

linked by a conjugated ligand. Electron transfer between two metals connected by a bridging ligand can occur at rates that range from very slow to so fast that a fully delocalized electronic description is appropriate. Understanding the structural features that control these intramolecular electron transfer rates has been an important theme in Taube's work.

Work on electron transfer and atom transfer reactions continues unabated in Taube's laboratory. For example, he and D. Geselowitz in 1980 demonstrated an unambiguous case of stereoselectivity in an outer-sphere redox reaction. Further, his recent study of the formation and stability of oxo ions (the "yl" ions) as a function of the electronic structure of the transition metal has given a real boost to attempts to understand the wide range of rates that are found for oxygen atom transfer reactions. The relevance of this new work to the development of mechanistic models for biological hydroxylating systems such as the cytochrome P-450 family is beginning to be recognized.

In relatively recent time Taube has been a leader in developing the chemistry of ruthenium and osmium complexes. He and D. E. Harrison demonstrated in 1967 that molecular nitrogen will displace water in  $\text{Ru}(\text{NH}_3)_5\text{H}_2\text{O}^{2+}$  to form the stable complex that A. D. Allen and C. V. Senoff had prepared by an indirect route in 1965. Shortly thereafter, he, Harrison, and E. Weissberg prepared

the first bridging dinitrogen complex,  $\text{Ru}(\text{NH}_3)_5\text{N}_2\text{Ru}(\text{NH}_3)_5^{4+}$ ; and, more recently, he and John D. Buhr demonstrated the oxidative coupling of ammonia ligands to form dinitrogen-bridged complexes. The finding that coordinated ammonias can be oxidized to dinitrogen is extremely important because it is the development of a useful model for the biological path.

Taube's work on dinitrogen complexes of ruthenium and osmium led him to explore metal-to-ligand  $\pi$  bonding (often called  $\pi$  backbonding) in Werner-type coordination complexes. (This involves mainly the study of  $\text{M} \rightarrow \text{N}$   $\pi$  interactions, in contrast to the more familiar  $\text{M} \rightarrow \text{C}$   $\pi$  backbonding that is the province of organometallic chemists.) His work has shown conclusively that dramatic changes in Brønsted acid-base equilibria, redox potentials, and other physical and chemical properties can be brought about by  $\pi$  backbonding interactions that are linked to changes in the oxidation state of certain central metals. It is a good bet that Taube's interest in backbonding stimulated him and R. A. Armstrong in 1976 to prepare a low-valent ammine complex of technetium, which in turn provided a strong stimulus to what has become an important area of chemistry. Technetium complexes are being increasingly used in the field of medicinal radiochemistry, especially in the diagnosis of diseased internal organs. Edward A. Deutsch of the University of

Cincinnati, who in one of Taube's former students, and Alan Davidson of the Massachusetts Institute of Technology are among the leaders in this rapidly growing field of research.

Henry Taube is a rare figure among internationally acclaimed scientists. He does little or no horn-tooting. Instead, he spends a great deal of time encouraging others, especially young people, to pursue research. (We admit that at certain of these times we have seen him savor a bit of sour-mash whiskey while listening to one of his wonderful old Maria Inögün phonograph records.) For these reasons, and more, he has been a real hero of ours for many years. It is truly delightful to have this opportunity to acknowledge the tremendous influence he has had on our lives and our work. And it is extremely heartening that the connection between Taube's brilliant elucidation of the fundamentals of inorganic solution redox chemistry and recent impressive advances in the understanding of electrode processes and biochemical redox phenomena is beginning to be recognized far and wide.—HARRY B. GRAY AND JAMES P. COLLMAN

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## The 1983 Nobel Prize in Economics

Gerard Debreu, the 21st scholar to receive the Nobel Prize in Economics since the award was established in 1969, is a mathematical economist's mathematical economist. Formed in the crack *École supérieure normale* of France, Debreu came to this country as a postdoctoral scholar, enriching the environment at the Cowles Foundation for Economic Research in its University of Chicago and Yale incarnations. For almost two decades he has graced the economics and mathematics departments at the University of California, Berkeley, serving as a magnet there for scholars from

all over the country and indeed the world.

I emphasize Debreu's French origins because of the deplorable headlines that stress the clean American sweep of the 1983 prizes. Credit always adds up to more than 100 percent: if you moved to Harvard 1 day before you got the prize, Harvard would bask in your reflected glory; and so would the kindergarten back in good old Champaign. Back in Paris, when Debreu was coming of age, Professor Maurice Allais, an engineer turned economist, had largely worked out on his own during the Occupation of

Paris the fundamentals of advanced economics. Himself of Nobel caliber, Allais gathered around himself after World War II a brilliant cadre of graduates from the engineering faculties which trace back to Napoleon. Marcel Boiteux, then a brilliant young mathematical economist and now the head of the state-owned electricity system in France, was one of them. And so was Debreu.

America did give Debreu's native genius and fine mathematical training the scope for its full development. Jacob Marschak and Tjalling Koopmans, themselves new Americans who gave and received here, recognized Debreu's talents. The names of the scholars Debreu worked with, or whose work he paralleled and built upon, tells much of his quality and opportunities: John von Neumann, Abraham Wald, Kenneth Arrow, Leo Hurwicz, John Nash, Samuel Karlin, Herbert Scarf, Robert Aumann, Roy Radner, Karl Vind, Werner Hilden-

brand, Robert Dorfman; the triumvirate A. W. Tucker, H. W. Kuhn, and D. Gale; Brouwer and Kakutani, W. Fenchel, S. Eilenberg, K. Menger, W. Blaschke, S. Smale, . . . Throughout there hovers over his shoulder the pervasive shade of N. Bourbaki (1).

### First Fame

Debreu's classic, *The Theory of Value: An Axiomatic Analysis of Economic Equilibrium*, was published when he was still in his 30's. Although barely 100 pages, it is a gem of terseness, elegance, and universality. A paragraph from the preface imparts its flavor.

The theory of value is treated here with the standards of rigor of the contemporary formalist school of mathematics. The effort toward rigor substitutes correct reasonings and results for incorrect ones, but it offers other rewards too. It usually leads to a deeper understanding of the problems to which it is applied, and this has not failed to happen in the present case. It may also lead to a radical change of mathematical tools. In the area under discussion it has been essentially a change from the calculus to convexity and topological properties, a transformation that has resulted in notable gains in the generality and the simplicity of the theory.

The words "contemporary formalist school of mathematics" are well chosen, for it is the mathematics à la Bourbaki

that has overtaken economic theory. Herbert Simon once observed that the level of mathematical rigor met in *Econometrica* exceeds that in *Physical Review*—a finding that Simon did not intend as a compliment to economics. I believe it was Debreu himself who speculated that the applied science which most nearly taxes the resources of sophisticated modern mathematics is, paradoxically, the soft discipline of economics. In a field not completely free of baroque preciousness, Debreu is known for his unpretentious no-nonsense approach to the subject.

### Existence of Equilibrium

In order to illustrate the new techniques let me discuss the problem of whether there is an equilibrium solution for goods' prices under competitive supply and demand.

Suppose we are many economic individuals, each endowed with one apple and three oranges. We all have the same tastes, which can be summarized by the following: Each of us persons always spends half of our income on apples and half on oranges.

Does there exist an equilibrium price ratio,  $P_{\text{apples}}/P_{\text{oranges}}$ , that clears competitive markets? Is the equilibrium unique?

For this trivial case the answers are

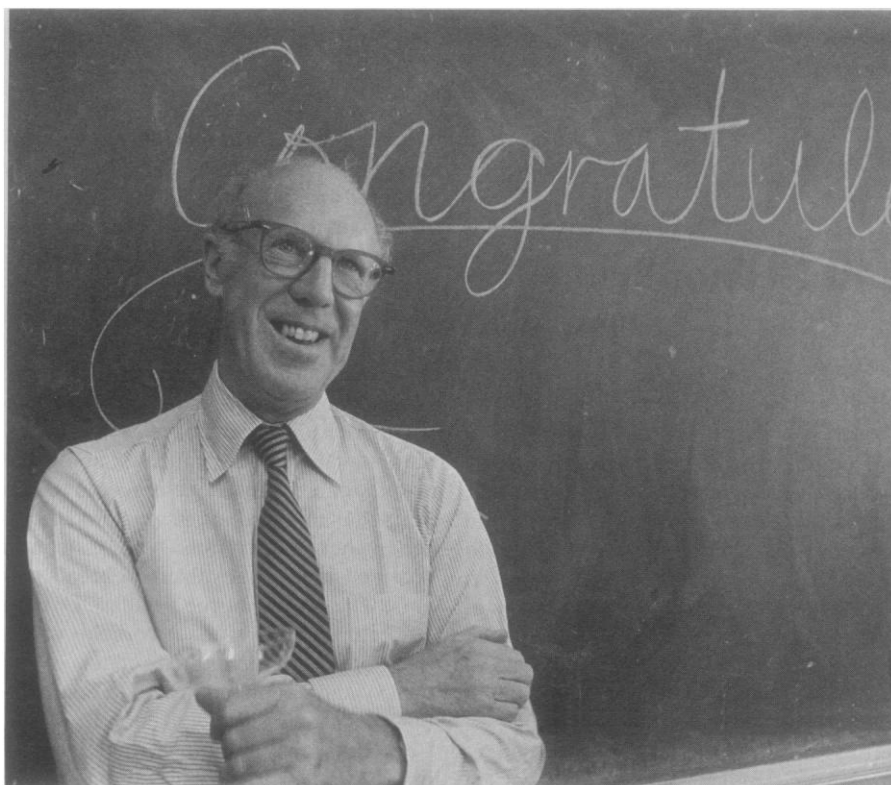
trivially clear: Yes, and Yes; at the equilibrium price ratio of 3.0, and only there, will the totals of supply and demand clear the market (and, actually, with each of us consuming what we initially own and being content to exchange nothing).

No Debreu or Bourbaki is needed for this conclusion. But, as the Swedish Royal Academy of Sciences hinted at in its citation of the award, it is by means of Brouwer and Kakutani fixed-point theorems in topology that Debreu, and independently Lionel McKenzie, proved that more complicated systems do have at least one nonnegative equilibrium solution.

Beautiful? Yes. Comprehensive? Yes. It was Sophie Tucker who said, "I've been rich. I've been poor. Believe me rich is better." Having lived as a scholar in both the pre- and post-Debreu era, I can testify that the modern proofs are better than what used to pass muster for demonstration of determinate economic equilibrium. Here's how our wave-of-the-hand expositions used to go. We used to count our number of unknowns—in this case one unknown price ratio. And then we counted our number of independent equations—in this case that the function of  $P_a/P_o$  representing aggregate supply of apples be equated to the function representing aggregate demand for apples. If we could show equality between number of equations and unknowns, we took that to be an adequate indication of a determinate economic equilibrium.

What's wrong with that? Plenty. Our functions, even the ones in this trivial problem, are not linear. There are no theorems on rank of matrixes that guarantee the existence of a solution of nonlinear relations. (Even for linear systems that are neither under- nor overdetermined, economists learned the hard way that meaningless negative solutions might be implied by the mathematical formalisms.)

For readers who were away from school the morning they dealt with fixed-point theorems, let us draw a square and pencil in the 45-degree diagonal connecting its southwest and northeast corners. Is there any way to draw a curve that goes from the square's east side to its west side without taking pencil off the paper, such that the curve and the diagonal have no single point in common? Brouwer's fixed-point theorem in one dimension proves that there is indeed no possible way. Similarly, under specified conditions about people's tastes and goods endowments, there is no way for



Gerard Debreu [University of California, Berkeley]

curves of supply and demand to be drawn without having at least one intersection point in common.

Warning: we must not purport to prove too much. Suppose people behave as Thorstein Veblen said they do, and value goods highly just because they are expensive in price. Then, the old-fashioned economist would argue, the higher its price goes the higher it will be further pushed, with no determinate equilibrium possible.

Let's test the new techniques on this case. Suppose in our earlier examples of apples and oranges that the fractions of income spent on each are not constants but instead are proportional to their respective absolute prices. McKenzie's existence theorem asserts that economic equilibrium is still determinate. "How can this be?" the literary economist asks. A postmortem reveals that  $P_a/P_o$  of infinity and zero are counted in by the mathematician as an existent solution, and it is precisely such odd results that the literary economists had in mind when speaking of indeterminateness of equilibrium price (2).

### Topology to the Rescue

It is not easy to convey to nonspecialists the intricacies of economic theory. Our oranges and apples case is one whose demand behavior can be generated by assuming that each of us acts to maximize a Weber-Fechner utility function of the form: log apples plus log oranges.

Even more generally, for you (two apples, three oranges) is better than (one apple, four oranges); and, more general-

ly, it can be shown that you will prefer ( $x$  apples,  $y$  oranges) to ( $X$  apples,  $Y$  oranges) whenever the product  $xy$  exceeds  $XY$ . What Debreu, and independently Hirofumi Uzawa, were able to show is this: if equality of  $xy$  and  $XY$  always implied indifference on your part between the two batches of goods, then there must exist a utility function,  $u(x,y)$ , whose maximization generates your demand functions. However, the same spending of half your income on each good could be the result of a preference structure on your part in which, when  $xy$  equals  $XY$ , you break the tie by preferring the goods batch having the most of apples. That would mean you have a *lexicographic* ordering that cannot be described by a scalar utility function. Using topological methods of Eilenberg and others, Debreu proves that a utility function must exist if both the following sets are *closed*: the sets of all points you deem at least as good as *any* prescribed (apples, oranges) batch; the sets of all points you deem no better than such a prescribed batch.

Here is still another topological success. In your Weber-Fechner utility function, the extra utility you get from adding simultaneously an apple and an orange is exactly equal to the sum of the extra utilities you get from an extra apple by itself and from an extra orange by itself. (This is in contrast, presumably, to the case where tea and lemon added together give you more than the sum of what each does separately, or the case of tea and coffee where the two together give less than the sum of each separately.) Nicholas Georgescu-Roegen, and Debreu independently, use topological methods to derive conditions for such

additive independence which are in the form of testable functional equations and are of a structure different from the differential equations used by Abel and 19th-century mathematicians.

### Style of the Scholar

Debreu writes sparingly. He never corrects errors of others, publishing only to establish novel results. In his adopted land where the art of lecturing is these days little cultivated and prized, Debreu is one of those who fills the blackboard with a neat sequence of axioms, lemmas, theorems, and corollaries. Eschewing faculty club and airline chefs, he prefers to dine Chez Debreu.

Gerard Debreu is not as American as apple pie. But he is as American as Enrico Fermi or Christopher Columbus.

—PAUL A. SAMUELSON

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### Notes

1. General N. Bourbaki is the mythical character under whose name the top French mathematicians have published a comprehensive series of studies designed to put all of mathematics on a rigorous and sophisticated foundation.
2. The problem is more than a semantic one. Years ago Milton Friedman advocated that the pound sterling be allowed to depreciate to restore equilibrium in Britain's balance of payments. Critics objected saying that, when demand elasticities are perverse, a depreciation might worsen the unbalance. To support Friedman's position, the late Egon Sohmen argued that a large enough depreciation would as a matter of logic alone have to produce an equilibrium. Jagdish Bhagwati and the late Harry Johnson produced counterexamples in which the so-called existent equilibrium involved a zero dollar price for the pound.