Reports

Turbulent Vertical Transport due to Thin Intermittent Mixing Layers in the Stratosphere and Other Stable Fluids

Abstract. *Pollution effects in the stratosphere and the ocean are exacerbated by buoyant stability. Turbulence in such media is confined to thin layers. To estimate vertical transport by such turbulence, one can view the situation as nature's way of simulating the finite-difference diffusion equation. This analogy finally yields a diffusivity parameter which is valid for this extremely inhomogeneous case.*

The effects of pollution in the stratosphere and the ocean are of increasing concern to society. Unfortunately, these environments have high static stability in the sense that a displaced fluid parcel tends to return to equilibrium height. This traps pollution so that it can, for example, catalytically destroy stratospheric ozone. It also severely restricts the vertical extent of turbulence so that the latter occurs in thin, horizontal, pancake-shaped layers which, in the stratosphere, are of the order of 100 m thick and several kilometers wide. In the ocean a thickness of 10 cm is typical. Such layered turbulence is extremely inhomogeneous in the vertical direction, and for this and other reasons it appears that vertical transport cannot be validly described by the usual eddy diffusion parameter of turbulence theory.

The purpose of this report is to develop a self-consistent bulk diffusion parameter, K_B, which is valid for layered turbulence in a stratified fluid and which can be related to experimental observables. It will be assumed that the (horizontal) flow between the turbulent layers is laminar and that molecular diffusion is relatively insignificant. Total mixing is assumed to take place within the mixing layers. The latter assumption is consistent with the observation of steplike structures in vertical temperature profiles in the upper ocean (*1*) and of similar structures in potential temperature profiles (over regions around 100 m thick) in the stratosphere (*2*) and upper troposphere. Another assumption is that the turbulent layers occur at random heights with random thicknesses (*1*). Finally, it is assumed that homogeneity in the horizontal direction permits use of a one-dimensional model.

A series of computer experiments is used to demonstrate that (i) vertical transport by random mixing layers can be regarded as a form of "discontinuous diffusion" and (ii) K_B is independent of initial and boundary conditions—that is, it is a valid diffusion parameter. In effect, the latter is the assertion that a series of random averages (over small regions) can serve as a Monte Carlo simulation of the diffusion equation.

The mechanism for vertical transport by the action of mixing layers is shown in Fig. 1. Let ϕ be, for example, the pollutant concentration or mixing ratio. For simplicity, ϕ is assumed to vary linearly with altitude, z. Let I denote a mixing layer. As a result of the mixing, ϕ will become constant within the layer, as shown by the dotted line. Material in region A can thus be transported to B. Once total mixing has occurred, there will be no further net vertical transport. The turbulence in I will eventually decay. Subsequently, a second mixing layer, II, could form which, by chance, overlaps the region originally occupied by I. It should be evident that this would cause further downward transport. The combined effect of I followed by II will thus cause some material to move from A to the bottom of II. Clearly this transport process has two essential aspects, namely mixing and overlap.

Digital computer simulations of the effects of random mixing layers on various initial distributions of ϕ were performed in the following way. The values of z were represented by 400 points. Initial values of ϕ were designated at these points (different assignments for the various cases). Two random number generators were used in order to (i) pick a random value of z on which to center the layer and (ii) pick a layer thickness, Λ, according to the following assignments of N, where N is a random number of uniform distribution from 0 to 99. When $0 \leq N \leq 53$, Λ was set equal to 3 points. Similarly, $53 < N \leq 74$ gave $\Lambda = 5$; $74 < N \leq 85$, $\Lambda = 7$; $85 < N \leq 92$, $\Lambda = 9$; and $92 < N \leq 99$, $\Lambda = 11$. This choice of probabilities was based on a distribution reported in (*2*), but which for present purposes should be regarded as arbitrary but reasonable. Based on this assignment, the root-mean-square of Λ was about 5.5 points. The effect of mixing was simulated by averaging. For example, the mixing of a 3-point layer covering points $z = 251$, 252, and 253 would be simulated by replacing the original values of ϕ at these 3 points by the average of ϕ over these points.

Figure 2 shows the time development of $\phi(z)$ for the first case. Its initial distribution was $\phi = 0$ for all 400 values of z except for the point $z = 200$. At that single point $\phi = 100$ initially. The curves started when 250 mixing events had been simulated and were drawn at intervals of 500 events, concluding when 26,256 events had been simulated. There is a

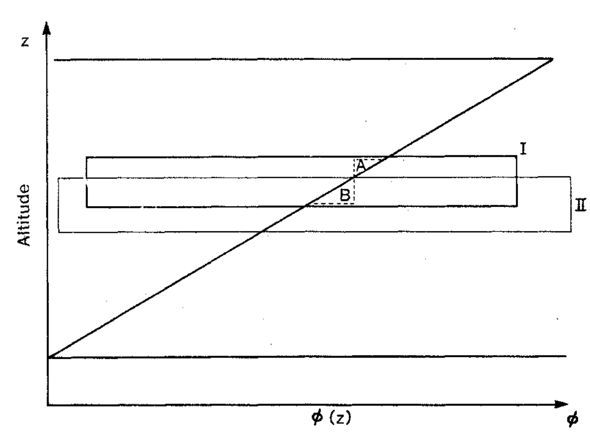

Fig. 1. Mechanism of transport perpendicular to turbulent mixing layers in a stratified fluid. The symbol ϕ denotes pollutant concentration or mixing ratio.

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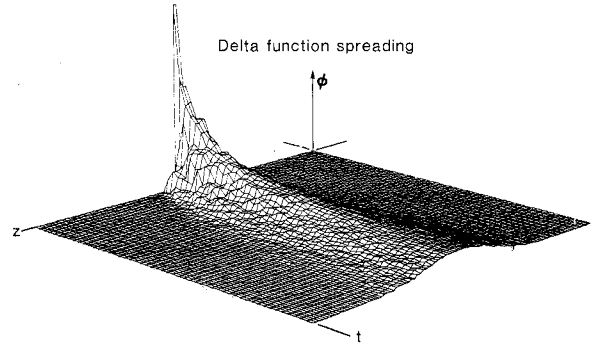

Fig. 2. Development of ϕ as a function of time as a result of random mixing layers simulated on a computer. The initial ϕ is a delta function.

striking similarity between this spreading behavior and the behavior seen in ordinary diffusion.

To calculate K_B for the process shown in Fig. 2 we consider the solution of the diffusion equation for the case where ϕ is initially a delta function centered at z_0. The well-known solution (one-dimensional, $-\infty < z < \infty$) is

$$\phi(z,t) = \frac{1}{\sqrt{4\pi Kt}} e^{-(z - z_0)^2/4Kt} \quad (1)$$

where K is a constant and t is time.

To determine K, one can measure $z - z_0$ such that ϕ has fallen by the amount $1/e$ from its peak value at z_0. Designating this value of $z - z_0$ by r_e, the bulk diffusion $K_B = r_e^2/4t$, as can be seen by setting the exponent of Eq. 1 equal to unity. This was done for the case where 30,400 mixing events had occurred and the result, in nondimensional form [in units of $(\overline{\Lambda^2})^{1/2}$ and Δt], was $K_{B\text{-delta}} = 1.45 \times 10^{-3}\ \overline{\Lambda^2}/\Delta t$. Here, $\overline{\Lambda^2}$ refers to mean square layer thickness and Δt stands for the average time interval between mixing events. The subscript B-delta indicates "bulk" diffusivity based on an initial delta-function distribution. Other subindices will be used for the alternative simulations below.

Since we are examining an unusual type of diffusion process, it is necessary to prove that it is a consistent one in the sense that the "constant" K_B is truly independent of the initial and boundary conditions on ϕ. We thus consider three more computer experiments which differ from the one above only in initial and boundary conditions. Case 2 was chosen to be the classic case of a semi-infinite slab with $\phi = 0$ inside and $\phi = 1$ outside initially. The computer model consisted of holding the value of points outside the slab fixed and letting the random mixing layers occur outside and inside the (finite) slab until the other end of the slab became involved. At the end of the 14,000 events, K_B was calculated by using the continuous solution $\phi = 1 - \text{erf}(y)$ where $y = z/2\sqrt{Kt}$ and erf(y) is the error function, which is tabulated in various places. The result was $K_{B\text{-int}} = 1.59 \times 10^{-3}\ \overline{\Lambda^2}/\Delta t$, where int is used for "intrusion." Case 3 was the "insulated slab" with $\phi = z/R$ $(0 < z < R)$ initially, where R is the slab thickness. The continuous solution (from Fourier-series expansion) shows that the first spatial harmonic, $A_1(t)$, decays as $(-4/\pi) \exp -K(\pi/R)^2 t$. By performing a fast Fourier transform on the computer solution to determine A_1 after 40,000 mixing events, it was determined that $K_{B\text{-insul}} = 1.50 \times 10^{-3}\ \overline{\Lambda^2}/\Delta t$. Finally, for case 4, a steady-state situation was simulated which consisted of calculating the flux of ϕ through a slab (region of z) 400 points thick in an "environment" 3000 points thick. The initial ϕ was set to the steady linear profile which resulted from the values of ϕ_{min} and ϕ_{max} at the minimum and maximum points of the environment. The program was devised by N. Grossbard in a manner which imposed no boundary conditions on the slab. The calculation of $K_{B\text{-flux}}$ involved Fourier's relation $K_{B\text{-flux}} = (\text{flux})/(\partial\bar{\phi}/\partial z)$, where the gradient was taken at the steady-state (calculated) value, and this gave the result 1.45×10^{-3} $\overline{\Lambda^2}/\Delta t$. A comparison of the various values of K_B above shows that, to an approximation, the "random layer" K_B is self-consistent and truly "diffusive" in nature. The presence of $\overline{\Lambda^2}/\Delta t$ for K_B could be immediately inferred from Fig. 1 by a dimensional argument.

Richardson number, Ri, profiles can be used to estimate both $\overline{\Lambda^2}$ and F, the fraction of the vertical dimension that is turbulent. By definition, $\text{Ri} = N^2/S^2$, where N and S are, respectively, the buoyancy (Brunt-Väisäla) frequency and the vertical shear of the horizontal fluid velocity. The usual criterion for dynamic (shear) instability is $\text{Ri} < 1/4$. It can be shown analytically (*3*) that $K_{B\text{-flux}}$ can be estimated from (*4*)

$$K_B = \frac{\overline{\Lambda^2}F}{8\Delta t_g} \quad (2)$$

The quantity Δt_g is the average time between an observation of the Ri profile and turbulent onset and may be measured by a combination of observations (*1, 5*) and theory (*3*). Richardson number profiles can be measured by in situ soundings [for example, from balloon (*1, 3, 6*)] and remote observations [for example, by radar (*5, 7–9*)] as well as with the help of smoke trail-derived velocity profiles (*3*). Allowance must be made for layer spreading effects (*1, 3*) in the estimates of Λ and F. To compare the results of Eq. 2 with the nondimensional estimates, one must use

$$\frac{\Delta t_g}{F} = \frac{\Delta t}{\overline{\Lambda}/R} \quad (3)$$

where R is the thickness of the fluid slab used in simulation. Equation 2 agrees exactly with the simulations, as can be shown with the help of Eq. 3.

In conclusion, a simple model of random mixing-layer transport in conjunction with computer experiments has led the way to a new bulk diffusivity parameter, K_B, which is valid for the type of stratified turbulence found in such fluids as the stratosphere and parts of the troposphere and ocean. This parameter can be estimated through Eq. 2 by means of measurements of Ri profiles and Δt_g in the environment. It is based on the idea that layered turbulence can be regarded as a sort of stochastic (Monte Carlo) simulation of the diffusion equation. The difficulties presented by the extremely inhomogeneous nature of stratified turbulence can thus be overcome.

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References and Notes

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10. I thank N. Grossbard for the computer programming and helpful suggestions. I also thank C. Stergis, J. Brown, A. Quesada, T. VanZandt, and J. O'Brien for their constructive criticisms. The image in Fig. 2 was generated by software provided by the National Center for Atmospheric Research and modified by P. Fougere.

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