The Quark-Hadron Transition in Cosmology and Astrophysics

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A transition from normal hadronic matter (such as protons and neutrons) to quark-gluon matter is expected at both high temperatures and densities. In physical situations, this transition may occur in heavy ion collisions, the early universe, and in the cores of neutron stars. Astrophysics and cosmology can be greatly affected by such a phase transition. With regard to the early universe, big bang nucleosynthesis, the theory describing the primordial origin of the light elements, can be affected by inhomogeneities produced during the transition. A transition to quark matter in the interior by neutron stars further enhances our uncertainties regarding the equation of state of dense nuclear matter and neutron star properties such as the maximum mass and rotation frequencies.

NORMAL NUCLEAR MATTER IS COMPOSED OF WHAT ARE known to be composite particles, namely, nucleons. Similarly the low-energy interactions of nucleons are described by the exchange of strongly interacting mesons such as π's, ρ's, and so on. At a more fundamental level, the theory of strong interactions is described by quantum chromodynamics (QCD) in terms of quarks and gluons. Experimentally, QCD is successful in describing the interactions of quarks and gluons at high energies. Particles in QCD are known to be asymptotically free, that is at high energies quarks and gluons are weakly coupled, while at low energies they appear to be confined in hadrons, that is, baryons (including nucleons) and mesons. Although the low-energy features of QCD and confinement have difficulties in predicting physical quantities such as baryon and meson masses, it is widely believed that at either high temperatures or high densities, normal (confined) nuclear matter undergoes a phase transition to an unconfined quark-gluon state (*1*).

The best quantitative evidence for a phase transition is found (*2*) by an application of lattice gauge theory to QCD. Monte Carlo simulations (*3*) of lattice QCD showed that indeed a phase transition takes place though the order of the transition will depend on the gauge group and whether or not fermions (quarks) are included in the calculations (*1*). Estimates for the critical temperature and density are $T_c \approx 150$ to 200 MeV, $\rho_c \approx 2$ to 3 GeV fm^{-3} (1 fm = 10^{-15} m).

There are three known physical environments in which the above conditions on the temperature and density can be achieved: heavy ion collisions (*4, 5*), the early universe (*6*), and the cores of neutron stars (*6*). The best chance for actually studying the properties of the quark-gluon phase are in the laboratory. Although current heavy ion experiments are approaching the necessary conditions, there has been no clear signal as yet that the quark-gluon phase has been observed (*5*). In this article, I will discuss our current understanding of the quark-hadron transition as it applies to the early universe and to the cores of neutron stars.

Conditions for a Quark/Hadron Transition

In the early universe (*6–10*), the transition from an initial unconfined quark-gluon phase to the confined hadronic phase took place at about $t \approx 10^{-5}$ s after the big bang. Recently (*11–18*), cosmological interest in the quark-hadron transition has been focused on the later time of about 1 s to 2 min, which is the era of nucleosynthesis (*19*). Perturbations formed during quark-hadron transition (*10, 20–24*) may affect the standard model predictions of the abundances of the light elements.

In the interior of neutron stars, the extremely high densities present may be sufficient to convert matter to the quark-gluon phase (*6, 25–30*). In this case the final product of the stellar evolution of massive stars would be a quark core with a (large) hadronic crust. Properties of neutron stars such as the maximum mass and rotation frequency are strongly dependent on the nuclear equation of state. The already uncertain nuclear equation of state at high densities is further complicated with the possibility of a phase transition to quark matter.

Before discussing the early universe or neutron star cores, it will be useful to briefly describe the (naive) thermodynamic picture for the quark-hadron phase transition. In the quark-gluon phase, one can use the simple bag model (viewing hadrons as a bag containing quarks and gluons) equation of state for the pressure, P_Q, and energy density, ρ_Q

$$P_Q = \frac{\pi^2}{90}\left[2\,(N_C^2 - 1) + \frac{7}{2} N_C N_f\right] T^4 - B \qquad (1)$$

$$\rho_Q = 3P_Q + 4B \qquad (2)$$

where N_C is the number of colors corresponding to a SU(N_C) gauge group for QCD ($N_C = 3$), N_f is the number of light quark flavors ($N_f = 3$ for up, down, and strange quarks), and B is the bag constant representing the difference in vacuum energy between the two phases. (I am using units such that $\hbar = c = k_B = 1$). One can in addition add higher order corrections to Eq. 1 (*31*). With this choice of an equation of state the critical temperature is almost

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completely determined by the value of B which takes reasonable values between 50 and 450 MeV fm^{-3}.

The equation of state for the hadron phase is far less certain. The simplest possibility is to compute the pressure for non-interacting hadronic states. In Fig. 1 (*32*), the pressure (P/T^4) is shown as a function of temperature for the quark-gluon phase (labeled Q) for two choices of the bag constant: B = 50 MeV fm^{-3} (solid) and B = 450 MeV fm^{-3} (dashed). The hadronic pressure for known particles is shown by the curve labeled H (solid, dashed, and dotted portions). Notice the steep rise in the hadronic pressure, which is due to the contribution of the multitude of known hadronic states.

In this naive Maxwellian construction for a first-order phase transition, phase equilibrium is achieved when $P_Q = P_H$ (the chemical potentials are all assumed to be zero). One notices immediately a problem inherent in Fig. 1. The quark and hadron curves cross twice, implying a second transition back to hadronic matter at very high temperatures. This is, of course, nonsense. It is highly uncertain what happens to the hadronic curve beyond the low-temperature crossing (if the hadronic phase even exists there).

One possibility is that our approximation of non-interacting hadrons at high density is bad. (This is most certainly true.) One can therefore try to incorporate interactions as mean fields as was done elsewhere (*8*). In Fig. 2, the same phase diagram is shown, but the hadronic curves assume (for both phenomenological and theoretical reasons) an exponential hadronic mass spectrum of the form (*33*)

$$n(m) \sim Cm^{-a}e^{m/T_0} \qquad (3)$$

where $n(m)dm$ is the number of hadronic states with masses between m and $m+dm$; C and a are constants and T_0 is known as the Hagedorn temperature (*33*). In Fig. 2, a = 1.5 T_0 = 160 MeV and C labels the two hadronic curves. Also included are mean field potentials (*8, 32, 34*) representing hard-core interactions. The striking difference between Figs. 1 and 2 is the flattening of the hadronic pressure. Now there is only a single crossing, and we have a picture of a physically sensible phase transition.

Another possibility is that there exists a critical temperature T_χ above which the hadronic phase no longer exists. One of the difficulties in discussing rigorously the confinement transition in terms of a low-energy effective field theory, is the absence of a local order parameter defining the phase. In lattice QCD, the Wilson loop (*1*) serves as an order parameter and it may well turn out that quark and gluon condensates which break the chiral and scale symmetries of the underlying QCD Lagrangian serve as the confinement order parameter as well (*35*). Campbell *et al.* (*32*) studied the behavior of the expectation value of the gluon condensate $\langle\chi\rangle$ in relation to the question of confinement. Hadronic masses are expected to scale with $\langle\chi\rangle$, so that the hadronic phase is only realized when $\langle\chi\rangle \neq 0$. The vanishing of the gluon condensate signals the end point of the hadronic phase. These points are shown in Figs. 1 and 2 by the X's on the hadronic curves. Thus even in the case for the known free hadronic states, the double crossing may be avoided simply because the hadronic phase ceases to exist above a temperature T_χ in which $\langle\chi\rangle = 0$.

The Early Universe

The one environment in which we can be most convinced that a transition to quark-gluon matter did occur is in the early universe. The standard big bang model is based on an initial hot and dense epoch, which expands and cools as the universe evolves. Indeed Einstein's equations and the conservation of energy together with a model of a homogeneous and isotropic radiation-dominated universe yield a simple relation between the temperature of the thermal radiation and time

$$t_s\, T^2_{MeV} \simeq 2.4\,[N(T)]^{-1/2} \qquad (4)$$

where the time t is measured in seconds, T in megaelectron volts, and N is the number of relativistic degrees of freedom of all particles present at temperature T (typically this is the number of spin and color degrees of freedom for particles with masses $m << T$).

At $T \simeq$ 200 MeV, we expect the confinement phase transition to have occurred so that at lower temperatures, the constituents of the universe are hadrons, leptons, and photons. The corresponding age of the universe at this time is $t \simeq 10^{-5}$ s. It is not until $t \approx 1$ or at $T \approx 1$ MeV that the processes leading to nucleosynthesis become important.

Second to the discovery of the microwave background radiation, the consistency between the calculated abundances of the light

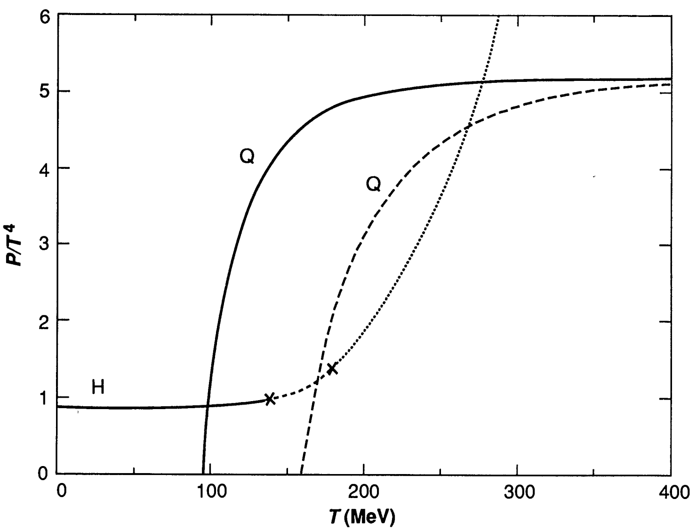

Fig. 1. The pressure P/T^4 versus T for hadrons (H) using known particles with $m \lesssim 2.5$ GeV and for quarks and gluons (Q) for B = 50 MeV fm^{-3} (solid), B = 450 MeV fm^{-3} (dashed). T_χ is represented by an X on the hadron curve at the end of the solid portion when B = 50 MeV fm^{-3} and at the end of the dashed portion when B = 450 MeV fm^{-3}. The dotted portion neglects the effect of a gluon condensate.

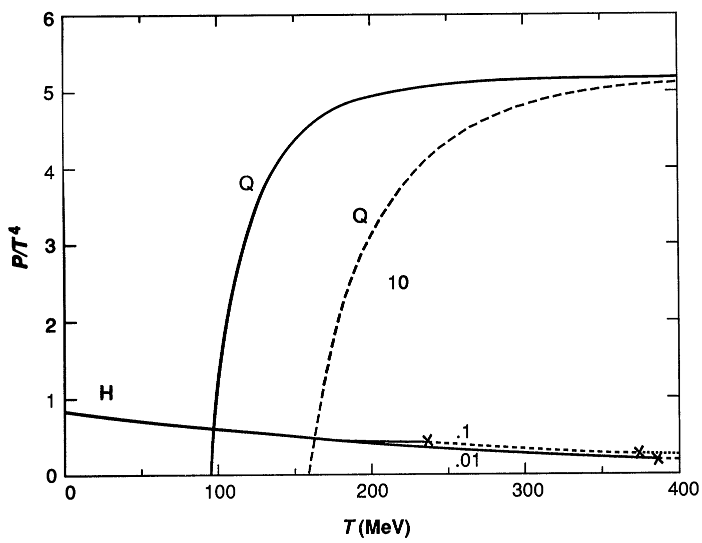

Fig. 2. The pressure P/T^4 versus T for hadrons (H) using a Hagedorn spectrum with a = 1.5 and T_0 = 160 MeV. The curves are labeled by their values of C. Mean field potentials have been included. The quark curves are as in Fig. 1.

elements D, ^{3}He, ^{4}He, and ^{7}Li, and the observational determination of these abundances is the strongest evidence supporting the big bang theory.

At high temperatures $T >> 1$ MeV the weak interaction rates for the processes

$$\begin{aligned} n + \nu_e &\leftrightarrow p + e^- \\ n + e^+ &\leftrightarrow p + \bar{\nu}_e \\ n &\leftrightarrow p + e^- + \bar{\nu}_e \end{aligned} \tag{5}$$

were all in equilibrium, that is, their interaction rates Γ were larger than the expansion rate of the universe which is determined by the Hubble parameter H. In equilibrium, the neutron to proton ratio is essentially controlled by the Boltzmann factor so that

$$(n/p) = \exp(-\Delta m/T) \tag{6}$$

where n and p are the number density of neutrons and protons, respectively, and $\Delta m = m_n - m_p$ is the neutron-proton mass difference. For $T >> \Delta m$, $(n/p) \approx 1$.

At temperatures $T >> 1$ MeV, nucleosynthesis cannot begin to occur even though the rate for forming the first isotope, deuterium, is sufficiently rapid. To begin with, at $T \gtrsim 1$ MeV deuterium is photodissociated because the photon energy $E_\gamma > 2.2$ MeV (the binding energy of deuterium; $\bar{E} = 2.7T$ for a blackbody). Furthermore, the density of photons is very high, $n_\gamma/n_B \approx 10^{10}$, and the high-energy tail of the photon distribution will continue to dissociate deuterium at lower temperatures. Thus the onset of nucleosynthesis will depend on the quantity:

$$\eta^{-1} \exp[-2.2 \text{ MeV}/T] \tag{7}$$

where $\eta = n_B/n_\gamma$. When this quantity becomes $\lesssim O(1)$, the rate for $p + n \rightarrow D + \gamma$ finally becomes greater than the rate for dissociation $D + \gamma \rightarrow p + n$. This occurs when $T \approx 0.1$ MeV.

Because the rates for processes in Eq. 5 freeze out at $T \approx 1$ MeV (that is, their interaction rate falls below the expansion rate), the neutron to proton ratio must be adjusted from its equilibrium value. When freeze out occurs, the ratio (n/p) is relatively fixed at $(n/p) \approx 1/6$.

The equilibrium value is adjusted by taking into account the free neutron decays up until the time at which nucleosynthesis begins. This reduces the ratio to $(n/p) \approx 1/7$. Because virtually all the neutrons available end up in a deuterium which gets quickly converted to ^{4}He, we can estimate the ratio of the ^{4}He nuclei formed compared with the number of protons left over

$$(N_{^4\mathrm{He}}/N_H) = (n/p)/2[1 - (n/p)] \tag{8}$$

or more importantly the ^{4}He mass fraction

$$Y_p = 2(n/p)/[1 + (n/p)] \tag{9}$$

for $(n/p) \approx 1/7$, we estimate that $Y_p = 0.25$ which is very close to the observed value. The actual calculated value of Y_p will depend on a numerical calculation which runs through the complete sequence of nuclear reactions.

The calculated abundances ^{4}He (by mass) D, ^{3}He (by number), and ^{7}Li (by number) are shown (*19*) in Fig. 3, as a function of the baryon to photon ratio η. The observational bounds: $Y_p = 0.23 \pm 0.01$, D/H $> 1.8 \times 10^{-5}$, (D + ^{3}He)/H $< 10^{-4}$, and ^{7}Li/H $< 1.4 \times 10^{-10}$ constrain the baryon to photon ratio to be $\eta = (2.8 \text{ to } 3.3) \times 10^{-10}$. [Uncertainties (*36*) in the nuclear cross sections may allow a slightly larger value of η, $\eta \lesssim 4 \times 10^{-10}$.] The fact that all of the light element abundances can be explained by a single set of parameter values in the simplest possible model (standard big bang) is strong evidence for the model itself.

The importance of the constraint on η is that it can be converted to an upper limit on the total baryon mass density or more importantly, on the fraction of closure density made up by baryons. A key question in cosmology today is the overall density of the universe, ρ. The cosmological density is often referred to with respect to a closure density $\rho_{c1} = 1.88 \times 10^{-29} h_0^2$ g cm^{-3} where $H_0 = 100\ h_0$ km Mpc^{-1} s^{-1} is the present value of the Hubble parameter (observationally, h_0 is known to be in the range ~ 0.4 to 1.0). The critical density is the density necessary to close the universe, that is the ratio $\Omega \equiv \rho/\rho_C$ distinguishes between closed ($\Omega > 1$), flat ($\Omega = 1$), and open ($\Omega < 1$) spatial geometries. It is widely believed (by theorists) that today $\Omega = 1$. One can relate η to the fraction of Ω in the form of baryons by

$$\eta \approx 2.8 \times 10^{-8} h_0^2 \Omega_B (2.7/T_0)^3 \tag{10}$$

where $T_0 \approx 2.7$ K is the temperature of the cosmic microwave background today. The upper limit of $\eta \leq 3.3 \times 10^{-10}$ implies an upper limit $\Omega_B \lesssim 0.08$ indicating that baryons make up less than 8% of the closure density. Thus if $\Omega = 1$, nucleosynthesis implies the need for nonbaryonic dark matter.

During the quark-hadron transition, it is expected (*20*) that perturbations in the baryon number density are formed which will lead to inhomogeneities in the *n/p* ratio at the time of nucleosynthesis. If we remember that the baryon chemical potential is small but not zero, then it is straightforward to compute the net baryon density in terms of the chemical potential. In a simple model

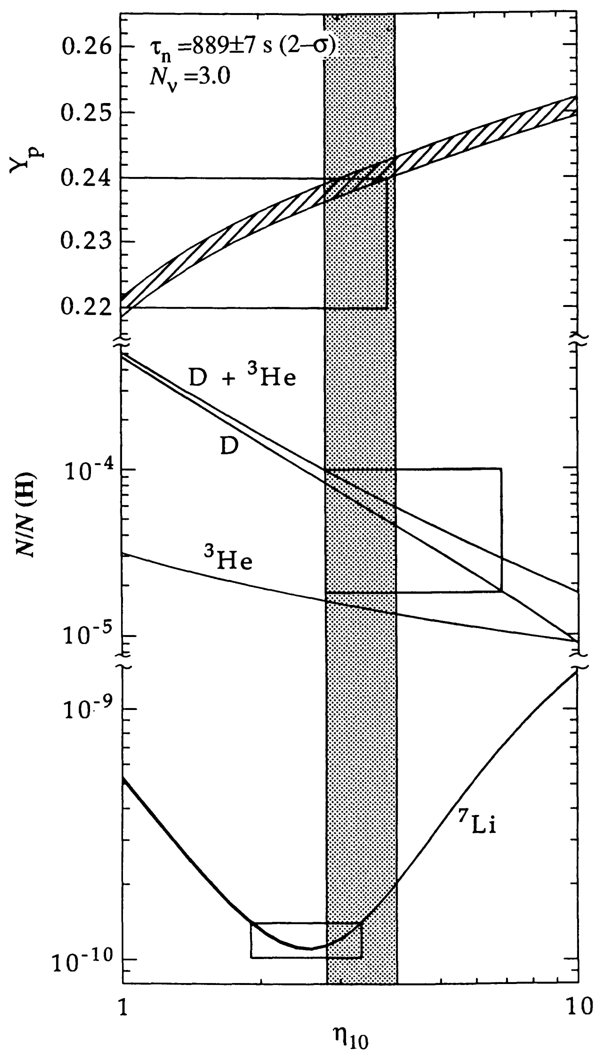

Fig. 3. The mass fraction of ^{4}He, Y_p, and the abundances by number of D, ^{3}He, D + ^{3}He, and ^{7}Li as a function of $\eta_{10} = \eta \times 10^{10}$. The vertical band delimits the range for η consistent with the observations.

containing only nucleons and massless up and down quarks, we would find:

$$n_{B_H} \simeq \frac{8\mu_H}{T}\left(\frac{m_N T}{2\pi}\right)^{3/2} e^{-m_N/T} \quad (11a)$$

$$n_{B_Q} = \frac{2}{9}\mu_H T^2 \quad (11b)$$

for the baryon densities in the hadronic and quark phases where chemical equilibrium ($\mu_H = 3\mu_Q$) has been assumed. The density contrast, n_{B_Q}/n_{B_H} is strongly dependent on the confinement transition temperature, and could be as large as ~200 for $T_c \simeq 100$ MeV. Realistically one would expect n_{B_Q}/n_{B_H} to range from ~7 to 100 for $T_c \geq 100$ MeV (*24*). These numbers of course are assumption-dependent and different assumptions regarding the nonequilibrium transport of baryon number near the phase boundry could increase or decrease these estimates (*22*).

Given the presence of baryon inhomogeneities in the early universe, because of the proton's electric charge, there is a preferential diffusion of neutrons versus protons out of the high density fluctuations (*11*). With a more uniform neutron density, the persisting inhomogeneity in the proton density results in a universe with both inhomogeneities and a variable *n/p* ratio. The result is that nucleosynthesis in the high density regions occurred with a low *n/p* ratio while the low density region had a high *n/p* ratio.

In the first set of calculations (*11*), it was claimed that such mixed conditions might allow $\Omega_B = 1$, while still fitting the observed abundances of ^{4}He, D, and ^{3}He, but with an overproduction of ^{7}Li. It was argued that perhaps depletion processes may have occurred to reduce an initial high primordial ^{7}Li abundance to one that conforms with the observations of old population II dwarf stars. Thus with only the additional assumption of an effective depletion of ^{7}Li, one of the main conclusions from nucleosynthesis could be altered. One should note that recent work (*37*) on the effects of depletion do not allow for an initial ^{7}Li abundance significantly above the observed population II abundances. Under certain conditions however, the ^{7}Li abundance may be reduced toward the end of nucleosynthesis by late time dissipation processes (*38*).

The initial calculations, however assumed that first neutron diffusion took place followed by the period of nucleosynthesis. It was later pointed out (*13*) that during nucleosynthesis, as neutrons are depleted at a higher rate in the high density regions, there will be neutron diffusion back into the high density regions. It was even argued that for certain phase transition parameters (such as separation of nucleation sites $\ell \approx 10$ m at the time of the transition) the back diffusion could lower the ^{7}Li abundance. However, more detailed diffusion calculations showed (*14–16*) that not only could ^{7}Li be affected but ^{4}He as well. Indeed it was found (*15*) that, by and large, nucleosynthesis with $\Omega_B = 1$ and with a baryon density contrast, $R \approx 100$, tended to overproduce both ^{4}He and ^{7}Li. As back diffusion evens out the effects of the initial fluctuation, the averaged result should approach the homogeneous value which leads to excesses in both ^{4}He and ^{7}Li for $\Omega_B = 1$. Furthermore, any narrow range of parameters, such as those which yield relatively low lithium and helium, are unrealistic because in any realistic phase transition there is a distribution of parameter values (distribution of nucleation sites, separations, density fluctuations, and so on). Therefore, narrow minima are washed out which would bring the ^{7}Li and ^{4}He values back up to excessive levels for parameter values with $\Omega_B \approx 1$. This is an important point because diffusive effects are only important in lowering the nuclear abundances in a narrow window of parameter space. The recent study by Meyer *et al.* (*39*) finds little change when averaging over a distribution of fluctuation separations.

Indeed the latest calculations (*15–18*) all showed that for $\Omega_B = 1$, only the D abundance can be brought into agreement with observations when the distance scale of the inhomogeneities, $\ell \geq 30$ m. Though the standard model constraints on η can be modified, the modification is rather limited (*17*) as can be seen in Fig. 4, allowing only a slight variation in the lower limit on η. One should bear in mind that these results assume that a significant baryon density contrast $R \geq 100$ is present. The value of R is also very sensitive to the transition temperature. The assumption of chemical equilibrium yields values of $R < 100$ for $T > 100$ MeV. Higher values of R require additional assumptions (*40*).

Furthermore, the value of ℓ is also very sensitive to T_c and the surface tension, σ, of the phase interface (*22, 41*); $\ell \simeq 3.7 \times 10^4$ m $(\sigma/\text{MeV}^3)^{3/2}\,(T_c/\text{MeV})^{-13/2}$. For values of $\sigma^{1/3} \simeq 70$ MeV estimated by Fahri and Jaffe (*42*) which agree with the effective field theory model estimates (*32*), $\ell \lesssim 0.7$ m for $T_c \gtrsim 100$ MeV. Thus it seems unlikely at this time to expect values of parameters such as R and ℓ so as to yield sizable modifications to standard big bang nucleosynthesis.

Neutron Stars

The quark-hadron phase transition may play an important role in the dynamics of neutron stars. Neutron star properties such as the maximum mass and rotation frequency are strongly dependent on the equation of state of dense nuclear matter. A phase transition (and hence a phase boundary) in the interior of a star will therefore greatly affect these two observable properties.

Given an equation of state, Einstein's equation together with the assumption of hydrostatic equilibrium lead to the Tolman-Oppenheimer-Volkoff equation (*43*)

$$\frac{dp}{dr} = -(\rho + p)(GM + 4\pi r^3 Gp)(r^2 - 2GrM)^{-1} \quad (12a)$$

$$\frac{dM}{dr} = 4\pi\rho r^2 \quad (12b)$$

where ρ, p, and M are all to be taken as functions of r, the radial distance to the center of the star. By solving these equations (typically numerically) using $p(\rho)$ as obtained from calculations, one can obtain $M(\rho)$, $p(r)$, and $\rho(r)$, for a given value of the central pressure p or energy density [it is assumed of course that $M(0) = 0$]. A solution to these equations does not, however, guarantee stability. Typically, stable configurations will be found when $dM/d\rho(0) > 0$.

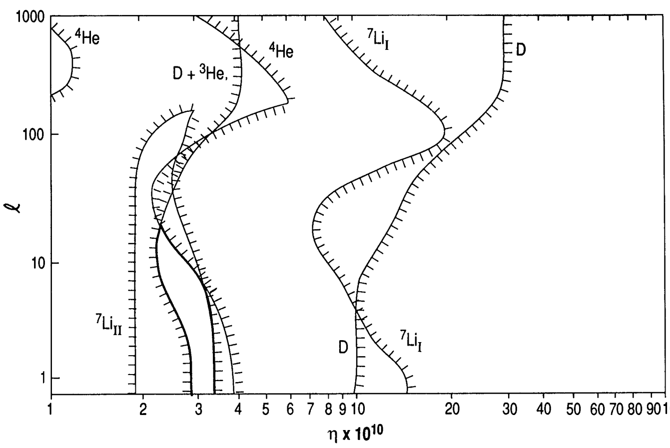

Fig. 4. Allowed region in the $\ell - \eta$ plane from the observational constraints on D, ^{3}He, ^{4}He, and ^{7}Li (from both population I and II stars). The area outlined by bold lines indicates the only region consistent with all observations.

Thus there will be a maximum central density and a maximum mass when $dM/d\rho(0) = 0$ and $d^2M/d\rho(0)^2 < 0$. At the maximum central density, it is straightforward to compute the maximum keplerian rotation frequency by balancing centripetal and gravitational forces at the surface of the star. The general relativistic form for the maximum rotation frequency takes the approximate form (*44*)

$$\Omega_K = 0.7 \times 10^4\ s^{-1}\ (M/M_\odot)^{1/2}\ (10\ km/R)^{3/2} \quad (13)$$

for a star of mass M and radius R.

The properties of normal nuclear matter near normal nuclear matter density ($n_0 \approx 0.15\ fm^{-3}$, $\rho_0 \simeq 3 \times 10^{14}\ g\ cm^{-3}$) are reasonably well studied (*45*). One can, for example, work in the context of a relativistic nuclear mean field theory (*46*) to obtain the equation of state at nuclear matter density. Interactions between nucleons can be accounted for by the inclusion of the vector meson exchanges of ρ's and ω's and two-pion exchange is simulated by the scalar field σ. The couplings of the meson fields to nucleons as well as the σ self-coupling can all be determined by nuclear matter properties. The resulting equation of state then resembles the well-known Friedman-Pandharipande (*47*) equation of state. In Fig. 5, the neutron star mass as a function of central density is shown (*30*) by the curve labeled N for this equation of state.

It is not realistic to expect this equation of state to be valid at higher densities when hyperons (excited baryon states) are expected to be present (*30, 48, 49*). The problem, however, with including additional baryon states (the entire low-lying baryon octet, for example) and φ-meson exchange for completeness, as was done by Kapusta and Olive (*30*), is that we have now inserted many more coupling constants which are extremely difficult to determine experimentally. Unfortunately, the neutron star mass curves are quite sensitive to these couplings.

In Fig. 5, the sensitivity to the σ-hyperon couplings are shown (*30*). The curves are labeled by the ratio $g_{\sigma\Lambda}/g_{\sigma N}$. The other couplings have been fixed by the SU(3)-expected values. Clearly the uncertainties in these couplings translate into large uncertainties in neutron star masses.

The possibility of a quark core within a neutron star adds to the uncertainty. Much of the work on neutron stars with quark cores was concerned with questions regarding the stability of these stars (*25–30*). The quark equation of state can be written as

$$P_Q = \frac{1}{4\pi^2}\mu^4 - B \quad (14)$$

assuming $T = 0$ and massless quarks, notice the similarity to Eq. 1. One can again add in higher order corrections (*31*). Using Eq. 14 one can construct a quark interior at high density again by a Maxwell construction, such that $P_Q = P_H$ with chemical equilibrium for both baryon number and electric charge. In Fig. 6, the neutron star mass is plotted versus the central density for the preferred equation of state, labeled 0.48φ [see (*30*) for details]. Also plotted in Fig. 6, are the neutron star masses assuming a quark interior. The curves are labeled by the critical baryon density for the quark phase in units of nuclear density $n_0 = 0.153\ fm^{-3}$. Similar to the case of a high-temperature transition, the value of the bag constant determines the critical baryon density. As one sees from the figure, in addition to the sensitivity of the neutron star masses, these hybrid stars are only stable for low ($n_c \lesssim 4n_0$, though the exact number is model dependent) critical densities.

Because the maximum mass and radius of the neutron star are dependent on the presence of a quark core, it is not surprising that the maximum rotation frequencies will also be sensitive to the star's interior. For example, for the models displayed in Figs. 5 and 6, the maximum rotation frequency varies from $\Omega_K = 0.76 \times 10^4\ s^{-1}$ for the quark core with $n_c = 3n_0$ to $\Omega_K = 1.3 \times 10^4\ s^{-1}$, for the case with no quark core but the soft equation state with (the probably unrealistically high value) $g_{\sigma\Lambda}/g_{\sigma N} = 1.187$. The pure nucleon equation of state gives $\Omega_K = 0.96 \times 10^4\ s^{-1}$.

Future Prospects

Despite the uncertainties concerning the details of the quark-hadron transition, it is possible to extract qualitative and in some cases quantitative effects of the transition in the early universe and in cores of neutron stars. Unfortunately, uncertainties in the nuclear (hadronic) equation of state will make it difficult for astronomical-observations to add much insight at this time on the nature of the transition. The immediate hope is that experiments at CERN and

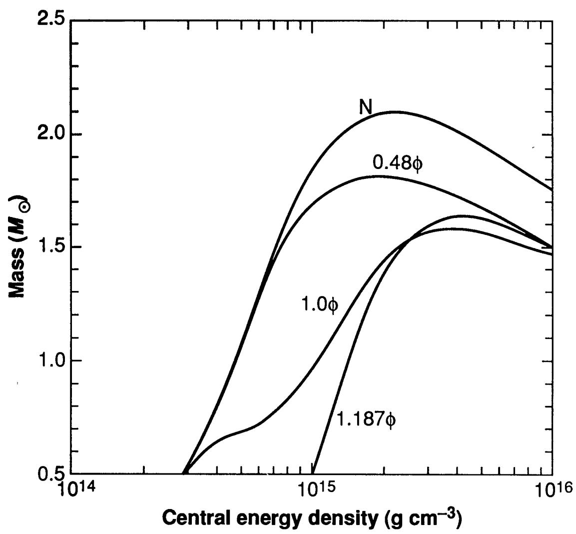

Fig. 5. Stellar mass as a function of the central energy density ρ(0), for a pure nucleon equation of state (N) and for equations of state which include hyperons. The curves are labeled by the ratio $g_{\sigma\Lambda}/g_{\sigma N}$. The φ signifies that vector φ exchange was included along with σ, ρ, and ω exchange as mean fields.

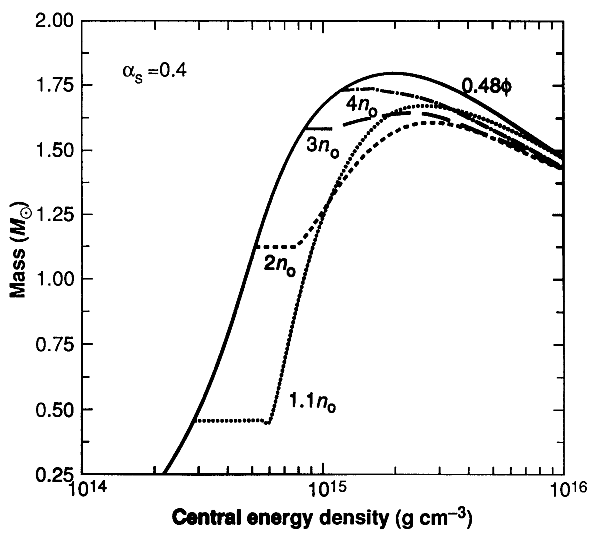

Fig. 6. Stellar mass as a function of the central energy density ρ(0) for the equation of state 0.48 φ with a first-order phase transition to quark matter at $n = 1.1n_0$ (dotted), $2n_0$ (small dashed), $3n_0$ (large dashed), $4n_0$ (dot-dashed), and with no transition at all (solid). The strong fine structure constant was taken as $\alpha_s = 0.4$.

future heavy ion facilities such as RHIC will be able to unambiguously discover a signal for the quark-gluon phase. Once the phase transition is better understood, it will clearly shed new light on the early stages of the universe and the final state of massive stars.

REFERENCES AND NOTES

1. For reviews, see: B. Svetitsky, *Phys. Rep.* **132**, 1 (1986); F. Karsch, *Nucl. Phys. (Proc. Suppl.)* **B9**, 357 (1989).
2. A. M. Polyakov, *Phys. Lett.* **B72**, 477 (1978); L. Susskind, *Phys. Rev.* **D20**, 2610 (1979).
3. M. Creutz, *Phys. Rev.* **D21**, 2308 (1980); L. D. McLerran and B. Svetitsky, *Phys. Lett.* **B98**, 195 (1981); J. Kuti, J. Polony, K. Szlachanyi, *ibid.*, p. 199; J. Engels, F. Karsch, J. Montvay, H. Satz, *ibid.* **B101**, 89 (1981); K. Kajantie, C. Montonen, E. Pietarinen, *Z. Phys.* **C9**, 253 (1981); for a recent review, see A. Ukawa, *Nucl. Phys. B (Proc. Suppl.)* **10A**, 66 (1989).
4. S. A. Chin, *Phys. Lett.* **B78**, 552 (1978); R. Anishetty, P. Koehler, L. McLerran, *Phys. Rev.* **D22**, 2793 (1980); J. D. Bjorken, *ibid.* **D27**, 140 (1983).
5. For recent experimental results, see *Z. Phys.* **C38** (1988) and *Nucl. Phys.* **A498** (1989).
6. J. C. Collins and M. J. Perry, *Phys. Rev. Lett.* **34**, 1353 (1975).
7. R. V. Wagoner and G. Steigman, *Phys. Rev.* **D20**, 825 (1979).
8. K. A. Olive, *Nucl. Phys.* **B190**, 483 (1981).
9. C. G. Kallman, *Phys. Lett.* **B112**, 213 (1981); E. Suhonen, *ibid.* **B119**, 81 (1982); J. Lodenquai and V. Dixit, *ibid.* **B124**, 317 (1983); D. N. Schramm and K. A. Olive, *Nucl. Phys.* **A418**, 289c (1984); T. De Grand and K. Kajantie, *Phys. Lett.* **B147**, 273 (1984).
10. M. Crawford and D. N. Schramm, *Nature* **298**, 538 (1982); C. Hogan, *Phys. Lett.* **B133**, 172 (1983).
11. J. H. Applegate and C. J. Hogan, *Phys. Rev.* **D31**, 3037 (1985); *ibid.* **D34**, 1938 (1986); _____, R. J. Scherrer, *ibid.* **D35**, 1151 (1987); C. Alcock, G. M. Fuller, G. J. Mathews, *Astrophys. J.* **320**, 439 (1987).
12. H. Reeves, in *Proceedings of the International School of Physics, Enrico Fermi*, Varenna Italy, J. A. Audouze and F. Melchiorri, Eds. (North-Holland, Amsterdam, 1987); K. A. Olive, *Nature* 330, 770 (1987).
13. R. A. Malaney and W. A. Fowler, *Astrophys. J.* **333**, 14 (1988).
14. H. Kurki-Suonio *et al.*, *Phys. Rev.* **D38**, 1091 (1988).
15. H. Kurki-Suonio and R. A. Matzner, *ibid.* **D39**, 1046 (1989); *ibid.* **D42**, 1047 (1990).
16. N. Terasawa and K. Sato, *Phys. Rev.* **D39**, 2893 (1989); *Prog. Theor. Phys.* **81**, 254 and 1085 (1989).
17. H. Kurki-Suonio, R. A. Matzner, K. A. Olive, D. N. Schramm, *Astrophys. J.* **353**, 406 (1990).
18. G. J. Mathews, B. Meyer, C. R. Alcock, G. M. Füller, *ibid.* **358**, 36 (1990).
19. J. Yang, M. S. Turner, G. Steigman, D. N. Schramm, K. A. Olive, *ibid.* **281**, 493 (1984); A. M. Boesgard and G. Steigman, *Annu. Rev. Astron. Astrophys.* **23**, 319 (1985); K. A. Olive, D. N. Schramm, G. Steigman, T. Walker, *Phys. Lett.* **B236**, 454 (1990); T. Walker, G. Steigman, D. N. Schramm, K. A. Olive, K. Kang, preprint UMN-TH-826/90 (University of Minnesota, Minneapolis, 1990).
20. E. Witten, *Phys. Rev.* **D30**, 272 (1984).
21. K. Iso, H. Kodama, K. Sato, *Phys. Lett.* **B169**, 337 (1986).
22. G. M. Füller, G. J. Mathews, C. R. Alcock, *Phys. Rev.* **D37**, 1380 (1988).
23. H. Kurki-Suonio, *ibid.*, p. 2104.
24. J. Kapusta and K. A. Olive, *Phys. Lett.* **B209**, 295 (1988).
25. G. Baym and S. A. Chin, *ibid.* **B62**, 241 (1976); G. Chapline and M. Nauenberg, *Nature* **264**, 235 (1976); R. L. Bowers, A. M. Gleeson, R. D. Pedigo, *Astrophys. J.* **213**, 840 (1977); E. Alvarez, *Phys. Rev.* **D23**, 1715 (1981); E. Alavarez, *Astron. Astrophys.* **98**, 390 (1981); K. A. Olive, *Phys. Lett.* **B116**, 137 (1982).
26. B. Freedman and L. McLerran, *Phys. Rev.* **D17**, 1109 (1978); V. Baluni, *ibid.*, p. 2092; M. B. Kislinger and P. D. Morley, *Astrophys. J.* **219**, 1017 (1978).
27. W. B. Fechner and P. C. Joss, *Nature* **274**, 347 (1978).
28. B. D. Serot and H. Uechi, *Ann. Phys.* **179**, 272 (1987).
29. N. K. Glendenning, *Phys. Rev. Lett.* **63**, 2629 (1989).
30. J. I. Kapusta and K. A. Olive, *Phys. Rev. Lett.* **64**, 13 (1990); J. Ellis, J. I. Kapusta, K. A. Olive, *Nucl. Phys.* **B348**, 345 (1991).
31. See for example, J. I. Kapusta, *Finite Temperature Field Theory* (Cambridge Univ. Press, New York, 1989).
32. B. A. Campbell, J. Ellis, K. A. Olive, *Phys. Lett.* **B235**, 325 (1990); *Nucl. Phys.* **B345**, 57 (1990).
33. R. Hagodorn, *Nuov. Cim. Suppl.* **3**, 147 (1965).
34. J. I. Kapusta and K. A. Olive, *Nucl. Phys.* **A408**, 478 (1983).
35. For discussion on the relationship between confinent and chiral symmetry-breaking see: R. Pisarski, *Phys. Lett.* **B110**, 155 (1982); R. Pisarski and F. Wilczek, *Phys. Rev.* **D29**, 338 (1984); J. Gasser and H. Leutwyler, *Phys. Lett.* **B184**, 83 (1987); *ibid.* **B188**, 477 (1987).
36. L. M. Krauss and P. Romanelli, *Astrophys. J.* **358**, 47 (1990).
37. C. P. Delyannis *et al.*, *Phys. Rev. Lett.* **62**, 1583 (1989).
38. C. R. Alcock, D. S. Dearborn, G. M. Fuller, G. J. Mathews, B. Mayer, *ibid.* **64**, 2067 (1990).
39. B. Meyer, C. R. Alcock, G. M. Fuller, G. J. Mathews, *Astrophys. J.*, in press.
40. G. M. Füller, G. J. Mathews, C. R. Alcock in *Dark Matter*, J. Audouze and J. Tran Thank Van, Eds. (Editions Frontières, Gif-sur-Yvette, France, 1989), p. 303.
41. K. Kajantie, L. Karkkainen, K. Rummukainen, *Nucl. Phys.* **B333**, 100 (1990).
42. E. Fahri and R. L. Jaffe, *Phys. Rev.* **D30**, 2379 (1984).
43. S. L. Shapiro and S. A. Teukolsky, *Black Holes, White Dwarfs and Neutron Stars* (Wiley, New York, 1983).
44. _____, I. Wasserman, *Astrophys. J.* **272**, 702 (1983); S. L. Friedman, J. R. Isper, L. Parker, *Phys. Rev. Lett.* **62**, 3015 (1989).
45. G. E. Brown, *Phys. Rep.* **163**, 1 (1988).
46. B. D. Serot and J. D. Walecka, *Adv. Nucl. Phys.* **16**, 1 (1986).
47. B. Friedman and V. R. Pandharipande, *Nucl. Phys.* **A361**, 502 (1981).
48. V. A. Amburtsumyan and G. S. Saakyan, *Sov. Astron.* **4**, 187 (1960); V. R. Pandharipande, *Nucl. Phys.* **A178**, 123 (1971); H. A. Bethe and M. Johnson, *ibid.* **A230**, 1 (1974); R. L. Bowers, A. M. Gleeson, R. Daryl Pedigo, *Phys. Rev.* **D12**, 3056 (1975).
49. N. K. Glendenning, *Phys. Lett.* **B114**, 392 (1982); *Astrophys. J.* **293**, 470 (1985); *Z. Phys.* **A326**, 57 (1987) and **A327**, 295 (1987); *Nucl. Phys.* **A480**, 597 (1988) and **A493**, 521 (1989).

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