Image Capture by Computer

A new use of fractal geometry and dynamical systems produces a technology for compressing and analyzing digitized images, with applications in biology and communications

HOW DO YOU PAINT A PICTURE over the telephone? How accurately can you describe a photograph? How quickly? And how many images can you fit onto a floppy disk? According to Michael Barnsley, a mathematician at the Georgia Institute of Technology, you should be able to describe as many as 30 pictures per second, as precisely as you like, and cram a large number of them into even a moderate-sized computer system—provided you use the proper box of mathematical tools.

Barnsley and colleague Alan Sloan recently formed a company, Iterated Systems Incorporated, to develop techniques and applications of data compression for efficient storage, rapid communication, and novel analysis of digitized images. They see a broad market in telecommunications, medical imaging, biological and terrain modeling, and possibly the upcoming technology of high-definition television.

An uncompressed digital image contains an enormous amount of data: a high-quality, 1024 × 1024 pixel image using 256 color or graytone gradations occupies a megabyte of memory. Twenty such pictures would fill up the typical hard disk drive on a personal computer. "This problem only emerged to public awareness with the age of the personal computer," Barnsley notes. "People . . . realized they were filling up their computers with data files that couldn't be stored."

But digital images contain less information than data, and it is really information that one would like to store and transmit. More precisely, images of real objects are tremendously redundant: neighboring pixels tend to be either the same or at least similar colors. A picture of a virgin Rubik's cube, for instance, can be boiled down to the location of its corners and the colors of the visible sides; one need not give a pixel by pixel description. Natural objects are more complex than a Rubik's cube, of course, but even a landscape can be represented by simpler shapes. How to do it?

According to Barnsley, the best way is to use fractal geometry and dynamical systems, with a bit of abstract measure theory thrown in for color. The new approach is an instance of pure mathematics creating a new technology, Barnsley says. "Our first papers were so abstract they just looked like they had nothing to do with the real world, and yet it's exactly that we're now using."

Mathematicians have known for some time that by iterating simple numerical rules they can produce complex geometric shapes. However, this system gives no control over the shape produced. Now the inverse problem is attracting a lot of interest: finding rules that will, under iteration, produce a particular geometric shape.

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One method uses systems known as contractive affine transformations and a random iteration algorithm. The random iteration algorithm creates a dotted image by randomly applying transformations from the system to points in the plane. In spite of the algorithm's random aspect, the picture that finally comes into focus is strictly determined by the system of transformation.

Any contractive system will produce a picture, called the attractor of the system, but the question is how to find a system that will produce, or reproduce, a particular picture. Part of the answer is provided by a result known as the Collage Theorem. The key ingredients of the theorem are a precise definition for the distance between any two geometric shapes and a numerical index for the contractivity of a system of mappings.

The theorem gives an estimate of the distance between the region of interest and the attractor of the system, based on the system's contractivity index and the distance between the original region and its collage of images under the contractive system. A highly contractive system and a close collage-to-region approximation imply a good attractor-to-region approximation. If the likeness is acceptable, then the region can be stored and communicated merely by specifying the data of the contractive system. This gives the high data-compression ratios.

The Collage Theorem also implies that small changes in the contractive system result in correspondingly small changes in the system's attractor, so that an image can be varied continuously by varying, for instance, the coefficients of the affine transformations that define a system. This opens the possibility of analyzing the similarity of two images by means of the data describing the contractive systems associated with each image. Dealing with the smaller data set could facilitate pattern recognition.

Graytone and color images can also be produced by the Random Iteration Algorithm. Although varying the probabilities with which the mappings of a contractive system are chosen does not change the shape of the attractor, it does change the amount of time that the point spends in different parts of the attractor. By counting the number of times that the point lands in each pixel, the algorithm can assign graytones or even colors to the pixels.

Moreover, this feature can be made mathematically precise by reformulating the Collage Theorem in terms of a subject called measure theory. Measure theory was introduced early this century as a foundation for modern integral calculus. It turns out to be the natural mathematical setting for probability theory. Barnsley says it is also the natural setting for fractal geometry.

Barnsley and Sloan have so far produced two systems which they plan to market: Vector Recurrent Iterated Function System (VRIFS) and the fractal transform. VRIFS improves on the Collage Theorem not only by incorporating measures, but also by introducing a vector recurrent technique: instead of treating a picture as a single object to be collaged—and instead of breaking it into separate objects to be collaged separately—VRIFS allows an operator to break a picture into separate objects, but then *every* object is used in the collage of all the objects.

As implemented on a Sun workstation, VRIFS displays one large (approximately 6 inches square) and four small (approximately 3 inches square) windows where the collaging is done. The system is highly interactive: an operator creates and adjusts the collages. According to Barnsley, VRIFS produces a "frighteningly high-quality" approximation to an original x-ray with a compression ratio of about 100 to 1, using approximately 100 contractive mappings.

VRIFS is specifically designed for biological modeling, Barnsley says. One system has been sold to a West European firm for an application to x-ray analysis. Hugo Rogers, a plant physiologist in the Soil Dynamics Laboratory of the U.S. Department of Agriculture in Auburn, Alabama, is interested in the ability of VRIFS to model realistic root systems. "The possibility of being able to quantify a root architecture and the spatial deployment of roots would offer us a lot," he says. One of the things that impressed Rogers during a visit to Iterated Systems, he says, "was a dirty old weed root system laying up on top of a half a million dollars worth of computers. . . . I figured that anybody that'd do that has to be on our level."

Whereas VRIFS is an interactive system, the fractal transform is a fully automated image-compression system. It features both speed and accuracy. As currently implemented on an AMT Distributed Array Processor, the fractal transform can compress a 256 × 256 pixel black and white image in under 10 seconds, and can regenerate the *exact* image in about the same amount of time. Barnsley estimates that putting the transform on a special chip would allow it to handle up to 30 pictures per second—a convenient figure for real-time animation.

Barnsley and Sloan are currently looking at applying their image-compression technology to the high-stakes game of high-definition television (HDTV), which will offer improved stereo sound and pictures with twice the resolution of standard TV. The Federal Communications Commission recently ruled that any HDTV broadcast must be viewable on current standard TVs (*Science*, 7 October 1988, p 29). The technical challenge is how to fit the extra information required by HDTV into the already clogged television spectrum.

Barnsley and Sloan think it might be unnecessary to change the signal. Their idea is to put a "fractal enhancement box" at the TV end, which would take an ordinary signal, "recode the image as a fractal, and decode it at twice the resolution—or higher," Barnsley says. Improving an existing picture is not a new idea, but using the fractal transform may allow it to be done at the speed and with the quality required for HDTV. ■ **BARRY A. CIPRA**

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How Females Entrap Males


C. H. Breenewalt/VIREP

During the springs of 1985 and 1986, Eivin Roskaft and two colleagues at the University of Trondheim, Norway, systematically removed the males from 20 pairs of breeding pied flycatchers, leaving the females "widowed." They then observed the females' response, particularly in relation to males, and learned something interesting: the females tried—and sometimes succeeded—to entrap visiting males into a relationship with them.

It is not news that males and females seek—and achieve—different goals in relationships with the opposite sex. But for ethologists, who describe and interpret animal behavior, these differences have increasingly become the focus of attention. Specifically, researchers are interested in reproductive success and how an individual's behavior might influence this crucial Darwinian parameter. Simply put, reproductive success is the number of offspring that an individual successfully contributes to the next generation. With even a slight advantage in reproductive success over its contemporaries, an individual's genes can—generation by generation—come to predominate in a population. Similarly, burdened with just a slight disadvantage in reproductive success, an individual's genes can eventually be consigned to oblivion. The overall question is, therefore, do individuals usually behave in ways that enhance their reproductive success?

For various reasons—some biological, some practical—birds have frequently been studied with this question in view. One of the biological reasons is that even within a single bird species a range of breeding systems—monogamy, polygyny, polyandry—can often be observed, depending on the ecological and social circumstances. In the pied flycatcher, for instance, some individuals mate monogamously while others engage in polygyny. For females, polygyny can be bad news, because they are sharing the attentions of a single male, who is trying to help rear the offspring. For flycatcher females—at least some of them—polygyny is even worse news, because of his two females, the male selects one as the primary female, who receives all the provisioning help, and a secondary female, who simply bears his offspring but benefits not at all from any paternal care. As a result, the reproductive success of secondary females may be reduced by as much as 50% of that of monogamous or primary females. Roskaft and his colleagues argue that, with a difference in reproductive success of this magnitude, there is great selection pressure for individuals to try to ameliorate the disadvantage, presumably through some behavioral strategy. It was signs of such a behavioral strategy in the female flycatchers that Roskaft and his colleagues were seeking with their experiment.

By widowing the 20 females, Roskaft and his colleagues effectively produced a cohort of secondary females, whose behavior they could monitor. Seventeen were visited by neighboring males, and six solicited copulations from the new males, three of which solicitations were successful. This behavior is interesting, because by this time none of the females were fertile, having laid their clutches of eggs. Such behavior is never seen in monogamous or primary females. The researchers interpret the secondary female's activities as attempts to encourage a male into believing that it might be fathering some offspring at the nest, and therefore perhaps staying and helping rear them, thus probably improving the female's reproductive success. This is the "male deception by females" hypothesis. "Males should be most easily deceived when the deception is most plausible," say Roskaft and his colleagues. "Thus, females widowed immediately after egg laying should be more successful in deception than females widowed when they have nestlings." ■ **ROGER LEWIN**

ADDITIONAL READING

J. O. Gjershaug *et al.*, "Marriage entrapment by solitary mothers: A study on male deception by female pied flycatchers," *Am. Nat.* **133**, 273 (1989).