Mathematics—Amid War—in San Francisco

San Francisco—*Gather mathematicians for a scientific meeting and the "real world" rarely intrudes. But there was no escaping the Gulf war at the joint meetings of the American Mathematical Society and the Mathematical Association of America in San Francisco from 15 to 20 January. Along with the rest of the country, mathematicians were glued to the tube; some took off from the meeting to join anti-war marchers on Market Street, where chants of "No mathematics for war!" were met with blank stares of incomprehension. At the sessions, the focus remained on math. But even there, topics came up that would have been unusual at a mathematics meeting a few years ago. Among them: the status of women mathematicians, the art of mathematics, and control theory. The latter actually brought the technical issues of the war into a session. Each of these subjects—along with others such as environmental modeling and historical topics—reflects mathematicians' broadening outlook on their discipline.*

Under Control

One of the technical marvels (or horrors) of the Gulf war has been the profusion of Nintendo-like TV images of "smart bombs" zeroing in on targets as precise as the elevator shaft of one particular building in downtown Baghdad. How do those smart weapons get to their targets? Part of the answer comes from a branch of mathematics known as control theory. Control theory involves continuously measuring the state of a physical system described by a set of differential equations and adjusting parameters in those equations to maintain an equilibrium position while directing the motion of the system.

At a session in San Francisco devoted to the topic, Hector Sussmann, a mathematician at Rutgers University, gave an overview of some of the directions control theory may be taking in the near future. And those directions have broad consequences, because the mathematics of control theory is finding increasingly wide application. Its principles underlie not only smart bombs and missiles but also the smooth operation of chemical plant processes, biomedical systems for drug delivery and other purposes, even anti-lock brakes. Some researchers are working on applications in robotic motion, while others have begun to develop new methods for weather forecasting based on "adaptive control" methods, in which estimates of fixed—but unknown—parameters are adjusted as observations of the system pour in.

But current applications are fairly straightforward compared with what Sussmann and his collaborators are interested in: control theory for nonlinear systems. Almost all real systems are, in fact, nonlinear, but most control theory applications are based on linear approximations. This is mainly because linear systems are easier to analyze—there are mathematical tools, such as Fourier analysis, which work well in linear settings but are of little help when nonlinear effects take over. One reason for studying the theory is that nonlinearity can, at least when control is not applied, lead to chaotic behavior—something that it is useful to avoid.

Part of the task for control theorists, Sussmann explained, is to see how much of the theory that has been developed to account for linear systems carries over into nonlinear ones. For example, take a situation that can be thought of as Bozo's approach to control theory: a circus clown wheeling around a stage, perched atop an outlandishly tall unicycle, balancing a broom in the palm of one hand. The clown's chief control problem—stabilization—is well understood for linear systems. In fact, stabilization can always be achieved, provided the coefficients of a linear system satisfy certain conditions that can easily be tested—such as verifying that the control parameter is actually "connected" to the system by a coefficient that differs from zero.

The corresponding problem for nonlinear systems, however, according to Sussmann, is "extremely far from having been solved." Control theorists have found theoretical instances where it's possible to achieve stabilization (for example, if a nonlinear system actually turns out to be a linear system in disguise), and others (such as the problem of parking a unicycle), where they can prove that stabilization is mathematically impossible. But a general understanding is lacking and the field is "wide open," Sussmann said.

Among the areas likely to benefit from a better understanding of nonlinear control, as well as other aspects of the theory, are drug delivery systems, where nonlinear effects are essential, and the design of complicated mechanical systems with interconnected parts, such as satellites with flexible antennas. Because the mathematics is not yet understood, the most ambitious examples are still in the future, but, Sussmann says, progress is being made and the future is "probably coming soon."

Women in Mathematics—"Riding the Wave"

Mathematician Jill Mesirov of Thinking Machines Corp. in Cambridge, Massachusetts, wasn't talking about an abstruse quantum mechanical wave function when she said: "We'll ride the wave while it's here, and take it as far as we can. It's possible we can take it far enough that it won't matter if it evaporates, because we'll already have made tremendous gains." Far from discussing a particular brand of mathematics, Mesirov was discussing the outlook for women in her discipline.

She was one of several female mathematicians *Science* interviewed in San Francisco on that subject. But the topic didn't come up just because *Science* did some interviews. At the meeting, the 20th anniversary of the Association for Women in Mathematics (AWM) was celebrated, and there were several sessions devoted to the problems of getting more women into mathematics—and keeping them there.

In part because of the AWM, of which Mesirov is president, women have made inroads into what used to an almost exclusively male preserve. Three of 13 invited addresses at San Francisco were by women—a proportion roughly in line with the number of female Ph.D.'s in math.

That figure represents a marked advance over the situation two decades ago, according to Leonore Blum, a mathematician at the International Computer Science Institute in Berkeley and currently vice president of the American Mathematical Society. Blum quoted some "advice" for women in a 1970 employment guide for mathematicians: "Women find the competitive situation in the government somewhat more advantageous to them, since it is relatively hard to secure a well-qualified mathematician for many higher level government jobs. In many such cases women are welcomed if their qualifications are better than those of the available men."

In 1991 the situation is far better, says Blum. Job listings routinely encourage women to apply—in part as the result of affirmative action laws—and women have gained prominent positions in the major mathematical societies. Mesirov, director of mathematical sciences research at Thinking Machines, agrees: "Enough barriers have broken down that there's a better climate now for women." The current crop of female mathematics Ph.D.'s is "much more self-confident," she says. "They are now gaining the fruits of a lot of the battles that we waged 10 or 20 years ago."

Yet Mesirov adds that such progress "doesn't mean that there isn't still discrimination." Although U.S. women earn about as many bachelor's degrees in mathematics as their male counterparts, things change dramatically at the post-graduate level, where women receive less than a quarter of the doctoral degrees (even less when U.S. Ph.D.'s granted to foreign students are counted in the total). And at the professional level, the imbalance can be even greater. In the mathematics departments of Berkeley, Chicago, Harvard, MIT, Princeton, and Yale combined, there is currently a total of one—that's right, one—tenured woman.

The goal of the Association for Women in Mathematics isn't just to get more women into jobs at those elite institutions, it's also to get more women studying math at all levels—including high school. "The fact that you don't take all 4 years of mathematics in high school can close off tremendous career possibilities," says Mesirov. Adds Mary Beth Ruskai, a mathematical physicist at the University of Lowell: "You see a lot of girls dropping out [of math classes] at the low B or high C level," whereas boys hang in "right at the C–D margin. It doesn't occur to people that encouraging those women to stay in doesn't mean that they're lowering standards. In fact, it's the other way around."

Making Mathematics Graphic by Computer and by Hand

Here's a mathematical teaser: What would intrigue an audience of mathematicians about a 10-minute video showing, through a remarkably detailed and intricate computer animation, solar energy being converted to chemical form via the complex choreography of photosynthesis, then utilized in the muscles of a grape-eating raccoon. There they were—3 dozen strong—at a session on math in graphic form. But where was the math?

It wasn't really in the advanced technology of the film from which this 10-minute strip was taken, although the technology is arresting. The film, which received its premier in Osaka last year, was made by Fujitsu, Ltd. for special presentation on a 3-D dome screen. The 3-D effect requires a special, $1-million projector and high-tech glasses with liquid-crystal shutters that allow images to alternate 96 times a second between the left eye and the right eye.

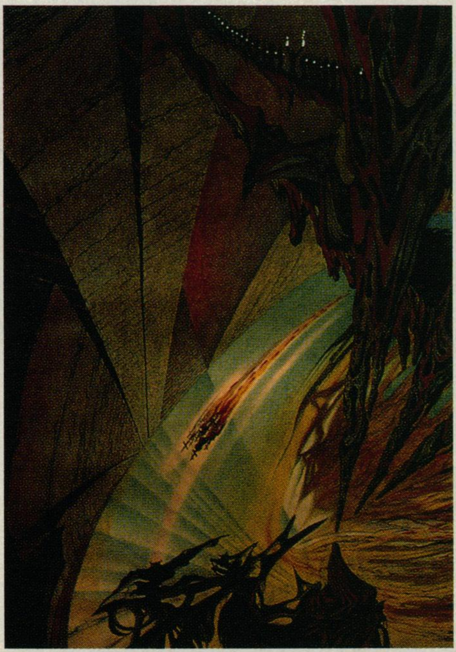
The mathematics, however, was in the making of the images, according to Nelson Max, a topologist at Lawrence Livermore National Laboratory who produced the film with his colleagues. "We had a supercomputer working on this for a year and a half," said Max.

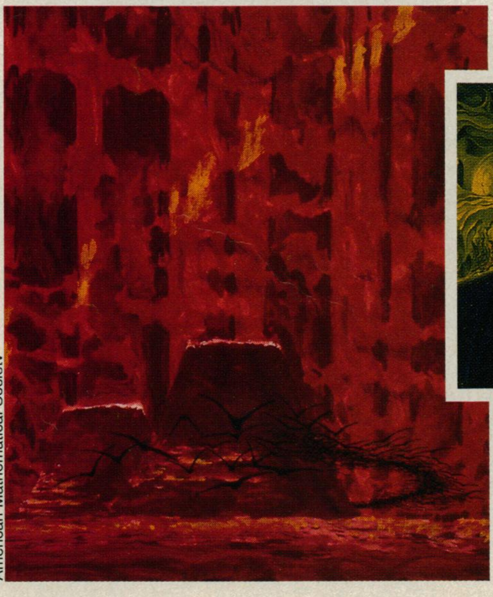
Among the things the supercomputer was busting its chops on was the challenge of orchestrating the animated workings of actin and myosin—the molecules that enable muscles to contract. The film had 225 myosin molecules in continuous motion, and the filmmakers decided they should bounce apart whenever they touched—but first they had to figure out when the collisions between the two types of biological reactants would take place.

To simplify the problem, Max and his collaborators pretended the rigid parts of each myosin molecule were enclosed in a set of capped cylinders. The simple shape enabled the researchers to derive an equation that specified when and where collisions would occur. But even that simplification resulted in a 10 degree polynomial—an equation still too hairy, considering the number of times it would need to be solved to produce the animation. By slightly altering the computed motion of the molecules between frames (something the viewer wouldn't see anyway), Max reduced the equation to a polynomial of degree 4—something Fujitsu's supercomputer could cope with.

While Max and his co-workers rely on Fujitsu's silicon wizardry to turn math into beautiful graphic forms, Anatolii Fomenko, whose work was also displayed at the meeting, aims for the same effect with an older technology: India ink on drawing paper. Fomenko, a mathematician at Moscow State University, makes use of algebraic and topological ideas to draw pictures reminiscent of Escher, Brueghel, or science fiction—as the display here suggests. Titles such as "A theorem in symplectic geometry," however, suggest that these aren't lightweight fantasies.

The American Mathematical Society has published a collection of Fomenko's work, with accompanying mathematical commentary. The book, *Mathematical Impressions*, is a slight departure from the society's normal run of mathematical monographs: It doesn't contain a single mathematical formula. ■ BARRY CIPRA





American Mathematical Society

Strange worlds. *Paintings by Anatolii Fomenko of Moscow State University on mathematical themes include "Motion of a heavy rigid body in space" (upper left), "The cylinder of a continuous mapping" (lower left), "Geometric fantasy on the theme of analytic functions" (right).*