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A Simple Description of the 3 K Cosmic Microwave Background

Paul S. Henry

Summary. An intuitive model for the expansion of the universe is developed in which special relativity is used to describe events seen by a hypothetical observer in a Lorentz frame of reference. The cosmic microwave background photons he sees are the red-shifted remnants of hot photons emitted from the matter flying rapidly away from him. This special relativistic model, also called the Milne model, represents the extreme case of a Friedmann (general relativistic) universe in the limit of vanishingly small density of matter. The special relativistic model approximates an open universe (one that expands forever) with increasing accuracy as time evolves.

The discovery of the 3 K cosmic microwave background, for which A. A. Penzias and R. W. Wilson won a Nobel Prize in 1978 (*1*), was rare, if not unique, among discoveries in physics, because it generated widespread interest outside the physics community. The front-page announcement of its discovery (*2*) as well as numerous popular articles and books (*3*) show that the cosmic microwave background and the big bang theory that it vindicates have an intellectual appeal extending well beyond the field of academic cosmology.

In this article basic physics and intuitive arguments will be used to develop a quantitative, though simplified, description of the cosmic microwave background. The goal is an explanation of how conditions that existed early in the history of the universe lead to the background radiation observed today.

Big Bang and Steady State

The radiation that Penzias and Wilson detected was a whisper of microwave energy left over from the big bang, the gigantic explosion in which our universe was created. Faint as it was, the very existence of the whisper was powerful evidence (*4*) that the universe indeed had a definite beginning in time; that it evolved from a hot, highly compressed state some 20 billion years ago into the condition we observe today.

With their startling results Penzias and Wilson sealed the fate of the competing view of the cosmos, the steady state theory. This theory held that the universe had always been, and always would be, much the same as it is now. Steady state, however, could not offer any convincing reason why the universe should be pervaded with 3 K microwave radiation. But in the big bang picture this radiation plays an important role, and its existence had been predicted in 1948 by George Gamow (*5*), an early proponent of big bang cosmology. Confronted with such a beautiful agreement between prediction and measurement, supporters of steady state retreated, leaving the field of cosmology to the big bang theory.

The Big Bang

Of events before the moment of the big bang, if there ever was such a time, we know essentially nothing, and of the first few milliseconds afterward, when the temperature of this "primeval fireball" (*6*) was still above 10^{11} K, our knowledge is only sketchy. But after this time it seems reasonably certain that the fireball expanded and cooled according to well-known laws of physics (*7*). As the expansion progressed, various elementary-particle and nuclear reactions took place, which created the ingredients from which the galaxies would evolve. By the time the temperature had fallen to 10^6 K, these reactions were complete. At this stage a typical observer inside the fireball would have seen all around him a hot mixture of protons, electrons, and photons, as well as a few other species not of interest here. The particles remained in close thermal contact through frequent scattering interactions, so they all cooled down together as the fireball expanded.

When the temperature of the fireball reached approximately 3000 K, roughly the filament temperature of an incandescent light bulb, the thermal energy of the electrons and protons was no longer enough to overcome their electrical attraction, and they started to form electrically neutral hydrogen atoms, which hardly interacted with the photons at all. In effect, the matter suddenly became transparent and the photons were released to their surroundings in a white-hot 3000 K flash. Some of the photons would travel through the transparent fireball for billions of years, only to arrive at a distant portion of it that had become Earth, where Penzias and Wilson were waiting.

To understand why the photons from the 3000 K flash now have a characteristic temperature of just 3 degrees above absolute zero, we need a more careful description of what an observer within the fireball sees as it evolves. This will be presented in the next two sections.

The author is a Member of Technical Staff, Bell Laboratories, Holmdel, New Jersey 07733.

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The Expanding Universe

Figure 1 shows the view seen by a typical observer, labeled A, located somewhere within the fireball. In addition to the chaotic motion associated with thermal energy, he sees a general motion of matter radially away from him, which he interprets as the expansion of the universe. In all directions he sees the same thing; the expansion is isotropic. This typical observer, often called a comoving observer, is one who "rides along" with the general expansion of the fireball, so there is no net flow of matter past him, only a flow away in all directions. The comoving observer need not be at the center of the fireball to see this picture; any comoving observer within the fireball sees a general expansion. The situation has been likened to the rising of a loaf of raisin bread. Each raisin sees the same thing: its neighbors receding uniformly in all directions. The notion that no observer within the fireball has a preferred view of its evolution is a basic tenet of modern cosmology called the cosmological principle. It states that all comoving observers see the universe around them evolve in the same way. Thus a second comoving observer in Fig. 1, labeled B, would also see the universe expanding isotropically around him, and would observe the same cooling of the fireball that A sees.

The cosmological principle, with its reference to observations made by a comoving observer, emphasizes the local view of the expansion. To use this principle for the study of the background radiation, a means is needed to relate the observations of events made by a local observer to the view of those same events seen by a distant observer. The general theory of relativity is the proper framework for a discussion of this problem. In this article, however, a simpler description is desired, so a model of the universe will be presented that does not require the full machinery of general relativity for its analysis. Intuitively, it seems reasonable that in such a model gravitational effects must be negligible, for these are the very effects general relativity was designed to deal with. This situation might be expected to exist in a universe in which the average density of matter is vanishingly small. General relativity confirms this intuition: the metric of an empty universe is the Minkowski metric (*8*); events within such a universe may be described by using special relativity. This cosmological model, in which gravitational effects are negligible, is known as the Milne model, after E. A. Milne (*9*).

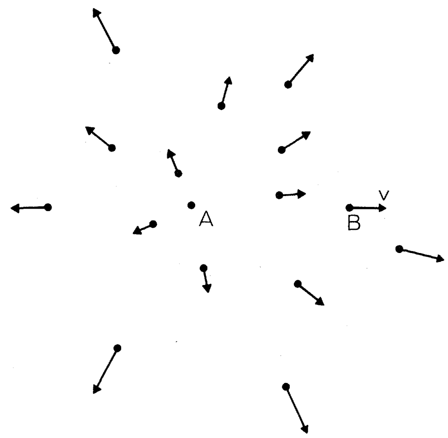


Fig. 1. Expansion of the universe. Comoving observer A sees all the matter receding radially away from him; B, another comoving observer, sees the same behavior.

Precisely because gravity is ignored, the Milne model cannot illustrate the intimate connection between matter and space-time that is expressed in general relativity. By choosing the limiting case of a model where general relativity degenerates into special relativity, we recognize at the outset that our picture will be incomplete. We are willing to forgo a comprehensive description of the expanding universe in return for a restricted description that explains observations made within that universe (*10*). We will return to general relativity later in order to estimate how accurately the Milne model describes our own universe, which is obviously not empty. For now, consider the Milne model a useful tutorial device.

Let observers A and B in Fig. 1 each be equipped with a clock, which indicates the elapsed time since the big bang, and a thermometer, which measures the local temperature. Associated with A, who will eventually become the detector of the cosmic background radiation, is a Lorentz frame of reference (*11*), which will be used to describe events of interest observed by A. We imagine a corps of "watchmen" stationed throughout space, their clocks synchronized with A, affixed to this frame. By observing events in A's frame, we mean that the watchman at the site of an event records the time on his clock at which it occurs. A sequence of events is described by examining the records of the watchmen and reconstructing what happened.

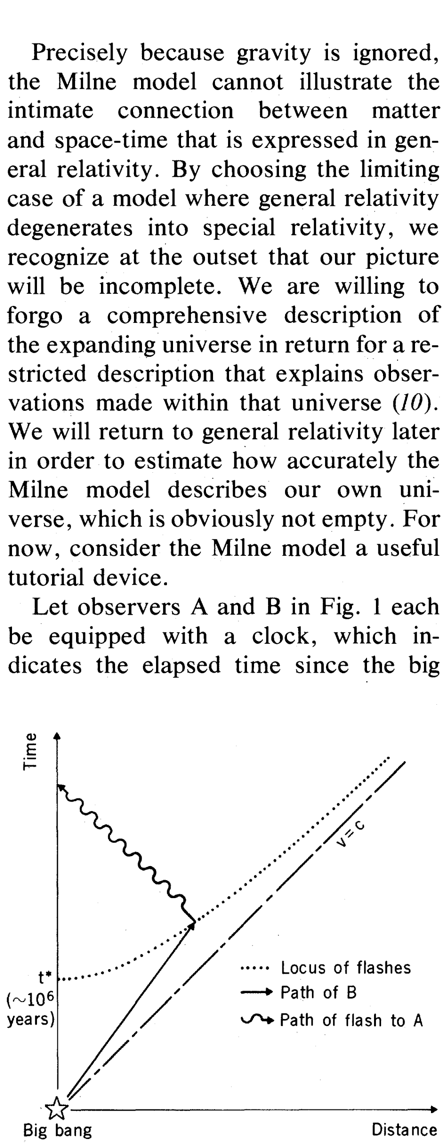


Fig. 2. Space-time diagram showing events seen by observer A. At $t = 0$, the big bang occurs. At $t = t^*$, A (moving along the time axis) emits his 3000 K flash. Because of time dilation he observes that the flashes from the receding matter take place at a time later than t^*. The red-shifted photons from these flashes are the cosmic microwave background.

During the expansion of the fireball, observer A sees the matter of the universe rushing away from him. Each particle moves at constant speed, because there are no gravitational forces. All the matter recedes as if it started from A at the instant of the big bang, so that at time t after the big bang A will observe B, who is moving at speed v, to be at the distance vt. The Hubble "constant," which is the ratio of speed of recession to distance, is simply $1/t$. At time t^* on his clock, A's thermometer has fallen to 3000 K and he emits his flash. [A general relativistic analysis of the expanding fireball indicates that $t^* \sim 10^6$ years (*7*).] He observes, however, that B's clock has not yet reached t^*, nor has B's thermometer fallen to 3000 K, because of relativistic time dilation. [This is the same effect that causes μ-mesons in flight to have a longer lifetime than those at rest (*11*).] At the moment of his own flash, A observes B's clock to read

$$t' = t^* \sqrt{1 - v^2/c^2} \qquad (1)$$

(Primed quantitites refer to measurements made on B's apparatus.) The time in A's frame at which B eventually emits his flash depends, through Eq. 1, on the velocity of B, and is shown by the hyperbola in the space-time diagram of Fig. 2. The horizontal and vertical axes represent distance and time observed by A. At $t = 0$, B starts out from the origin and travels along a line of slope $1/v$. Photons from the flash emitted when B reaches the hyperbola then travel back toward A at speed c.

In an isotropic expanding universe with a continuous spread in recession velocities ($0 < v < c$), A sees a spherical shell of flashes originating at ever greater distances. For contemporary observations, where $t \gg t^*$, the observed flashes originate well out on the hyperbola, where it is asymptotic to the line $v = c$. In this case the time interval between the big bang and the flash is approximately equal to the travel time of the returning photons, and the sum of the two is the present age of the universe or roughly 20 billion years. Thus in our frame of reference the background photons detected today were emitted approximately 10 billion years ago, well after our local temperature dropped through 3000 K. This illustrates a general feature of the Milne model: the farther out one looks, the greater is the recession velocity and hence the greater is the relativistic time dilation. Distant portions of the universe are observed to evolve more slowly than the immediate neighborhood. Even now, at a distance of approximately 20 billion light-years, portions of the universe are just cooling through 3000 K. Photons from the resulting flashes will arrive here about 20 billion years from now.

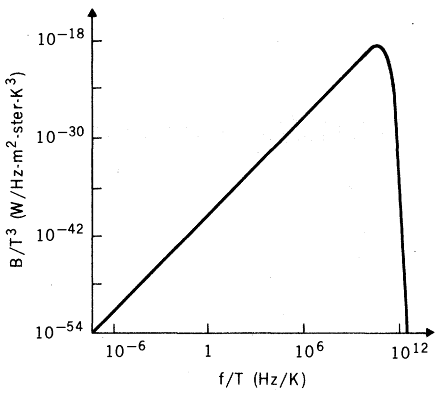


Fig. 3. Brightness (B) of a blackbody as a function of frequency (f). The shape of the spectrum is independent of the temperature (T), but the frequency scale and brightness level vary as T and T^3, respectively.

Blackbody Radiation and the Red Shift

To observer B at the site of a 3000 K flash, the emitted photons display a spectrum, shown in Fig. 3, which is characteristic of radiation from a blackbody. (A blackbody is an object that absorbs all the radiation incident on it. At room temperature it appears black, hence the name.) The abscissa is proportional to frequency, and the ordinate to brightness, an important quantity in the description of thermally generated radiation. Brightness is a measure of power (energy per unit time) per unit bandwidth per unit area per unit solid angle. As shown in Fig. 4, the brightness (in meter-kilogram-second units) of a source at frequency f is the amount of energy received from the source in 1 second by a detector with an active area of 1 square meter and a bandwidth of 1 hertz centered on f, divided by Ω, the solid angle contained in the acceptance cone of the antenna.

The brightness spectrum of Fig. 3 is a universal curve, valid for blackbody radiation at any temperature T. It can be seen from Fig. 3 that a change in the temperature of a blackbody does not alter the shape of its spectrum, but merely changes the scale factors of the coordinate axes. So as an incandescent blackbody cools it gets redder, because the frequency of peak brightness is proportional to temperature, and dimmer, because brightness varies with the cube of the temperature. For example, if the temperature of a blackbody were reduced by a factor 2, the frequency at the peak of the spectrum in Fig. 3 would be cut in half and the corresponding brightness would be reduced by a factor 8. This scaling property of the blackbody spectrum is important in the following discussion of the microwave background.

Consider the photons in the 3000 K flash. Relative motion of the source and the detector causes the spectra measured by the two corresponding observers (B and A, respectively) to differ. Specifically, a photon emitted at frequency f' measured by B will be observed by A at a lower frequency given by the Doppler relation

$$f = f' \sqrt{\frac{1 - v/c}{1 + v/c}} \equiv f' \frac{1}{1 + z} \qquad (2)$$

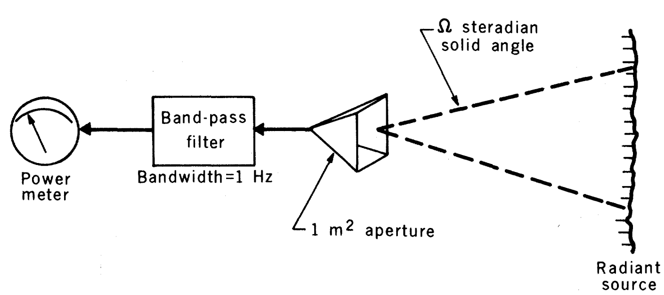
The term on the right is included to establish correspondence between the notation of this article and conventional astronomical nomenclature; the quantity $1 + z$ is called the red shift. The frequency reduction in Eq. 2 arises from two effects: the classical "stretching out" of wavelength due to source recession, and the apparent reduction in source frequency due to relativistic time dilation. The flash photons observed by A will all be red-shifted by the factor $1 + z$, so the shape of their spectrum will not change, but its frequency axis will be compressed by $1 + z$. This is reminiscent of the frequency compression associated with the cooling of a blackbody that was discussed earlier.

Indeed, when all kinematic effects are considered, the brightness spectrum observed by A is exactly the spectrum of a blackbody at a temperature of (3000 K)/$(1 + z)$ as we now show. In a burst of photons emitted from B toward A, all quantities associated with time intervals—that is, frequency, bandwidth, and rate of arrival—will be modified according to Eq. 2. These effects will cause the observed brightness to be reduced by $1 + z$, because the reduction in photon energy (proportional to frequency) is offset by the same reduction in bandwidth, leaving only the reduced rate of arrival. But in addition to this rate effect there is a geometric effect that causes a further brightness reduction of $(1 + z)^2$. This phenomenon, known as the headlight effect, is the optical analog of what might be called the shotgun effect in classical mechanics. The solid angle defined by the blast of a shotgun fired forward from a moving flatcar appears smaller to an observer on the ground than to the gunner himself.

Similarly, the ground-based observer sees an increase in solid angle when the gun is fired toward the rear. In the case of the 3000 K flash, A observes the solid angle of the narrow cone of photons headed toward him from B to be larger than the solid angle measured by B by the factor $(1 + v/c)/(1 - v/c)$, which is $(1 + z)^2$. The area density (number per unit area) of photons and hence the brightness seen by A is thus reduced by $(1 + z)^2$. The headlight effect and the rate effect mentioned earlier combine to produce a net reduction in brightness of $(1 + z)^3$.

The frequency and brightness reductions of the burst of photons by the factors $1 + z$ and $(1 + z)^3$, respectively, are precisely the same effects that are observed when the temperature of a blackbody is reduced by $1 + z$. That is, the radiation from a blackbody at temperature

Fig. 4. Operational definition of brightness. The portion of the source "seen" by the detector is defined by the acceptance cone of the antenna, which contains a solid angle of Ω steradians. The brightness is the power in watts received in a 1-Hz bandwidth by the 1-m² aperture, divided by Ω.



T receding with a red shift of $1 + z$ is identical to that from a stationary blackbody at temperature $T/(1 + z)$.

The brightness of the microwave background measured by Penzias and Wilson at 4 gigahertz (*12*) was equal to the brightness of a stationary blackbody at 3 K. Later measurements at other frequencies (*4*) showed that indeed the radiation had a blackbody spectrum, just as the big bang theory predicted. These measurements imply that the photons, which started out at 3000 K, have undergone a red shift of $1 + z = 1000$, which corresponds to a recession velocity in Eq. 2 of $v/c \sim 1 - (2 \times 10^{-6})$.

The temperature of the microwave background in the Milne model (and in all expanding big bang models) is a decreasing function of time, because as time increases in Fig. 2 the origin of the background photons moves farther out on the hyperbola, corresponding to a greater red shift. From Eqs. 1 and 2 it can be shown that in the Milne model the red shift increases linearly with time, resulting in a background temperature of

$$T = 3000\ \mathrm{K}\left(\frac{t^*}{t}\right) \qquad (3)$$

In the last two sections the cosmic microwave background has been described in the context of the Milne model of an expanding universe. The most important features of the background that emerge from that discussion are:

1) The background photons were emitted in a 3000 K flash and traveled undisturbed until detection.

2) The apparent cooling of the photons from 3000 K is due to the red shift associated with the recession of the source from the receiver.

3) The temperature of the background varies inversely with time.

Validity of the Model

The Minkowski metric, on which the preceding special relativistic discussion of the microwave background is based, can only be an approximation to the structure of space-time in our universe, because it does not include the gravitational effects of matter. The accuracy of this approximation is the subject of this section.

In the simplest cosmological models consistent with general relativity (the Friedmann models with zero pressure and no cosmological constant), the metric at any point, such as the site of observer A, can be cast in Minkowski form (*13*), and this metric will approximate the true metric within a sphere of radius R surrounding that point, provided

$$\frac{4\pi}{3}\frac{G\rho R^2}{c^2} << 1 \qquad (4)$$

where G is the universal gravitational constant ($G = 6.67 \times 10^{-11}$ m^3/kg-sec^2) and ρ is the average density of matter in the universe. Equation 4 is consistent with the intuitive notion mentioned earlier that the Minkowski metric should be valid in the limit $\rho \rightarrow 0$ (*14*).

For an observer in the Milne universe at time t, the maximum radius of interest is $ct/2$. (Photons emitted at distances greater than $ct/2$ arrive later than t.) Equation 4 can then be written (neglecting factors of order unity) as

$$\frac{\rho}{\rho_c} << 1 \qquad (5)$$

where ρ_c is the "critical density" that will produce just enough gravitational force eventually to halt the expansion of the universe (*15*). At the present time ρ/ρ_c is thought to be $\sim 1/4$ (*16, 17*), which has two important implications for the validity of the Milne model. First, the fact that ρ/ρ_c is not a great deal less than unity means that the model can provide a useful, but not very precise, description of the present universe. Second, and more important, ρ/ρ_c less than unity means that the universe is open (it will expand forever), and in all open universes ρ/ρ_c evolves from a value near unity asymptotically toward zero. That is, sooner or later the Milne model will be applicable to our universe.

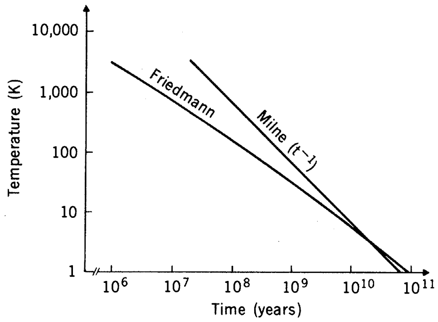


Fig. 5. Temperature of the cosmic microwave background according to the Milne (special relativistic) and Friedmann (general relativistic) models. As the universe evolves, the predictions of the two models for the time dependence of the temperature agree more and more closely.

The trend of the universe toward the Milne model is illustrated in Fig. 5, which shows the temperature of the microwave background radiation as a function of time for the Milne (special relativistic) model and for the Friedmann (general relativistic) model corresponding to the present value of $\rho/\rho_c = 1/4$. Each curve has been scaled so that it passes through the point corresponding to the present state of the universe: 3 K at 2×10^{10} years. As expected from the preceding discussion, the slopes of the two curves differ most at early epochs, when ρ/ρ_c is at its largest value. At the 3000 K flash, for example, the temperature in the Freidmann model has a $t^{-2/3}$ time dependence, compared with the t^{-1} dependence of the Milne model. But as time evolves, the two models converge. In the present universe the Friedmann time dependence is $t^{-0.83}$, and in the distant future it will approach the t^{-1} behavior of the Milne model.

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18. I thank my colleagues at Bell Laboratories and P. J. E. Peebles at Princeton University for helpful discussions and advice.