but most of the short presentations were extracted from authors' longer written papers reviewed in (*1*).

We learned that most of the algorithm Khachian used is the work of other Soviet mathematicians: D. B. Yudin and A. S. Nemirovsky of Moscow, and N. Z. Shor of Kiev, whose article (*2*) states the algorithm in its clearest form and also its best form for practical computation—not only for LP, but for the much more general "convex programming" problem. Because of the multiple authorship and the basic idea—the generation of numbers which, geometrically, describe a sequence of ellipsoids which must all contain a solution and which shrink, so that it is eventually identified—we agreed to refer to it and its near relatives as the "ellipsoid algorithm" (EA). (Khachian showed that, for one EA variant, each step, and thus the whole algorithm, could in principle be executed in "polynomial time" on a suitable computer. That was the finding which excited theoretical computer scientists, since the simpler versions of the generally used method for solving LP problems, Dantzig's "simplex method," have been shown not to run in polynomial, but in exponential time, in the worst case. "Worst case" behavior is always the easiest to study; a theory of "average" behavior, which would explain the fact that, in practice, the simplex method acts like a highly efficient polynomial-time algorithm, does not exist. Misconceived as applying to the traveling salesman and other "NP-complete" problems, Khachian's result overexcited some journalists.)

There is agreement that Khachian's variant will not be used. In both theory and practice it can take thousands of steps to solve trivial problems. A typical real problem of good size might require, say, 1 hour of high-speed computer time to be solved by the simplex method. The computer time for Khachian's variant would be about 50,000,000 years (it is an algorithm for which the "worst case" and "average" behavior are not far apart).

The idea of the EA, though, is intriguing and admits a large number of possible improvements, to which most of the papers at our workshop were directed: the starting ellipsoid chosen can be much smaller than early work indicated; the sequence can often be made to shrink much more rapidly (the way of doing that was discovered independently by 16 of the 42 authors); there are better ways of formulating the LP problem for solution, ways to reduce its size as the EA progresses, improved "stop rules," and proposals for hybrid methods that would use the EA to locate a rough solution which could then be polished off by the simplex method.

Perhaps in combination these ideas will improve the computing speed by several orders of magnitude, and a few hope that, after enough development, the EA will overtake the simplex method for LP. Since the simplex method was invented in 1947, improvements in the algorithm itself are thought to have speeded it up by no more than a factor of 10; the more important development has been the exploitation of "sparseness," the fact that almost all the data in the large matrices used are zeroes and need not enter into the arithmetic. This could account for a factor of 100 in the problem mentioned above, which thus might require 1000 hours, or 0.1 years, using the original version of the simplex method (and 10,000 years on a 1947 model computer). It seems the EA would not have competed with the simplex method had it been available in 1947; what's worse, no one has yet seen how to use the valuable "sparseness" devices in the EA.

One can, however, say this for the EA: it solved a significant theoretical problem and can be used to solve others; it may still be practically useful for the difficult nonlinear problems that Shor deals with; and it certainly brought a new kind of excitement to one area of applied mathematics.

PHILIP WOLFE
Office of the Chairman, Mathematical Programming Society, IBM Research Center 33-2, Post Office Box 218, Yorktown Heights, New York 10598

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1. "A bibliography for the ellipsoid algorithm," March 1980. Available on request from P. Wolfe, IBM Research Center, P.O. Box 218, Yorktown Heights, N.Y. 10598.
2. N. Z. Shor, *Kibernetika* **13**, 94 (January-February 1977); translated in *Cybernetics* **13**, 94 (September 1977).

A Second "Random Walk"

For a second edition of *A Random Walk in Science* (Crane-Russak, New York, 1974), I should welcome contributions of humor in science, historic and contemporary: anecdotes, witty accounts, cartoons, self-deceptions, and hoaxes. Especially sought are items which, while humorous, also have value for history and insight—in all fields of science. Please identify fully the sources of contributions.

ROBERT L. WEBER
104 Davey Laboratory, University Park, Pennsylvania 16802