 6 reduces to the optimum SNR. Note that in this case the back-action noise is absorbed by the source resistor R_S and is never seen again. Further, the full available signal power P_S is delivered to the input port of the amplifier.

When the input impedance of the amplifier is different from the source impedance one can still match impedances with an impedance transformer (Fig. 3B). The impedance transformer makes the input impedance of the amplifier appear to have the same value as the source impedance, as seen by the source through the impedance transformer. Impedance transformation can be accomplished by a

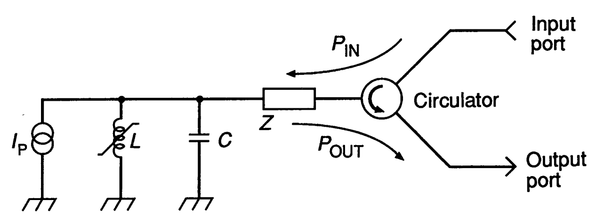

Fig. 5. A parametric amplifier using a time-dependent inductance. The time-dependent inductance is created by injection of a pump current I_P through the nonlinear inductor L. The inductor L and capacitor C form the electrical equivalent of the pendulum of Fig. 4. The amplifier is a reflection amplifier; that is, the incoming signal and the amplified outgoing signal propagate in opposite directions along the transmission line Z. A circulator, a device that directs power flow, is used for separation of the input port from the output port.

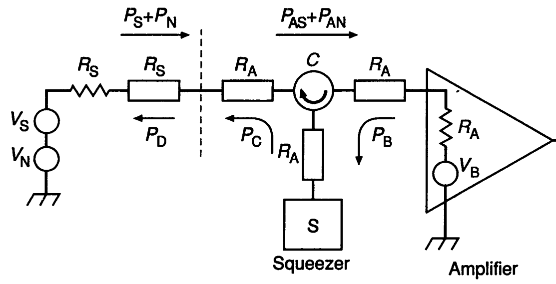

Fig. 6. A circuit that incorporates back-action evasion to overcome the degradation of the SNR resulting from an impedance mismatch. A phase-sensitive amplifier or squeezer S is used for deamplification of one phase of the back-action noise coming from the detector amplifier. If the deamplification is sufficiently great, there will be negligible back-action noise left to be reflected off of the impedance mismatch.

number of methods. For example, a conventional transformer, neglecting losses and assuming perfect coupling, with turns ratio $r = N_s/N_p$ (where N_s is the number of turns in the secondary winding and N_p is the number of turns in the primary winding), will take an impedance R_p connected across the primary winding and transform it to an impedance $R_s = R^2R_p$, as seen across the secondary winding. A transmission line one-quarter wavelength long, $\lambda/4$, can be used to match the impedances of the source and the amplifier, provided the $\lambda/4$ transmission line is chosen to have an impedance that is the geometric mean $\sqrt{R_S R_A}$ of the source impedance R_S and amplifier impedance R_A. Resonant circuits constructed from capacitors and inductors are also often used as impedance transformers, particularly at radio frequencies. Generally, impedance transformers have losses or other nonidealities that degrade their performance so that the optimum SNR is not realized. This is particularly true when the source impedance and the amplifier impedance are very different. It is thus worth exploring other methods of achieving the optimum SNR when there is a large impedance disparity.

Amplitude Components and Phase-Insensitive Amplification

At a particular point along a transmission line, the voltage of a signal propagating, say, to the right can be written in the form

$$V = V_1 \cos(\omega t) + V_2 \sin(\omega t) \tag{7}$$

where V_1 and V_2 will be referred to as the amplitude components, $\omega = 2\pi\nu$ is the angular frequency of the carrier, and t is time. Quantum mechanically (*10*), the amplitude components are noncommuting operators that satisfy the uncertainty relation

$$\Delta V_1 \, \Delta V_2 \geq Rh\nu B/2 \tag{8}$$

where ΔV_1 and ΔV_2 are the root-mean-square (rms) uncertainties or noise of V_1 and V_2, respectively, R is the impedance of the transmission line, and B is the bandwidth of the detector used to measure V_1 and V_2. Equation 8 thus sets a lower bound on the precision with which V_1 and V_2 can be simultaneously measured.

A consequence of Eq. 8 is that a noiseless phase-insensitive linear amplifier does not exist. In a noiseless phase-insensitive linear amplifier, the amplitude components of the output would be given by $V_1^{OUT} = G^{1/2}V_1$ and $V_2^{OUT} = G^{1/2}V_2$, where G is the amplifier's power gain (assuming that the amplifier's input and output impedances are the same). Suppose now that one performs a measurement on the output to the maximum precision allowed by Eq. 8, that is

$$\Delta V_1^{\mathrm{OUT}}\, \Delta V_2^{\mathrm{OUT}} = Rh\nu B/2 \tag{9}$$

Then, dividing through by the amplifier's gain to refer the signal to the input, one has made a measurement of V_1 and V_2 with an uncertainty product given by

$$\Delta V_1 \Delta V_2 = \frac{Rh\nu B}{2G} \tag{10}$$

This equation clearly contradicts Eq. 8 when G is greater than unity. A phase-insensitive linear amplifier must thus add noise to the amplified output to prevent violations of the Heisenberg uncertainty relation Eq. 8, with a noise power of $(G - 1)h\nu B/2$. The physical mechanism that gives rise to this noise is different for different amplifiers, but this noise must be there. For example, in laser and maser amplifiers, this noise arises from spontaneous emission. In nondegenerate parametric amplifiers, this noise is the frequency-converted and amplified noise coming from the idler resistor. If one considers a mixer as a kind of phase-insensitive linear amplifier (which does frequency conversion down to the intermediate frequency), this noise arises from the image port termination. Some other physical mechanism will be responsible for this noise in transistor amplifiers.

Besides adding noise to the amplified output, a phase-insensitive linear amplifier must emit noise from its input port. The amplifier can be viewed as a blackbody absorber which, when its impedance is matched to a signal source, will swallow up all the power delivered to it. Under the best of circumstances, the amplifier must thus at least emit vacuum fluctuations from its input port with a noise power of $h\nu B/2$. Phase-sensitive amplifiers will now be described that have better output noise properties than the phase-insensitive amplifiers (*11, 12*).

Phase-Sensitive Amplification

Consider an amplifier that amplified V_1 by a factor of $G^{1/2}$ and deamplified V_2 by the same factor:

$${}_1^{\mathrm{OUT}} = G^{1/2} V_1$$

$$V_2^{OUT} = \frac{V_2}{G^{1/2}} \tag{11}$$

This amplifier is phase-sensitive. To carry out such a transformation, the amplifier must have an internal clock that decides which amplitude component to amplify and which amplitude component to deamplify.

There will be no contradiction with Eq. 8 this time, because the decrease in the uncertainty of V_1, when the measurement of V_1^{OUT} is referred to input, is compensated for by the increase in the uncertainty of V_2 when V_2^{OUT} is referred to input. Phase-sensitive amplifiers can thus, in principle, be noiseless (*12*).

A simple mechanical phase-sensitive amplifier (*13*) is depicted in Fig. 4. The stick figure periodically changes the length of the pendulum in an effort to amplify the pendulum's motion. The stick figure pulls up on the string every time the pendulum is closest to the ground. The pendulum is moving fastest at this position and, consequently, work is done against centrifugal force as the stick figure pulls up on the string. This work increases the kinetic energy of the pendulum. When the pendulum momentarily comes to rest at the maximum excursion of its swing, the stick figure lowers his right hand, lengthening the string. Because the pendulum is momentarily at rest here, no work is done against centrifugal force. Thus, the stick figure continuously feeds energy into the pendulum, amplifying the initial swinging motion. This is an amplifier because the amount of swinging motion developed after a definite time has elapsed is proportional to the initial amount of swinging motion. (The output is proportional to the input.) This is a phase-sensitive amplifier because, if the stick figure chose to lengthen the string when the pendulum was closest to the ground and to shorten the string when the pendulum momentarily came to rest at the end of its excursion, energy would be extracted from the pendulum and the pendulum's motion would be decreased, that is, deamplified. Note also that the stick figure lifts the string twice for each round trip of the pendulum. The parametric amplifier is said to be pumped at twice the signal frequency. Clearly the pump, in this case the stick figure pulling up and down on the string, is the clock that decides which amplitude component of the initial swinging motion will be amplified and which component will be deamplified. Although this is a one-shot mechanical amplifier, it embodies the essential physics of degenerate parametric amplifiers.

An example of a parametric amplifier capable of continuous operation is shown in Fig. 5. The amplifier uses a time-dependent inductor, the inductance of which is varied at twice the resonant frequency of the tank circuit formed by the capacitor and inductor. The time-dependent inductance plays the same role as the time-dependent moment of inertia in the mechanical parametric amplifier described above. A circulator, a nonreciprocal device that works on the basis of the Faraday effect, directs incoming signals toward the tank circuit. Depending on the phase relation between the time variation of the inductor and the incoming signal, the signal is either amplified or deamplified and then delivered out the output port of the circulator. Such an amplifier is termed a negative resistance reflection amplifier, because the incoming signal enters the amplifier through the same port through which the amplified signal leaves the amplifier. The circulator thus serves the function of separating the input from the output. A parametric amplifier of this kind has recently been operated with very low noise (*14*). The time-dependent inductor consisted of a Josephson junction, the bias current of which was modulated at twice the signal frequency. The components of the electrical parametric amplifier described here can, in principle, be lossless. Phase-sensitive parametric amplifiers thus need not add any noise to the inputs they process.

When noise is fed into the input port of a parametric amplifier, with P_1 denoting the noise power in the V_1 amplitude component and P_2 denoting the noise power in the V_2 component, the noise power in the two amplitude components at the output is given by

$$P_1^{\mathrm{OUT}} = GP_1$$

$$P_2^{\mathrm{OUT}} = \frac{P_2}{G} \tag{12}$$

A phase-sensitive linear amplifier is thus capable of deamplifying one component of the incoming noise. This is called noise squeezing. If the incoming noise consists of vacuum fluctuations, the noise delivered out the output port will have less noise than the vacuum in one phase. In this case, the output is referred to as a squeezed state. Squeezed states have been successfully generated with optical degenerate parametric amplifiers (*15–20*). More recently, squeezed states have also been generated at microwave frequencies (*14*).

Back-Action Evasion

The back-action evader for which its operation is easiest to explain is shown in Fig. 6 (*21, 22*). Between the source and the detector amplifier, a circulator has been inserted. One port of the circulator is terminated with a phase-sensitive amplifier operated in the negative resistance reflection mode. Let P_{B1} and P_{B2} denote the power in the two amplitude components of the back-action noise emitted by the detector amplifier. This back-action noise is directed by the circulator to the phase-sensitive amplifier. Letting P_{C1} and P_{C2} denote the power in the amplitude components of the noise leaving the reflection amplifier, one obtains

$$P_{C1} = GP_{B1}$$

$$P_{C2} = \frac{P_{B2}}{G} \tag{13}$$

This noise now propagates towards the impedance mismatch. A fraction R of this noise is reflected off of the impedance mismatch. A fraction T of the noise coming from the signal source is transmitted through the impedance mismatch. Letting P_{N1} and P_{N2} denote the power of the two amplitude components of the noise coming from the signal source, the noise power P_{AN1} and P_{AN2} in the amplitude components of the signal propagating toward the detector amplifier is then

$$P_{AN1} = TP_{N1} + RGP_{B1} \tag{14}$$

$$P_{AN2} = TP_{N2} + R\frac{P_{B2}}{G}$$

A fraction T of the signal power is transmitted through the impedance mismatch. Letting P_{S1} and P_{S2} denote the power in the two amplitude components of the signal propagating from the source and letting P_{AS1} and P_{AS2} denote the signal power transmitted through the impedance mismatch, one has

$$P_{AS1} = TP_{S1} \tag{15}$$

$$P_{AS2} = TP_{S2}$$

The signal and noise propagating from the impedance mismatch to the circulator is directed by the circulator toward the input port of the detector amplifier. Hence, at the input of the detector amplifier, the two amplitude components have the following respective SNRs:

$$SNR1 = \frac{TP_{S1}}{TP_{N1} + RGP_{B1}} \tag{16}$$

$$SNR2 = \frac{TP_{S2}}{TP_{N2} + R\frac{P_{B2}}{G}} \tag{17}$$

As the gain G of the reflection amplifier is made large, *SNR1* for the V_1 amplitude component goes to 0; that is, the signal becomes buried in the noise amplified by the reflection amplifier. However, from Eq. 17, *SNR2* for the V_2 amplitude component comes very close to the ideal SNR, given by Eq. 1, as G becomes large. Thus, by the squeezing of one amplitude component of the back-action noise, the SNR for this component can be made as good as that which can be realized by impedance matching, but the signal in the other amplitude component must be sacrificed. If the detector amplifier is a phase-sensitive amplifier that amplifies V_2, then the amplifier need not add noise to this amplitude component, and the SNR given by Eq. 17 could be realized at the output of the amplifier with large power gain. Also, as a side benefit, the amplifier would deamplify the noisy amplitude component V_1. To accomplish this, one must synchronize the internal clocks that determine which amplitude components are amplified and deamplified by the noise squeezer and the detector amplifier. This is most easily accomplished by pumping of the squeezer and the detector amplifier with the same oscillator.

The back-action evader described above is perhaps the simplest to explain. It should be remembered that the transmission lines in Fig. 6 are not essential. They are included for easy visualization of the power flow and of the events at the impedance mismatch. The length of the transmission lines can be shrunk to 0. A variety of back-action evading circuits, distinct from that of Fig. 6, have been devised (*6–8, 10, 21–25*). Many of these circuits do not use circulators. Back-action evasion has been demonstrated at optical frequencies (*26–28*), and work is in progress on electromechanical back-action evaders with signal frequencies in the kilohertz frequency region (*29–32*). The development of low-frequency back-action evaders is motivated by the problem of gravitational radiation detection.

Finally, back-action evasion is capable of a few tricks that would be impossible to accomplish by impedance matching. When operated in a regime where vacuum fluctuation noise is the dominant noise, back-action evasion sets up strong quantum mechanical correlations among the V_2 amplitude components of the signals on either side of the impedance mismatch. To see this, consider an impedance mismatch for which the transmission coefficient and the reflection coefficient are equal, $T = R = 1/2$. Then when the back-action noise is highly squeezed, the V_2 amplitude component of the signal transmitted through the impedance mismatch becomes essentially a carbon copy of the same amplitude component reflected off of the impedance mismatch back toward the source. Quantum mechanically, these two signals are highly correlated, and, with the use of circulators, one can have access to both. It has been shown that the strong quantum mechanical correlations of these signals can be used, in principle, to generate quantum superpositions of macroscopically distinct quantum states called Schrodinger cats (*33, 34*).

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