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Origin and Mechanism of Large-Scale Climatic Oscillations

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In the geological history of the earth, long warm epochs have alternated with comparatively short cold epochs. The last warm epoch spanned an interval from approximately 180 million to 30 million years ago. During that time the climate at low latitudes was hotter than at present, at middle latitudes it was like that typical of the tropics and subtropics today; and at high latitudes it was similar to our modern temperate climate. A characteristic feature of the epoch was the apparent absence of significant climatic oscillations; the warm climate continued steadily. Thirty million years ago the climate began to cool, and the cooling increased as the present state was approached. During the last million years the temperature reached a low with the epoch of the great glaciations of the earth. The cold epoch is characterized by changeability and unsteadiness of the climate and large-scale oscillations of temperature, wind velocity, hydrological cycle, sea level, and other parameters of the environment.

What were the causes of these large-scale oscillations of the climate and glaciation during the last million years? Within the last 100 to 150 years, a number of hypotheses have been suggested to answer this question (*1*, *2*).

The majority of hypotheses explaining the origin of glacial oscillations are based on postulated causes such as variations of the solar constant, changes in solar activity, passage of the solar system through an interstellar gas-dust cloud, variations in the velocity of the earth's rotation, and gigantic surges of the Antarctic ice sheet. However, it is impossible to estimate the probability of occurrence of these causes or to prove that they occurred.

Summary. Numerical experiments with a simplified thermodynamic model of the glacier-ocean-atmosphere global system have been performed. Characteristic regimes of the system are auto-oscillations with periods varying between 20,000 and 80,000 years. The longer climatic waves are generated by the influence of variations of the earth's orbital parameters. Computed changes of glacial area, temperature, sea level, and other climate characteristics have values within expected ranges. The transition from a relatively warm epoch (when continental ice sheets are absent) to conditions characteristic of the Pleistocene is modeled. The calculated curves show how weak temperature fluctuations have been followed by large-scale oscillations.

Another group of hypotheses is based on analysis of changes in the earth's orbital parameters. These changes are considered to cause glacial and climatic oscillations through their direct effect on incoming solar radiation. However, if external disturbances play a major role in generating climatic and glacial oscillations, an important question still arises: Why were large-scale climatic oscillations with characteristic periods of 10,000 to 100,000 years absent during the last 100 million years while changes in the earth's orbital parameters were approximately the same?

Other hypotheses to explain the origin of glacial oscillations are based on consideration of an interaction between glaciers and the ocean. These hypotheses are relatively recent and consider feedback mechanisms for climate-forming processes. However, they are extremely speculative and usually contain an analysis of only some processes, while other, no less significant processes receive no attention and external disturbances are not taken into account. Therefore these hypotheses have not yet been widely recognized. More detailed examinations of the hypotheses of the origin of glacial and climatic oscillations are contained in Schwarzbach (*1*) and Sergin and Sergin (*2*).

Modeling the Glacier-Ocean-Atmosphere Global System

The work discussed here is concerned with the numerical modeling of large-scale oscillations of the climate and glaciation of the earth over the last million years. Figure 1 shows a qualitative curve of the spectral density of oscillations of the earth's average temperature in the Northern Hemisphere. The curve is based on the analysis of a large number of spectra; the data used were from instrumental observations of temperature, dendrochronology, time series based on isotope relationships, and estimates of the characteristic times of various geophysical and geological processes (*2*, *3*). The lowest frequency processes express the evolution of the earth as a planet—formation of the atmosphere and hydrosphere, monotonic change in the earth's rate of rotation, and so on. These processes have a characteristic time greater than 2 billion to 4 billion years. Orogenic rhythms and the alternation of warm and cold geological epochs are characterized

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by a time on the order of 30 million to 300 million years. The large-scale oscillations of climate and glaciation during the Pleistocene are characterized by periods of 10,000 to 100,000 years. The other oscillations have periods one order of magnitude less. The large sharp peak in Fig. 1 represents seasonal variations.

The simulation of physical processes with periods $\tau_0 = 10^4$ to 10^5 years must include the description of all internal processes having relaxation times $\tau \leq \tau_0$. Therefore the model of large-scale oscillations of the climate and glaciation of the earth must include the atmosphere, ocean, ice sheets, and land surface. This system is called the glacier-ocean-atmosphere (GOA) global system. The problem of hydrodynamic modeling of the GOA system was considered by V. Sergin (*4*). Numerical solution of the complete GOA hydrodynamic model has not yet been accomplished. We consider here a simplified thermodynamic model of the GOA global system and the performance of some numerical experiments.

We are interested in the dynamics of a system with a characteristic time of 10^4 to 10^5 years. The variables of significantly slower processes are treated as parameters in the dynamic equations of the GOA model. The gaseous composition of the atmosphere, the earth's rate of rotation, and the land distribution are considered invariant. They form some average state, deviations from which are expressed by the system under study.

When the variables of the system under study change, the processes of the climatic and meteorological spectrum, which occur much more quickly, may be considered to be in a steady state (that is, the time lag of faster processes can be ignored, which makes it possible to lower the order of the differential equations with respect to time). Therefore only the time lags represented by the ocean, continental ice sheets, and processes of isostatic compensation need to be taken into account.

Zonally averaged fields of variables are considered. The thermal field of the earth's surface may be represented by the average temperature and by the temperature difference between equator and pole. Other fields (such as wind velocity or cloudiness) may be represented by their characteristic values. When this is done, the field of geometry is fixed (for example, the vertical structure of the variables can vary in time with changes in their characteristic values, while retaining the same shape). This type of formulation considerably diminishes the number of degrees of freedom, and the

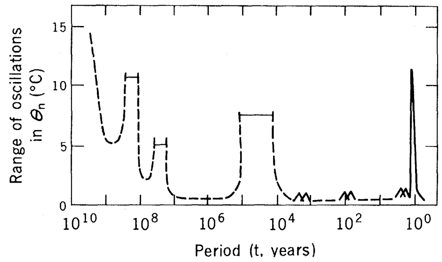


Fig. 1. Qualitative curve of spectral density of oscillations of average surface temperature θ_n in the Northern Hemisphere.

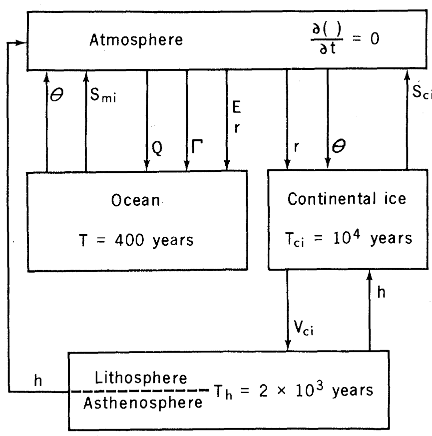


Fig. 2. Block diagram of the simplified GOA model. Symbols: θ, average surface temperature; S_{mi}, area of maritime ice; Q, total heat flux; Γ, equator-to-pole temperature difference; r and E, precipitation and evaporation rates, respectively; S_{ci}, area of continental ice; V_{ci}, volume of continental ice; h, sea level; and T, T_{ci}, and T_h, relaxation times for the ocean, continental ice sheet, and isostatic compensation, respectively.

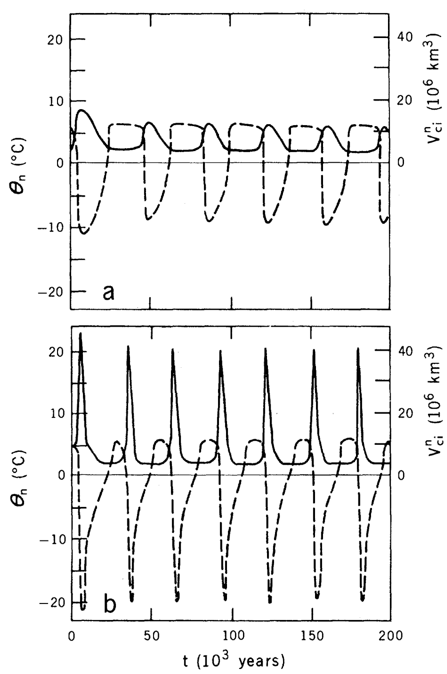


Fig. 3. Variation of average surface temperature θ_n (- - -) and of continental ice volume V^n_{ci} (——) in the Northern Hemisphere, calculated for nominal values of the parameters and two different choices for the depth of the ocean: (a) 2 km and (b) 8 km.

system can be described by ordinary differential equations.

All variables are expressed as average annual deviations from the corresponding quantity in the current climatic epoch. The first simplified thermodynamic model of the GOA global system was constructed by Sergin and Sergin (*2, 3, 5, 6*). A block diagram of the GOA global system (Fig. 2) shows the exchange between all components of the complete model. The lithosphere and asthenosphere are included in the GOA model because the land surface is pressed down if an ice sheet grows on the land mass and rises if an ice sheet disappears. Because sea level and the ratio of the sea and land areas on the earth change, the albedo of the earth and the rate of evaporation also change.

In calculating deviations of heat inflow into the GOA system, the following features are taken in account: variations in the rate of evaporation and turbulent heat exchange, variations in surface temperature and degree of cloudiness, variations in albedo caused by changes in the area of ice cover and extent of the ocean (associated with glacioisostatic fluctuations of sea level), and variations in energy transport across the equator. The global hydrologic cycle is also calculated. The GOA model is represented by a system of ordinary sixth-order nonlinear differential equations (*2, 7*).

Numerical Experiments

The first series of numerical experiments was performed for the Northern Hemisphere, with an impenetrable wall assumed at the equator. The experiments were carried out without external perturbations and with a broad variation in the values of selected parameters (*2, 7*). The curves in Fig. 3 depict the variations of the average surface temperature and of the volume of continental ice obtained with nominal values of the parameters and two different choices for the depth of the ocean. The amplitudes of these variations increase with increasing ocean depth. This means that the ocean should be deep enough to generate auto-oscillations.

The influence of the greenhouse effect on climate variations was also studied. The influence of water vapor on the absorption of long-wave radiation was prescribed as 50 percent less and 50 percent more than in the real atmosphere. After an initial model response, the increased greenhouse effect eliminates the auto-oscillation of the GOA model (Fig. 4); that is, the greenhouse effect helps to estab-

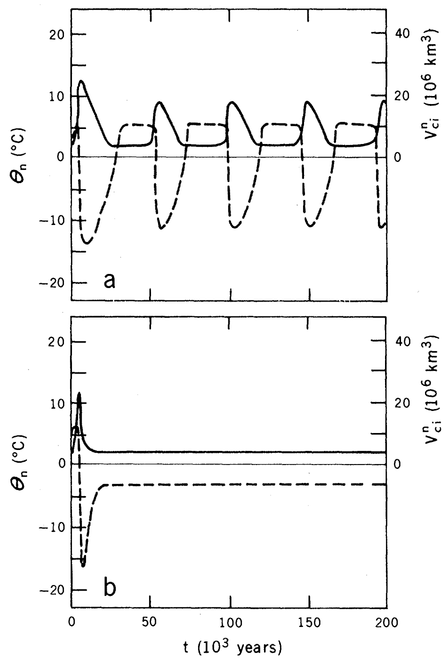


Fig. 4. Calculated variations of θ_n (- - -) and of V^n_{ci} (——) in the Northern Hemisphere with the influence of water vapor on the absorption of long-wave radiation prescribed as (a) 50 percent less and (b) 50 percent more than in the real atmosphere.

lish a nonoscillatory climate. This result may be useful for understanding the past climates of the earth and possibly the climates of other planets.

Cloudiness provides negative feedback in the model of the GOA system. Figure 5 presents the results of calculations for the case when changes in cloudiness do not influence the radiation budget and for the case when they influence the radiation budget to the maximum possible degree (2).

The auto-oscillations in these experiments are a typical regime of the GOA system for the Northern Hemisphere. Natural oscillations arise in this hemisphere as a result of nonlinear interactions between two large inertial components, namely, the heat capacity of the ocean and the accumulation of mass in continental ice sheets. The computed amplitudes and periods of the climate oscillations have reasonable values, and it is important that the temperature deviations from the present level toward cooling are two to four times greater than those toward warming. Current paleoreconstructions tend to confirm such oscillatory behavior.

In modeling the global system, the equations of the physical processes for the Southern Hemisphere and the Northern Hemisphere are the same, but with different numerical values for the parameters. The Antarctic ice sheet is treated as the single ice unit in the Southern Hemisphere and has some peculiarities. Its horizontal dimensions are limited on all sides by the ocean, and its area therefore cannot increase sharply. Moreover, because of the oceanic environment, the inflow of precipitation to the ice sheet is quite stable. Further, the geographic location of Antarctica ensures low temperatures. Therefore even during significant climatic variations, the size of the ice sheet would not be expected to decrease sharply. This is confirmed by the fact that it has already existed for at least 20 million years. According to current estimates, the variations in the size of the Antarctic ice sheet over the last million years have been about ±7 percent (8). In this case, there is only negligible feedback in the GOA system with respect to energy inflow due to the variation of the ice sheet's area. Thus practically no interaction takes place between the ocean and continental ice in the Southern Hemisphere and auto-oscillations are not generated there. Figure 6 presents the results of numerical experiments for the Southern Hemisphere. The GOA system of that hemisphere is characterized by aperiodic movements.

Figure 7 shows the variations of the GOA system including both the Northern and Southern Hemispheres, using the two values 0.1 and 0.5 for the coupling coefficient k_e for energy transport across the equator (in the real atmosphere $k_e = 0.4$). The scale of the temperature variations in the Southern Hemisphere and the characteristic periods of the natural oscillations increase with increasing k_e. Oscillations in the Southern Hemisphere are induced and occur as a result of energy transport across the equator.

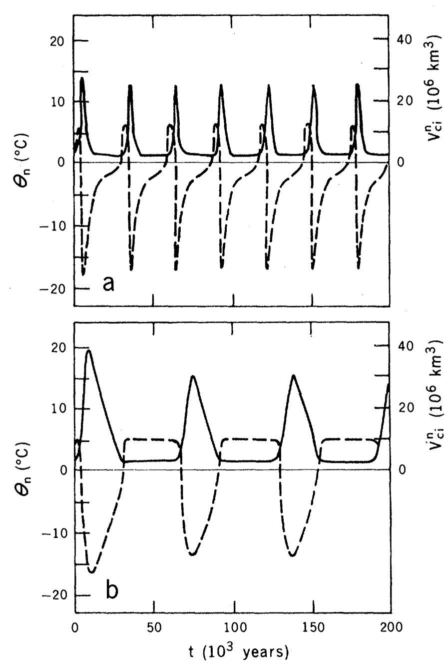


Fig. 5. Calculated variations of θ_n (- - -) and of V^n_{ci} (——) in the Northern Hemisphere when changes in cloudiness (a) do not influence the radiation budget and (b) influence the radiation budget to the maximum possible extent.

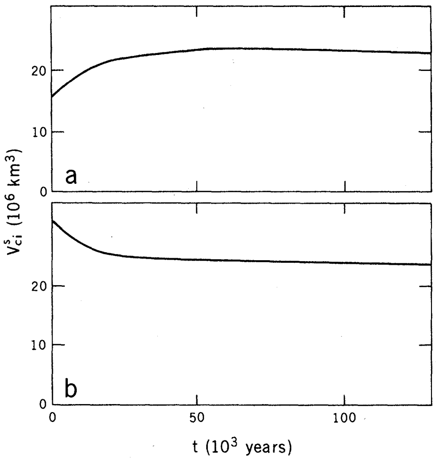


Fig. 6. Calculated curves of the volume of continental ice V^s_{ci} in the Southern Hemisphere for different initial conditions: (a) $V^s_{ci}(0) = 15 \times 10^6$ km^3 and (b) $V^s_{ci}(0) = 30 \times 10^6$ km^3.

Other results of the numerical experiments are presented in (2). The periods of calculated auto-oscillations vary from 20,000 to 80,000 years. The range of oscillations of the earth's average surface temperature, temperature difference between equator and pole, volume of continental ice, area of maritime ice, and other values are of the same order of magnitude as empirical data.

Characteristic qualitative features of climatic and glacial oscillations appear to be correctly reproduced in the model. For instance, as was the case for simulations for the Northern Hemisphere alone, the calculated temperature deviations from the recent condition are two to four times greater toward cooling than toward warming. Also, the variations in the mean temperature of the earth's surface in the Northern Hemisphere are approximately three times greater than those in the Southern Hemisphere.

Influence of External Perturbations

We consider next the influence of external perturbations on climatic oscillations with characteristic periods of 10,000 to 100,000 years. At present there are neither experimental data nor reliable theoretical calculations that indicate significant effects associated with the movement of the solar system through gas-and-dust nebulae, long period variations in the solar constant, solar activity, or the earth's rate of rotation. We therefore do not take these factors into account. The effect of sporadic discharges of volcanic ash, averaged over a long time interval, seems to be negligibly small (*2*). In addition, there are as yet no reliable quantitative data on this effect and we therefore neglect it. On the basis of similar considerations we also disregard the effect of meteoric matter entering the atmosphere.

Potentially significant disturbances include (i) deviations of the influx of solar radiation connected with variations in the orbital parameters of the earth and (ii) tectonic movements of the earth's crust. Variations in the orbital parameters of the earth [obliquity of the ecliptic, precession of the earth's axis, and eccentricity of the earth's orbit (*9*)] induce oscillations of the latitudinal distribution of insolation, which should give rise to variations in the temperature difference between low and high latitudes. In addition, they cause variations in the amplitude of the seasonal insolation curve, which should affect the amplitude of the seasonal temperature variation. The tectonically induced increase in the level of land during the Pleistocene was approximately 100 meters, while the elevation of mountainous regions was about 1 to 3 kilometers (*10*). At that time, the surface area of the continents increased by approximately 10^7 km^2, according to available data on the hypsometry of land and the bathymetry of the ocean (*2*). An increase in the earth's albedo (due to the difference between the albedo of land and that of water) and a more continental climate (range of the seasonal temperature variation) should result from such changes.

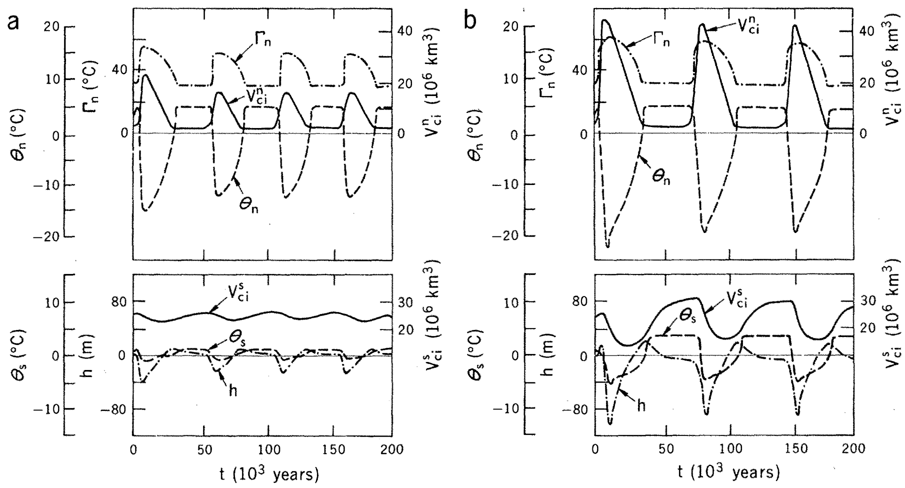


Fig. 7. Calculated variations of climatic characteristics in the global GOA system. Here θ_n, Γ_n, and V^n_{ci} are Northern Hemisphere average surface temperature, equator-to-pole temperature difference, and continental ice volume, respectively; θ_s and V^s_{ci} are Southern Hemisphere average surface temperature and continental ice volume, respectively; and h is the variation of sea level. The coupling coefficient k_e describing energy transport across the equator is 0.1 in (a) and 0.5 in (b).

Numerical experiments on the dynamics of the GOA system were performed taking into account the variations in the earth's orbital parameters and the tectonic movement of the earth's crust (*2*). Characteristic curves of the variation of average temperature, temperature difference between the equator and the pole, continental ice volume, and sea level are illustrated in Fig. 8. External perturbations have a noticeable modulating effect on the auto-oscillation of the model. The length of climatic waves generated in the presence of external disturbances exceeds the auto-oscillation period of the model. Tectonic disturbances in particular bring on a monotonic change of variables. Further results of the numerical experiments are presented in (*3*).

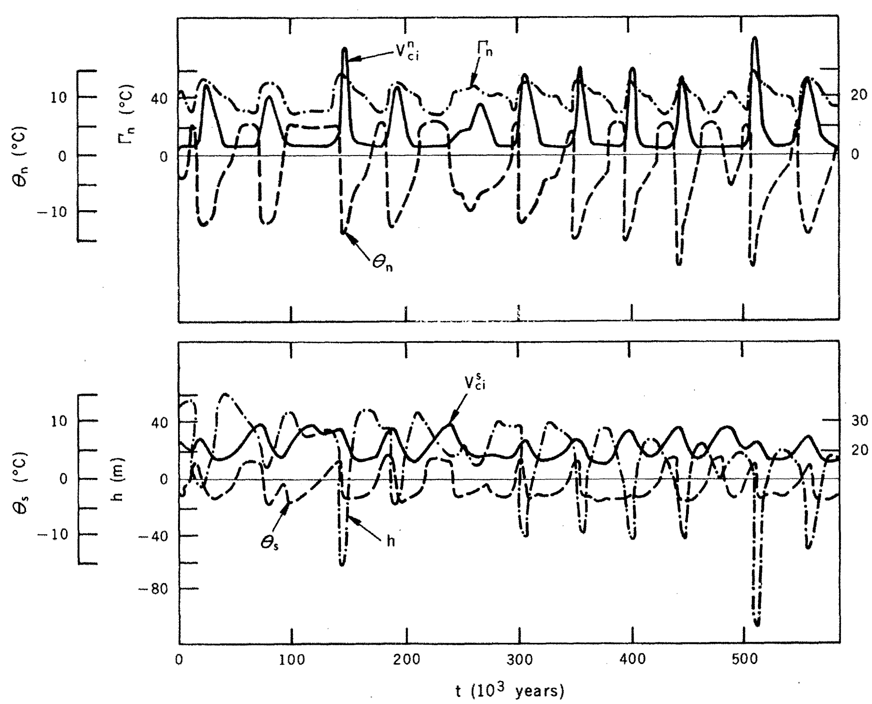
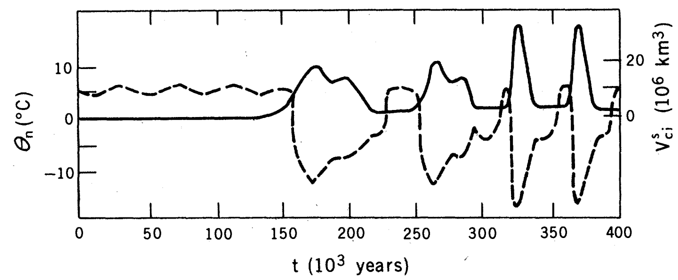


Fig. 8. Calculated variations of climatic characteristics in response to changes in the earth's orbital parameters and tectonic changes in land height and area. Numerical values of the model parameters are nominal. The key to the results is the same as for Fig. 7.

Modeling of the Beginning of Large-Scale Oscillations

According to contemporary paleogeographic data, ice cover appeared in the Antarctic 10 million to 30 million years ago and in Greenland about 3 million years ago—that is, in the Neogene. But large climatic and glacial oscillations began later in the Pleistocene. To explain this fact, a special experiment was performed.

It should be kept in mind that the Antarctic and Greenland ice sheets are of the island continent type. An increase in the dimensions of ice sheets of this type is limited by the ocean depth and a decrease is limited by the amount of precipitation, which is quite large. Fluctuations of their dimensions therefore cannot be sufficiently large to have a significant effect on the global climate. After coming to "rest" in their growth, such ice sheets enter a quasi-steady state. Their primary



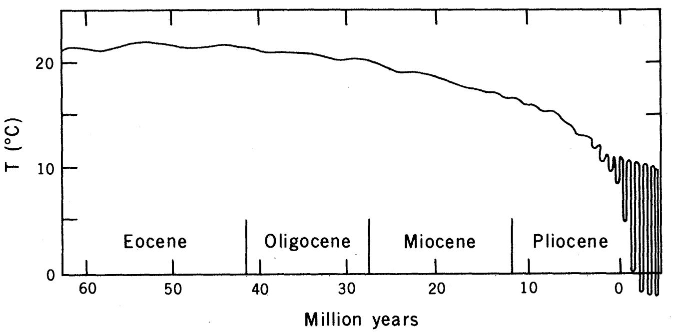


Fig. 9 (left). Simulation of the transition from a comparatively warm epoch to a typical cold epoch (the Pleistocene). Fig. 10 (right). Curve of the temperature change in central Europe (*11*). The time scale for the Pleistocene is larger than for the Tertiary. Temperature oscillations are insignificant in the warm epoch because of the aperiodic properties of the climatic system, which contains only one large inertial component (the ocean). The range of oscillations increases with decreasing temperature and becomes largest in the Pleistocene. This is caused by the growth of the second large inertial component (continental ice sheets), which changes the dynamic properties of the climatic system; the system becomes auto-oscillatory.

role is to lower the overall temperature of the earth's surface.

Large fluctuations in size are characteristic of ice sheets of the continental type, which do not have mechanical limitations due to purely external factors. Among them are the Pleistocene North American and Eurasian ice sheets. The effect of their appearance on the global climate can be tested by setting up a special experiment with the GOA model. Absence of unrestricted glaciers in the model can be expressed by eliminating their effect on the other components of the system. To do this, it is technically necessary to introduce a coefficient (designated as π) in the continental ice formation block for the Northern Hemisphere and to assume initially that $\pi = 0$. The transition to the appearance of continental ice sheets can then be roughly simulated by varying π from 0 to 1.

To save some computer time, we selected 400,000 years as the length of the transition period. During the experiment, we allowed external perturbations associated with the effect of variations in the orbital parameters of the earth to serve as inputs to the model.

The results of this experiment (Fig. 9) were obtained from the following variations of the coefficient: $\pi = 0$ for the time interval 0 to 100,000 years, π increases uniformly from 0 to 1 for the time interval 100,000 to 300,000 years, and $\pi = 1$ thereafter. When unrestricted glaciers are absent in the system, only small temperature fluctuations (with a range of about 1°C) due to astronomical factors are observed. With the onset of the appearance of unrestricted glaciers a slow decline in temperature occurs, and when the temperature reaches the level characteristic of interglacial periods ($\pi \approx 0.4$) large-scale oscillations of climate and glaciation set in. This experiment directly demonstrates the role of the second inertial component in the system. Only with the appearance of continental glaciation do the dynamic properties of the climatic system change—the system becomes oscillatory. The effect of astronomical factors is then intensified and manifested in increased deviations of climatic characteristics. The overall decline in average surface temperature is 6° to 7°C in the Northern Hemisphere and about 2°C in the Southern Hemisphere. Figure 10 shows the empirical curve of temperature changes in central Europe (*11*). The calculated results are in qualitative agreement with paleoclimatic reconstruction data, although the time scales are quite different because of the choice of π.

Conclusions

Simulation of the GOA system and its variations, taking into account the effects of external disturbances, provides the basis for the following conclusions. The characteristic regimes of the system are auto-oscillations, which have been shown to occur with realistic numerical values of all parameters. Auto-oscillations occur in the Northern Hemisphere because of nonlinear interactions between the two large inertial components determined by the heat capacity of the ocean and mass accumulation in continental ice sheets. In the Southern Hemisphere the system is aperiodic; oscillations are induced and occur because of energy and mass transfer across the equator. External disturbances influence the period and range of climate oscillations. The calculated periods and variations in the mean temperature of both hemispheres, the sizes of marine and continental ice covers, the ocean level, and other values are of the same order of magnitude as the empirical data. Present climatic conditions are unstable for any realistic values of the model parameters. However, the growth of the total area of continental and maritime ice is limited to about 20 to 30 percent of the hemispheric area. Variation of the model parameters does not lead to increased glaciation, and this indicates that complete glaciation of the earth is not possible.

The calculated curves for the transition from a relatively warm epoch to the Pleistocene show how weak temperature fluctuations are succeeded by large-scale oscillations. Thus the simulation results not only answer the question of the cause of oscillations in the earth's climate and of glaciation during cold epochs, they also explain the absence of oscillations during warm epochs; in the absence of the second inertial component (continental ice), the system is essentially aperiodic. Because no assumption about the cause of the climatic cooling that led to the formation of unrestricted ice sheets was introduced in this work, the results are valid regardless of the cause of the initial cooling (for example, orogeny and the elevation of continental platforms, or continental drift).

Some recent work tends to confirm the idea that variations in the earth's orbit are the fundamental cause of the succession of late Pleistocene ice ages (*12*). An overview of climatic variability and its causal mechanisms was given in (*13*). Our work (*2–6*) represents a stricter physical approach to elucidating the causes of large-scale oscillations of the earth's climate and glaciation in the Pleistocene. In this approach the central problem is construction of a mathematical model of the global system including the continental glaciers, ocean, and atmosphere. Even if we conclude that external disturbances play a leading part in generating climatic oscillations, a mathematical GOA model is needed to calculate the transformation of external disturbances into the climatic and glacial variations observed.

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Early Two-Dimensional Reconstruction and Recent Topics Stemming from It

Allan M. Cormack

In 1955 I was a lecturer in physics at the University of Cape Town when the hospital physicist at the Groote Schuur Hospital resigned. South African law required that a properly qualified physicist supervise the use of any radioactive isotopes and since I was the only nuclear physicist in Cape Town, I was asked to spend $1^1/_2$ days a week at the hospital attending to the use of isotopes, and I did so for the first half of 1956. I was placed in the Radiology Department under J. Muir Grieve, and in the course of my work I observed the planning of radiotherapy treatments. A girl would superpose isodose charts and come up with isodose contours, which the physician would then examine and adjust, and the process would be repeated until a satisfactory dose distribution was found. The isodose charts were for homogeneous materials, and it occurred to me that since the human body is inhomogeneous these results would be distorted by the inhomogeneities—a fact that physicians were, of course, well aware of. It occurred to me that in order to improve treatment planning one had to know the distribution of the attenuation coefficient of tissues in the body, and that this distribution had to be found by measurements made external to the body. It soon occurred to me that this information would be useful for diagnostic purposes and would constitute a tomogram or series of tomograms, though I did not learn the word tomogram for many years.

At that time the exponential attenuation of x- and gamma rays had been known and used for over 60 years with parallel-sided homogeneous slabs of material. I assumed that the generalization to inhomogeneous materials had been made in those 60 years, but a search of the pertinent literature did not reveal that it had been done, so I was forced to look at the problem ab initio. It was immediately evident that the problem was a mathematical one, which can be seen from Fig. 1. If a fine beam of gamma rays of intensity I_0 is incident on the body and the emerging intensity is I, then the measurable quantity $g = \ln (I_0/I) = \int_L f\, ds$, where f is the variable absorption coefficient along the line L. Hence if f is a function in two dimensions, and g is known for all lines intersecting the body, the question is, Can f be determined if g is known? Again, this seemed like a problem which would have been solved before, probably in the 19th century, but again a literature search and inquiries of mathematicians provided no information about it. Fourteen years would elapse before I learned that Radon had solved this problem in 1917. Again I had to tackle the problem from the beginning. The solution is easy for objects with circular symmetry, for which $f = f(r)$, r being the radius. One has Abel's equation to solve, and its solution

$$f(r) = -\frac{d}{dr}\left[\frac{r}{\pi}\int_r^\infty \frac{g(s)\, ds}{s(s^2 - r^2)^{1/2}}\right] = -\frac{dI(r)}{dr} \qquad (1)$$

has been known since 1825. In 1957 I did an experiment in Cape Town on a circularly symmetrical sample consisting of a cylinder of aluminum surrounded by an annulus of wood. The results are shown in Fig. 2. Here $I(r)$ is plotted against r and the constant slopes indicate the constant values of the absorption coefficient in wood and aluminum. Even this simple result proved to have some predictive value, for it will be seen that the three points nearest the origin lie on a line of a slightly different slope from the other points in the aluminum. Subsequent inquiry in the machine shop revealed that the aluminum cylinder contained an inner peg of slightly lower absorption coefficient than the rest of the cylinder.

Further work occurred intermittently over the next 6 years. Using Fourier expansions of f and g, I obtained equations like Abel's integral equation but with more complicated kernels, and I obtained results like Eq. 1 but with more complicated integrands. However, they were also integrals from r to ∞, a point which I shall return to. These integrals were not too good for data containing noise, so alternative expansions were developed which dealt with noisy data satisfactorily. By 1963 I was ready to do an experiment on a phantom without circular symmetry with the apparatus shown in

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The author is professor of physics at Tufts University, Medford, Massachusetts 02155. This article is the lecture he delivered in Stockholm on 8 December 1979 when he received the Nobel Prize in Physiology and Medicine, which he shared with Godfrey N. Hounsfield. The article is published here with permission from the Nobel Foundation and will also be included in the complete volume of *Les Prix Nobel en 1979* as well as in the series Nobel Lectures (in English) published by Elsevier Publishing Company, Amsterdam and New York. Dr. Hounsfield's lecture will be published in a forthcoming issue.