

The Discovery of Charge Conjugation-Parity Asymmetry

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Physics as a science has made incredible progress because of the delicate interplay between theory and experiment. Astonishing predictions based on theories devised to account for known phenomena have been confirmed by experiment. Experiments probing previously unexplored areas often reveal physical effects which are completely unanticipated by theoretical conjecture. The incorporation of the new effects into a theoretical framework then follows.

This year, Professor Cronin and I are being honored for a purely experimental discovery, a discovery for which there were no precursive indications, either theoretical or experimental. It is a discovery for which, after more than 16 years, there is no satisfactory accounting. But showing as it does a lack of charge conjugation-parity (CP) symmetry and, correspondingly, a violation of time-reversal invariance, it touches on our understanding of nature at its deepest level.

The discovery of failure of CP symmetry was made in the system of K mesons. This observation is especially interesting because it was the study of these same particles that led to the overthrow of parity conservation, the notion that interactions and their mirror-reflected counterparts must be equal.

My own interest in K particles started in 1952 and 1953, while I was at Columbia working with Jim Rainwater on μ -mesonic atoms. At that time the strange behavior of the particles newly discovered in cosmic rays (1) was a major topic of conversation in the corridors and over coffee. By strange behavior I am refer-

ring to the copious production but slow decay. Protons bombarded by pions would result in the production of Λ^0 's at 10^{13} times the rate of their decay back to pions and protons. Pais (2) came to Columbia and talked of his ideas on associated production to explain this anomaly. Gell-Mann (3) visited and discussed the scheme which he and, independently, Nakano and Nishijima (4) had devised to account for associated production.

Their idea was implausible and daring in the face of available data. The scheme assigned the K mesons to two doublets, K^+ and K^0 and the antiparticles K^- and \bar{K}^0 . The natural assignment would have been the same as for pions, a triplet of particles, K^+ , K^0 , and K^- . Nishijima also assigned quantum numbers, subsequently called strangeness, which were conserved in the strong interaction but not in the weak. The K^+ and K^0 were assigned +1, the \bar{K}^0 and K^- as well as the Λ^0 , -1.

Standing alone among the particles with positive strangeness were the K^+ and K^0 mesons, and I idly thought that if the situation was ever to be understood these objects might be the key. Most often, experiments in physics are long and difficult. It takes some special tweaking of interest to make the commitment to a new area of research. The original motivation is, in the end, apt to appear naïve. However, I did, in fact, join the Princeton cosmic-ray group headed by George Reynolds, and spent the summer of 1954 on a mountain in Colorado learning about the ongoing experiments. During the same period the energy of the cosmotron at Brookhaven

was being raised to 3 GeV. Associated production was clearly seen by Shutt and his group at Brookhaven (5), and K mesons produced in the cosmotron were identified in photographic emulsion (6). By the end of the summer I reluctantly decided the future was not in studying cosmic rays in the mountains I loved, but with the accelerators.

The following fall, with Bob Motley, a graduate student, we began to design an apparatus to detect K mesons, using purely counter techniques at the cosmotron. As this work progressed the cascading interest in the tau-theta puzzle (7) led us naturally to explore the lifetime of the K particles as a function of their decay mode. We were successful with our detectors, and Motley and I published our results simultaneously with those from the Alvarez group at Berkeley, which was using the bevatron as a source (8, 9). These results showed the degeneracy in the lifetime of the tau and theta mesons. Independently, the masses of tau and theta had been shown to be the same to within 1 percent (10). The situation then set the stage for the famous work of Lee and Yang (11), followed by the experiments with the striking results showing maximal parity violation in the weak interactions (12). This remarkable story was told by Lee and Yang on this occasion in 1957.

At about this time there appeared a paper by Landau (13), written before the results of the beta decay experiments were known. Addressing the tau-theta problem, he observed that a simple rejection of parity conservation would create difficult problems in physics. However, with what he called "combined inversion," that is, space inversion and the simultaneous transformation of particle into antiparticle, the difficulties would be avoided. Indeed, this is a path

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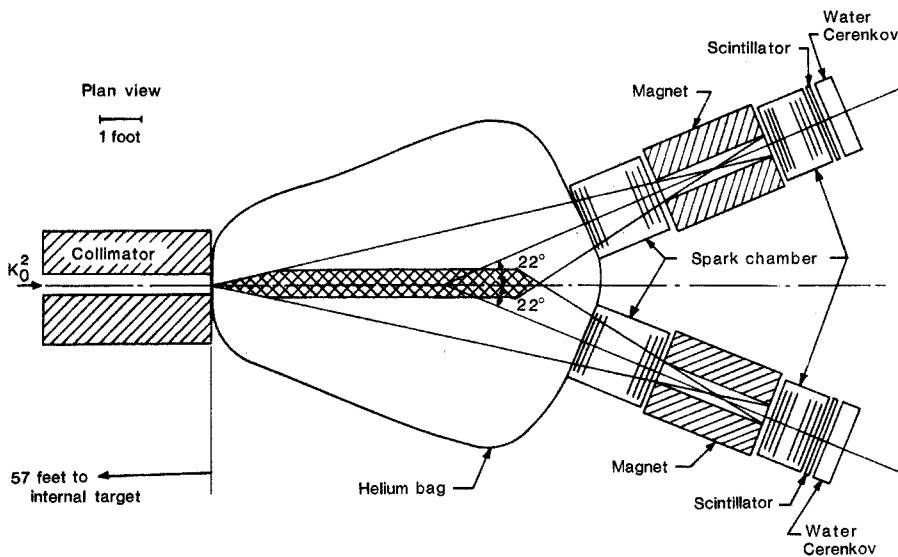


Fig. 1. Plan view of the apparatus as located at the Brookhaven AGS.

that nature appeared to take. Subsequent experiments showed that parity violation was compensated by a failure of charge conjugation. The weak interactions were therefore invariant under the combined operations of particle-antiparticle interchange and mirror reflection, charge conjugation-parity.

One symmetry had been shown to be invalid but had been replaced by a still deeper one. This new symmetry was especially appealing because of the CPT theorem. This theorem, which is based on little more than special relativity and locality and which is the foundation of all quantum field theory, says that all interactions must be invariant under C, P, and T (time reversal) all combined. If CP is good, so also is T, in complete accord with all experimental data. The subject was left in a highly satisfactory state. "Who would have dreamed in 1953 that studies of the decay properties of the K particles would lead to a new revolution in our understanding of invariance principles?" wrote Sakurai in 1963 (14). But then in 1964 these same particles, in effect, dropped the other shoe.

It is difficult to give a better example of the mutually complementary roles of theory and experiment than in telling the story of the neutral K mesons. For a physicist the pleasures are special because there is scarcely a physical system which contains so many of the elements of modern physics. Two-state systems, of which this is an example, abound, but this one has special properties which give it a unique beauty. I hope that I can convey to you some of the reasons why this system has held such a fascination for us. The story begins with the isotopic spin-strangeness assignment of Gell-Mann and Nishijima. The assignment of

the K mesons to two doublets makes the K^0 and \bar{K}^0 distinct entities. But both particles decay to two π mesons. If the physicist sees π^+ and π^- mesons in his detector, which is the source, the K^0 or \bar{K}^0 ? The problem was solved through the remarkable insight of Gell-Mann and Pais in their 1955 paper (15). In the spirit of quantum mechanics it is necessary that the source of the π^+ and π^- mesons be some linear combination of K^0 and \bar{K}^0 states. They observed that a $\pi^+\pi^-$ final state is even under charge conjugation. By even, we mean that the wave function does not change its algebraic sign upon interchange of particle and antiparticle. This evenness condition is obviously met by the combination $K^0 + \bar{K}^0$. This they called the K_1^0 (16). If this is the case, there must be another state equally probable, $K^0 - \bar{K}^0$, the K_2^0 , which is odd under charge conjugation and, correspondingly, is forbidden to decay to $\pi^+\pi^-$. But it can decay to many other states, three-body states such as $\pi^+\pi^-\pi^0$. It was expected that the decay to the three-body states would be substantially inhibited compared to the two-body one. The particle corresponding to the K_2^0 would have a longer lifetime than the K_1^0 by about 500. In addition, it was expected that the K_1^0 and K_2^0 would have somewhat different masses, even though the masses of K^0 and \bar{K}^0 are strictly equal by the CPT theorem.

This long-lived neutral K meson, predicted by Gell-Mann and Pais, was then looked for and found by a Columbia group working at the Brookhaven cosmotron (17). The theoretical model, based on the notion of charge conjugation invariance in the weak interactions, had been confirmed. Then suddenly parity was found to be violated in the weak

interactions along with charge conjugation. This dark cloud was almost immediately removed with the observation that one had only to replace C with CP and the story of the neutral K mesons would remain the same (13). With CP invariance the K_2^0 would continue to be absolutely forbidden to decay to two pions. The successful description of the neutral system of K mesons has been characterized by Feynman (18) as "one of the greatest achievements of theoretical physics."

Additional features of the $K^0\bar{K}^0$ system become evident if we write the wave function including the lifetime and energy terms for the case of production of a K^0 at time $t = 0$

$$\Psi(t) = \frac{1}{\sqrt{2}} \{ |K_1^0\rangle e^{-i/2\tau_1 + i\omega_1 t} + |K_2^0\rangle e^{-i/2\tau_2 + i\omega_2 t} \}$$

$$|K_1^0\rangle = \frac{1}{\sqrt{2}} [|K^0\rangle + |\bar{K}^0\rangle]$$

$$|K_2^0\rangle = \frac{1}{\sqrt{2}} [|K^0\rangle - |\bar{K}^0\rangle]$$

It is seen that after a time, long compared to the K_1^0 lifetime and short compared to the K_2^0 lifetime, the state that was originally a pure K^0 will become a K_2^0 , which in turn is an equal mixture of K^0 and \bar{K}^0 . To give a measure of the magnitudes involved we should point out that the K_1^0 meson, in a typical experimental situation, travels an average of a few centimeters before it decays, whereas the K_2^0 travels tens of meters. At a distance greater than about 1 meter from the point of production of a K^0 , a nearly pure K_2^0 beam will be present.

Another important characteristic of the system becomes apparent when we consider the interaction of K_2^0 's with matter. The K_1^0 's and \bar{K}_1^0 's, by virtue of their opposite strangeness, have quite different interaction cross sections. Passing a beam of K_2^0 's through a block of material will result in a mixture of K_1^0 's and \bar{K}_1^0 's which, because of differential absorption of the two components, is no longer 50-50, but instead a mixture equivalent to a new combination of K_1^0 's and K_2^0 's. The newly produced short-lived K_1^0 's decaying to $\pi^+\pi^-$ will appear behind the material. This phenomenon is called regeneration (19). In the case of the absorbing material being completely transparent to K_1^0 's and opaque to \bar{K}_1^0 's, the intensity of the K_1^0 's after the absorber will be one-fourth the initial intensity of the K_2^0 incident on the absorber.

In the late 1950's M.L. Good (20) observed that with a very small mass

difference between the K_1^0 and K_2^0 the regeneration phenomena just discussed would result in a coherent process. By coherent we mean that the scattering process of K_2^0 to K_1^0 would not be from individual nuclei but from the whole block of scattering material. That is, the block of material would remain in its initial quantum mechanical state during the scattering process. In this case, as with ordinary light passing through glass, the regeneration material could be treated as having an index of refraction. The K_1^0 's regenerated coherently would have precisely the same energy as the incident K_2^0 's and an angular distribution identical to that of the incident beam but broadened by diffraction effects determined by the size of the regenerating material perpendicular to the beam. A characteristic wavelength for the K_2^0 mesons in a typical experiment is about 10^{-13} cm. The transverse dimensions are typically 10 cm. The corresponding diffraction pattern has a width of the order of 10^{-14} radian. In addition, the coherent addition of K_1^0 waves has been observed over distances greater than 10^{14} wavelengths. The unique feature of this coherently regenerated K_1^0 beam is that it can be distinguished from the original beam since it decays with a short lifetime to $\pi^+\pi^-$. To my knowledge, it is the only

instance where a forward coherently scattered beam can be distinguished from the original beam.

It has become evident to physics students in the audience that the $K_1^0 K_2^0$ story has an analogy in polarized light. The K_1^0 and K_2^0 correspond to the left and right circularly polarized light, and the K^0 and \bar{K}^0 states are equivalent to the x and y components of linear polarization. The passage of a K_2^0 beam through a block of condensed material is equivalent to the passage of left circularly polarized light through a doubly refractive medium like calcite, which has a different index of refraction for the x and y components of polarization. The general picture of regeneration, coherent and incoherent, was confirmed in a definitive bubble chamber experiment (21).

There are many associated phenomena still to be explored. For example, experiments coherently regenerating K_1^0 's from the planes in crystals have yet to be done. At the particle momenta commonly available the Bragg angles are exceedingly small, and the extinction factor, the Debye-Waller factor, comes into play at correspondingly small angles, but the experiments could be done.

Unexpectedly, the $K^0 \bar{K}^0$ system provides us with important and highly precise information about the gravitational interaction. It relates to the question of strong universality, that is, whether different objects, in this case particle and antiparticle, with the same inertial mass behave the same in a gravitational field. As observed by M. L. Good (22), if the K^0 and its antiparticle, the \bar{K}^0 , had opposite gravitational potential energy, the $K^0 \bar{K}^0$ system would mix so quickly that the long-lived particle would never be seen. By analyzing the system in more detail, one can show that if the gravitational interaction of particle and antiparticle differ by a fraction κ , then κ must be less than 10^{-10} if we are dealing with the gravitational field of the earth, 10^{-11} for the solar system, and 10^{-13} for the galaxy.

Voyages of discovery can be made in new uncharted waters, but also in the familiar bays close to port, provided one has observing apparatus that can see familiar objects with detail greater than that previously possible. In 1963 we had the opportunity to investigate the neutral K meson phenomena with resolution greater than that permitted before. The introduction of spark chambers as charged particle detectors permitted precise track position determination, but also they could be selectively triggered on appropriate classes of events.

Using such new devices with our col-

leagues Jim Christenson and Rene Turlay, Jim Cronin and I initiated a systematic study of (i) the regeneration phenomena, (ii) what we called CP invariance, and (iii) neutral currents. We were interested in the regeneration phenomena in particular because of an anomaly that had just been reported by a group studying the passage of K_2^0 's through a liquid hydrogen bubble chamber (23). Not many of our colleagues would have given us much credit for studying CP invariance, but we did anyway, and neutral currents, of long interest, were discussed by Sheldon Glashow on this occasion 1 year ago.

A plan view of the apparatus we used for these studies is shown in Fig. 1. It is a two-armed spectrometer, each arm with spark chambers before and after a magnet for track delineation. Cerenkov and scintillation counters in both arms operated in coincidence provided the signals to trigger the spark chambers, which were recorded photographically. The apparatus was placed in a beam of neutral particles at the Brookhaven alternating gradient synchrotron (AGS) at a distance such that K_1^0 's would have decayed away, leaving K_2^0 's. The angle between the spectrometer arms was chosen to optimize the detection of K^0 mesons decaying to two π mesons. In the regeneration studies blocks of various solid materials were placed in the neutral

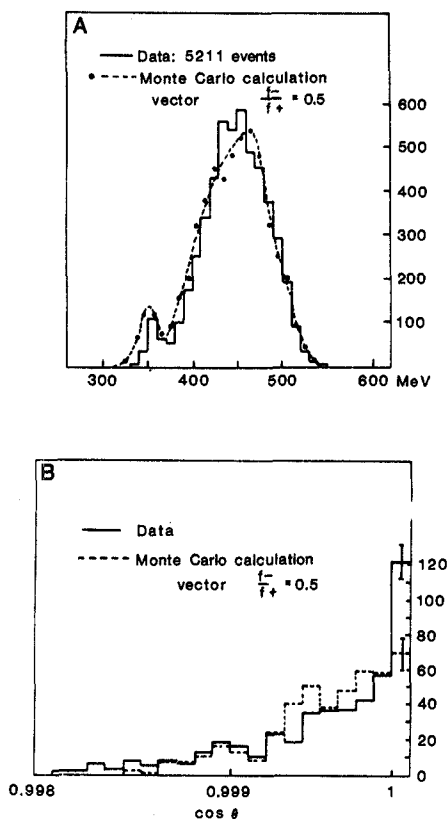


Fig. 2. Angular distribution of the events in the appropriate mass range measured by a coarse machine.

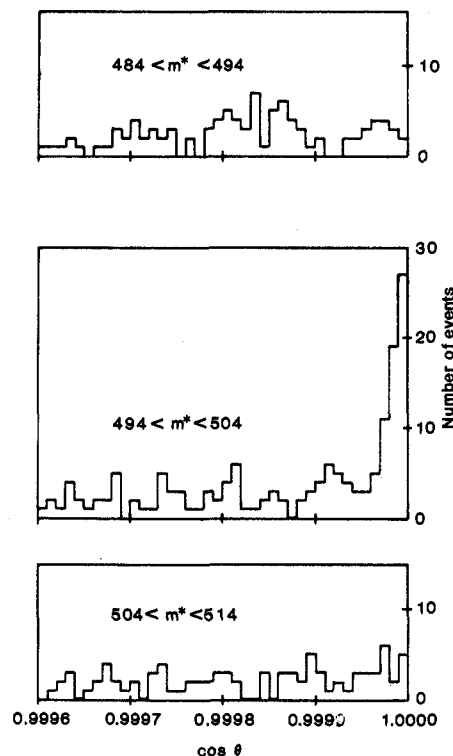


Fig. 3. Angular distribution of the events after measurement by a precise machine in three relevant mass regions.

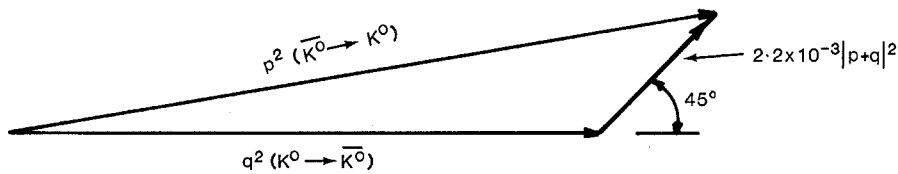


Fig. 4. Vector diagram showing schematically the difference in the amplitudes for $K^0 \rightarrow \bar{K}^0$ and $\bar{K}^0 \rightarrow K^0$.

beam. In the studies of the free decay of $K^0 \rightarrow$ two pions, the decay volume was filled with helium gas to minimize the interactions.

The decay to two pions is distinguished from the copious three-body decays in two ways. The sum of the momenta of the two detected particles must line up with the direction of the incident K^0 's. In general, this will not happen for three-body decay. In addition, the mass computed for the parent particle must match the mass of the K^0 meson. The original data are shown in Figs. 2 and 3. Figure 2 shows the data after measurement of the photographic records on a relatively coarse measuring machine. The presence of the peaking of events along the beam line stimulated more precise measurements, and these results are shown in Fig. 3. Clearly there are about 56 events in the forward peak in the proper mass interval where the background is 11. From these data we established that the branching ratio of K^0_2 to two pions relative to all the charged-mode decays was 2×10^{-3} . Here was the first evidence for the decay completely forbidden by CP conservation (23). We were acutely sensitive to the importance of the result and, I must confess, did not initially believe it ourselves. We spent nearly half a year attempting to invent viable alternative explanations, but failed in every case.

The study of coherent regeneration was important for the CP measurement for several reasons. First, the results we found were entirely consistent with expectations; there were no anomalies. The measured coherent regeneration rates in tungsten, copper, carbon, and liquid hydrogen enabled us to show that coherent regeneration in the gaseous helium which filled the decay volume would produce a totally negligible contribution to the signal we observed. Second, the coherent regeneration of the K^0_1 's, which subsequently decayed to π^+ and π^- mesons, provided an invaluable calibration of the apparatus.

It is appropriate now to look at the neutral K system in a somewhat more quantitative way (24). Because of the mixing of the K^0 and \bar{K}^0 through the weak interaction, the time rate of change

of a K^0 wave will not only depend on the K^0 amplitude but also on the \bar{K}^0 amplitude, namely

$$-\frac{dK^0}{dt} = AK^0 + p^2\bar{K}^0$$

and

$$-\frac{d\bar{K}^0}{dt} = B\bar{K}^0 + q^2K^0$$

We have let the particle symbol stand for the amplitude of the corresponding wave. With invariance under CPT, particle and antiparticle masses and lifetimes must be precisely identical. In terms of the equations above, A must be equal to B . Now CP violation can, in fact, occur in two ways, either through terms in the set of equations above or in the amplitudes for the decay. Subsequent experiments show that most, if not all, of the violation is in the equations above, involving the so-called mass-decay matrix. Professor Cronin will discuss the ramifications of the effect being present also in the decay terms. Suffice it to say here that any departure of p^2 from q^2 will result in the decay $K^0_2 \rightarrow$ two pions. With CP nonconservation the short- and long-lived particles are no longer the K^0_1 and K^0_2 previously defined, but rather

$$K^0_S = \frac{1}{\sqrt{p^2 + q^2}} \{p|K^0\rangle + q|\bar{K}^0\rangle\}$$

and

$$K^0_L = \frac{1}{\sqrt{p^2 + q^2}} \{p|K^0\rangle - q|\bar{K}^0\rangle\}$$

The fact that K^0_L decays to two pions shows that the amplitude for particle-to-antiparticle transitions, in this case $K^0 \rightarrow \bar{K}^0$, does not quite equal the reverse, $\bar{K}^0 \rightarrow K^0$, and indeed we now know rather precisely that not only are the magnitudes somewhat different but there is a small phase angle between the two amplitudes (see Fig. 4). We indicated earlier that, through the CPT theorem, a violation of CP is equivalent to a violation of time reversal invariance. As Professor Cronin will show, the CPT theorem has been shown to hold in the neutral K system independently, so in a self-contained way a violation of time reversal invariance is demonstrated.

We are all familiar with the time asymmetry associated with entropy. Entropy in a closed system increases with time. This kind of time asymmetry results from the boundary conditions. But for the first time we have in the neutral K mesons a physical system that behaves asymmetrically in time as a result of an interaction, not a boundary condition.

Since the microscopic physical laws had always been thought to be invariant under time reversal, this discovery opens up a very wide range of profound questions. Professor Cronin will go into some of these questions in greater detail. I will mention two. Can this effect be used to decrease the entropy of an isolated system? We look out from the earth and see a highly ordered universe. With entropy always increasing, how can this be? Is CP violation an effect that can be used, in effect, to wind up the universe? The answers to these questions appear to be no (25).

At the same time, we look out from the earth and see the remains of an earlier, much hotter universe. In that earlier time one expects that matter and antimatter would condense out in equal amounts and eventually annihilate to gamma radiation. However, no evidence of antimatter is seen. The gauge theories described on this occasion 1 year ago allow for the possibility of proton (and antiproton) decay. This process, coupled with CP violation, drives the universe toward a preponderance of matter over antimatter and can account for the observed ratio of the amount of matter to radiation (26).

Lewis Thomas, whose essays on science grace our literature, has written, "You measure the quality of the work by the intensity of the astonishment." After 16 years, the world of physics is still astonished by CP and T noninvariance. I suspect that the Nobel Committee was motivated by considerations similar to those of Thomas in awarding to Professor Cronin and myself this highest of honors.

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Distal Regeneration and Symmetry

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The way in which the activities of the cells of an embryo are coordinated in space and time is one of the great remaining enigmas of biology, and it is a problem which has not, so far, benefited significantly from recent advances in the methodology of molecular biology. Instead, our efforts at understanding the formation of spatial pattern in developing animals are still comparable to the pre-Mendelian stage of genetics; we are

structive in gaining an understanding of the functioning of the normal genome. The details of many naturally occurring abnormalities were described as long ago as 1894 by Bateson (1), who was the first to attempt to define a set of rules to describe the symmetry relationships of extra appendages. Since then, the abnormalities resulting from experimental manipulation have been thoroughly analyzed in several experimental systems.

Summary. A revision of the "polar coordinate model" shows how pattern formation in diverse regenerating systems can be understood in terms of strictly local cell interactions.

still searching for the formal "rules" by which we can predict the behavior of embryos under various experimental treatments. Only when we understand these principles at a formal level, do we expect to be able to frame appropriate questions for a molecular analysis.

Although we seek to understand the events occurring in normal embryonic development, it is the bizarre mistakes made by the developing organism, either spontaneously or in response to experimental manipulations, that are most instructive, just as mutant genes were in-

We showed several years ago (2) that most of the pattern regulation (developmental responses to surgical intervention) seen in the appendages of amphibians and cockroaches as well as the imaginal discs of *Drosophila* could be understood in terms of a single set of rules, the polar coordinate model. Some parts of this model made use of local interactions between cells, but one of the rules (the "complete circle rule") implied that more long-range interactions could occur between cells. In this article, we propose an alternative formulation taking into account recent experimental findings, and the revised model involves only interactions that occur on a strictly local level between cells and their nearest neighbors.

The Polar Coordinate Model

We begin from Wolpert's (3) idea that spatial patterns result from cells acquiring information about their physical positions in the developing cell population. We consider this positional information to be specified along polar coordinates in two dimensions (that is, in a cell layer), so that each cell would have a positional value with an angular and a radial component. In the limb or imaginal disc, the radial sequence of positional values corresponds to the (presumptive) proximal-distal axis of the appendage, with the distal tip at the center of the field (Fig. 1), while the circumferential sequence of positional values corresponds to the circumferential position on a cross section at each proximal-distal level.

The most important feature of this model is the proposition that tissues have the general property of intercalation. When cells from two normally non-adjacent radial or circumferential positions are confronted with one another as a result of either grafting or wound healing, the discontinuity in positional values stimulates local cell division, and during this growth, those positional values are generated which normally lie between the confronted positions. If the confronted cells differ in circumferential position, then intercalation generates the set of positional values that separates the two points by the shorter, rather than the longer, of the two possible circumferential routes (the "shortest intercalation rule"). This proposition is supported by direct evidence from studies of the cockroach leg (4, 5) and from indirect evidence from studies of imaginal discs and amphibian limbs (2). An important consequence of intercalation is that it results in patterns that show continuity, in which all pattern elements are adjacent to their normal nearest neighbors or to extra copies of themselves, even though globally the pattern may be bizarre (6). It is important to note that the systems we discuss are those which show epimor-

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