Reports

Thermal Evolution of Plutons: A Parameterized Approach

Abstract. *A conservation-of-energy equation has been derived for the spatially averaged magma temperature in a spherical pluton undergoing simultaneous crystallization and both internal (magma) and external (hydrothermal fluid) thermal convection. The model accounts for the dependence of magma viscosity on crystallinity, temperature, and bulk composition; it includes latent heat effects and the effects of different initial water concentrations in the melt and quantitatively considers the role that large volumes of circulatory hydrothermal fluids play in dissipating heat. The nonlinear ordinary differential equation describing these processes has been solved for a variety of magma compositions, initial termperatures, initial crystallinities, volume ratios of hydrothermal fluid to magma, and pluton sizes. These calculations are graphically summarized in plots of the average magma temperature versus time after emplacement. Solidification times, defined as the time necessary for magma to cool from the initial emplacement temperature to the solidus temperature vary as* $R^{1.3}$, *where* R *is the pluton radius. The solidification time of a pluton with a radius of 1 kilometer is* 5×10^4 *years; for an otherwise identical pluton with a radius of 10 kilometers, the solidification time is* $\sim 10^6$ *years. The water content has a marked effect on the solidification time. A granodiorite pluton with a radius of 5 kilometers and either 0.5 or 4 percent (by weight) water cools in* 3.3×10^5 *or* 5×10^4 *years, respectively. Convection solidification times are usually but not always less than conduction cooling times.*

For the characterization of the geothermal energy potential of crustal magma bodies (*1*) and models describing the origin and development of economically important ore deposits in plutonic rocks (*2*), information on the thermal evolution of convecting magma chambers is necessary. Aspects of theoretical importance such as the development and interpretation of metamorphic aureoles, the influence of cooling rate on the grain size distribution of minerals in granitoid rocks, and the role of magmatic differentiation and the origin of compositional zoning within magma chambers also depend on knowledge of the thermal history of cooling plutons (*3*). In principle, finite-difference or finite-element methods may be used to compute the spatial and temporal dependence of convection rates and melt temperature for appropriate sets of initial and boundary conditions and for a variety of melt compositions. In practice, however, the presence of rather thin chemical and thermal boundary layers (centimeter to meter scale) within magma bodies makes such a procedure costly and time-consuming. In addition, the large number of factors that govern the thermal evolution of a pluton such as the depth of pluton emplacement, the local wall-rock environment (that is, injection into wet or dry country rock), the size and shape of chambers, and the temperature and compositional dependence of the physical and thermodynamic properties of silicate melt suspensions cannot be easily varied in finite-difference calculations. Parameterized methods for the solution of geophysical and petrological problems have become increasingly popular since the pioneering work of the last decade (*4*). Their popularity springs from the relative ease in which complex phenomena involving many parameters may be treated. This is usually accomplished by the solution of relatively simple ordinary differential equations which preserve the essential physics of the processes under investigation. The main disadvantage of a parametric approach is that the field variables are spatially averaged over the entire system. Consequently, parametric methods are of little use in predicting the spatial dependence of temperature within a convecting magma chamber. To this end, other methods must be used (*5*). I report here a simple parameterized approach to quantitatively model the thermal evolution of plutons of various compositions and sizes emplaced into widely contrasting geologic environments.

The parameterized model accounts for heat transfer by conduction and convection within the chamber and into the surrounding country rock. Hydrothermal fluid circulation within a permeable or fractured country rock accounts for most heat loss when magma is emplaced into water-bearing country rock. In dry country rock, thermal conduction is the primary mode of heat loss. The nucleation and growth of crystallizing phases liberate enthalpy of crystallization, which adds to the heat available for dissipation. The rate at which heat leaves the pluton depends on the depth of pluton emplacement and the scale of the magmatically induced hydrothermal system. The development of a hydrothermal flow regime depends on the availability of fluids and the presence of permeable or highly fractured country rock. Large hydrothermal systems tend to occur in the upper parts of the crust where meteoric water is more plentiful. An important aspect of the model described here is the inclusion of a strongly temperature-dependent apparent viscosity for the magma. The motivation for this feature arises from the experimental rheology of two-phase polymer systems (*6*). As crystallization proceeds, the apparent viscosity (*7*) of a magmatic suspension increases dramatically; thermal boundary layers increase in thickness, and the rate of heat transport out of the pluton decreases. In the calculations presented herein, I assume a simple linear dependence of the volume melt fraction (θ) with temperature above the solidus temperature ($T - T_s$). This is not likely to be a good assumption; indeed, one might expect from kinetic considerations (*8*) that pluton solidification rates are dependent upon large number of parameters including the absolute temperature, the degree of supercooling, activation energies for flow and mass diffusion, and surface free-energy differences between each of the crystalline phases and the melt. More realistic temperature- and time-dependent crystallization rates that explicitly consider the processes of nucleation and growth should be used, but the necessary kinetic data are not available at present.

A conservation-of-energy equation which accounts for the heat loss from a convecting spherical pluton of radius R and volume V undergoing crystallization is

$$\rho C_p V \frac{dT}{dt} + 4\pi R k\, Nu(T - T_c) = -\rho \Delta H V \frac{d\theta}{dt} \qquad (1)$$

A derivation of this expression may be found in (*9*); symbols are defined in (*10*). The Nusselt number, Nu, is a dimensionless heat flux parameter indicative of the

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vigor of convective mixing within the magma body. Experimental, theoretical, and numerical studies of thermal convection in fluids suggest that Nu is related to the thermal Rayleigh number (Ra) (11) according to

$$Nu = a(Ra)^b \qquad (2)$$

where a and b are constants related to the specific geometry, flow regime, and boundary conditions of a given experiment (12). If a linear relationship between θ and $T - T_s$ is assumed, then Eqs. 1 and 2 may be combined to give

$$\frac{dT}{dt} + \frac{3\kappa a(T - T_c)^{1+b}}{R^2[1 + \Delta H/C_p(T_\ell - T_s)]} \frac{\alpha g R^3}{\kappa \nu_{T_\ell}} \times \exp[bs(T - T_\ell)] = 0 \qquad (3)$$

Equation 3 describes the spatially averaged magma temperature as a function of time. At $t = 0$ (time of emplacement) it is assumed that $T = T_i$, which may or may not be equal to T_ℓ. When $T_i < T_\ell$, some fraction of crystals will initially be present, if we assume equilibrium crystallization. As time passes, crystallization proceeds with decreasing rapidity since the apparent viscosities increase dramatically with crystallinity and so the rate of heat loss from the body decreases (that is, Ra and Nu decrease). According to Eq. 3, the thermal history of a pluton is most sensitively dependent upon (i) the depth of pluton emplacement, (ii) the heat-transfer characteristics of the local environment (for example, emplacement into hydrous or anhydrous country rock), (iii) the size of the pluton, and (iv) the bulk composition of the melt. Parametrically, factors (i) and (ii) may be modeled by varying T_c. Plutons cooling deep in the crust (relatively high T_c) do so more slowly than those emplaced under a thin overburden (low T_c). Attendant with the emplacement of magma into a permeable, fluid-saturated environment will be an efficient hydrothermal circulatory system capable of dissipating large amounts of magmatic heat. Alternatively, in the case of magmatic intrusion into relatively dry country rock (for example, intrusion into a gabbroic or granulitic lower crust), T_c assumes a value considerably greater than that expected for the unperturbed temperature at the depth of emplacement.

In the simple one-dimensional case of solidification of magma initially at T_i in contact with dry country rock at T_{CR}, T_c (13) is given by

$$T_c = \frac{1}{2}(T_i + T_{CR}) \qquad (4)$$

Equation 4 is a fair approximation of the effective T_c up to a Fourier number ($Fo = \kappa t/R^2$) of about 10^{-1}. In dimensional terms, Eq. 4 provides an accurate (within about 20 percent) estimate of T_c until a time after emplacement given by

$$t \sim 10^4 R^2 \qquad (5)$$

with R given in kilometers and t in years. For example, T_c values computed from Eq. 4 for a pluton with $R = 10$ km emplaced into dry country rock are approximately valid up to about 10^6 years. Thereafter, T_c values slowly fall. For magma emplacement into an environment where hydrothermal circulation significantly contributes to the dissipation of magmatic heat, an absolute minimum T_c is T_{CR}, the unperturbed country rock temperature at the depth of pluton emplacement. Naturally, in real situations T_c will always be somewhat greater than T_{CR}. The T_c values may exceed country rock solidus temperatures if T_i and T_{CR} are high enough, and consequently partial melting may be significant along contacts between magma chambers and country rock. It is expected that partial fusion effects are important in the ascent of magmatic diapirs (14). If these ascent rates are controlled primarily by the balance between buoyancy and viscous forces (that is, if Stokes law is applicable), then the absence or presence of partial fusion along contacts is critical in determining the magnitude of the ascent rate.

The essential results of this study are presented in Figs. 1 and 2. Figure 1a shows the effect of initial T_c on the cooling history of a quartz monzonite pluton [+2 percent (by weight) water] ($2R = 10$ km) emplaced at a depth of about 7 km. Solidification times (the time interval between liquidus and solidus) vary from 2×10^5 to 3.6×10^6 years as T_c goes from 500° to 700°C. I conclude that emplacement depths and the scale of hydrothermal circulatory systems are first-order parameters in determining the cool-

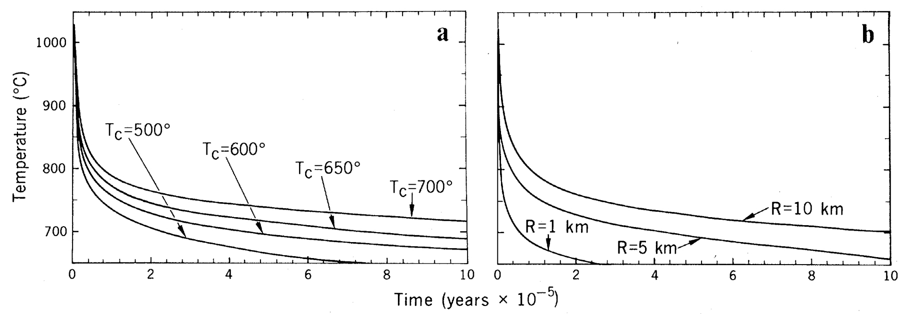

Fig. 1. (a) Influence of T_c, the temperature of the magma chamber–country rock contact, on the cooling history of a quartz monzonite pluton +2 percent water (by weight) (diameter = 10 km) crystallizing at a depth of about 7 km; T_c depends on the geothermal gradient, the emplacement depth, and the hydrothermal fluid/magma volume ratio. The spatially averaged temperature of the magma is plotted on the ordinate. The initial magma temperature is taken to be 1030°C. Data on the thermal and thermodynamic properties of melts were taken from (15). (b) Spatially average magma temperature versus time after emplacement as a function of pluton radius (R) for granitic plutons [same composition as in (a)] emplaced at 1030°C and a depth of about 7 km. The value of T_c has been assumed constant and equal to 600°C; $p_t = 2$ kbar. A pluton with $R = 1$ km cools to the solidus temperature in 60,000 years; a pluton ten times larger requires nearly 1.1×10^6 years or nearly 20 times the time period in which to cool to the same temperature.

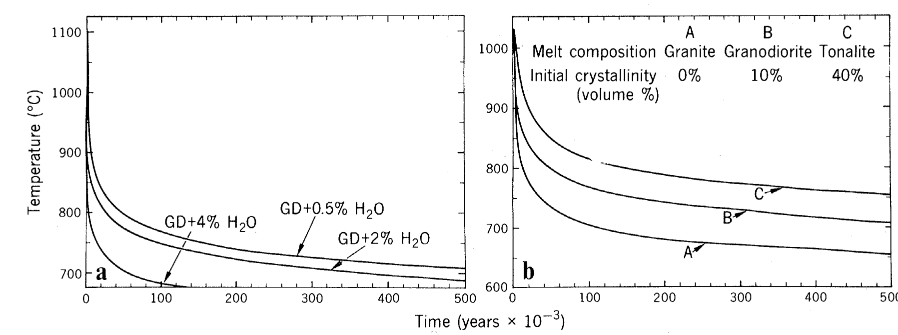

Fig. 2. (a) Influence of the magma water content on the cooling history of granodioritic (GD) plutons ($2R = 10$ km; $T_c = 600$°C; $p_t = 2$ kbar). Water plays a central role in determining the viscosity of the melt at T_ℓ and also the melting interval, $T_\ell - T_s$. Increasing the water content by a factor of 2 [from 2 to 4 percent (by weight)] decreases solidification times by a factor of about 7 because of the lower magma viscosity. (b) Influence of initial crystallinity and melt composition on the cooling history of plutons ($R = 5$ km). In all cases $T_i = 1030$°C, $T_c = 600$°C, $p_t = 2$ kbar, depth of emplacement is 7 km, and all melts contain approximately 2 percent water. Plutons that initially are partly crystallized cool more slowly than aphyric ones.

ing time of large plutons. Figure 1b explores the influence of pluton size on the cooling history of a water-bearing granitic melt. Conduction cooling times vary with the square of R, whereas in convective cooling the solidification time varies approximately according to $R^{1.3}$ (Eq. 3).

Figure 2a shows the remarkable role water plays in determining the cooling time of granodioritic plutons. A pluton with 0.5 percent water (R = 5 km) cools to the solidus temperature in 330,000 years, whereas one with 4 percent water cools in only 50,000 years, other factors remaining constant. This is a graphic illustration of the importance of rheology in determining the thermal and chemical evolution of a magmatic system. For instance, mixing in a chemically (for example, water content) zoned magma chamber may be severely restricted due to a variation of an order of magnitude in the melt viscosity with depth. The influence of initial crystallinity and melt composition (excluding water) is summarized in Fig. 2b, where the thermal trajectories for granitic, granodioritic, and tonalitic plutons (R = 5 km) are given. The more mafic magmas have longer cooling histories because they are initially (that is, at $T = T_i$) partially crystallized and more viscous and consequently cool more slowly than related aphyric melts.

The parametric approach described here enables one to efficiently and simply evaluate the influence of various geometric, thermodynamic, thermophysical, and compositional factors on the cooling history of large volumes of magma emplaced within the crust. The calculations reported here are broadly compatible with the more detailed (in a spatial sense) and more laborious computations carried out by other works (*5*).

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7. The apparent viscosity is defined by the ratio $\sigma/\dot{\epsilon}$, where σ and $\dot{\epsilon}$ represent, respectively, the shear stress and strain rate of a single or multiphase fluid at a fixed temperature, pressure, and composition. The ratio $\sigma/\dot{\epsilon}$ is equal to the viscosity for a Newtonian fluid.
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10. Magma density, $\rho = 2.3$ g cm^{-3}; magma heat capacity, $C_p = 0.28$ cal g^{-1} sec^{-1}; thermal conductivity, $k = 5 \times 10^{-3}$ cal cm^{-1} sec^{-1} K^{-1}; contact temperature, T_c; heat of crystallization, ΔH = 80 cal g^{-1}; volume fraction of melt in the pluton, θ; temperature of the melt, T; time after the initiation of crystallization, t; isobaric expansivity, $\alpha = 5 \times 10^{-5}$ K^{-1}; acceleration, g = 981 cm sec^{-2}; kinematic viscosity at the liquidus temperature, ν_{T_ℓ}; the viscosity parameter, $s = 0.08$, is defined by
$$\nu = \nu_{T_\ell} e^{s(T_\ell - T)};\ \kappa = \frac{k}{\rho_\ell C_p} = 10^{-2}\ \mathrm{cm^2\ sec^{-1}}$$
T_ℓ, T_s, T_c, and T_i represent the melt liquidus, melt solidus, contact, and initial melt temperatures, respectively; p_ℓ is pressure.
11. The definition of Ra appropriate to boundary layer thermal convection about a vertical pluton-country rock contact is
$$Ra = \frac{\alpha g (T - T_c) R^3}{\kappa \nu_{T_\ell}} \exp[s(T - T_\ell)]$$
This definition accounts for a strongly temperature-dependent magma viscosity.
12. For free convection about a vertical flat plate held at constant temperature, typical values are as follows: $a = 0.31$ and $b = 0.24$ (laminar flow); $a = 0.13$ and $b = 0.33$ (turbulent flow). Values for the uniform heat flux case are $a = 1.87$ and $b = 0.20$ (laminar flow). The boundary between turbulent and laminar convective flow regimes coincides roughly with $Ra = 10^{10}$ for high Prandtl number ($Pr = \nu/\kappa$) melts. In the numerical experiments reported here, $a = 0.305$ and $b = 0.239$. Data were compiled from W. M. Rohsenow and H. Y. Choi, *Heat, Mass, and Momentum Transfer* (Prentice-Hall, Englewood Cliffs, N.J., 1961); B. Gebhart, *J. Fluid Mech.* **14**, 225 (1962); K. Stewartson and L. T. Jones, *J. Aeronaut. Sci.* **24**, 379 (1957); M. J. Lighthill, *Q. J. Mech. Appl. Math.* **6** (Part 4), 399 (1953); R. Krishnamurti and F. B. Cheung, *Int. J. Heat Mass Transfer* **20**, 499 (1977).
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Corotation Lag in Jupiter's Magnetosphere: Comparison of Observation and Theory

Abstract. *Voyager 1 plasma flow data are compared with a recent theory that predicted measurable departures from rigid corotation in Jupiter's magnetosphere as a consequence of rapid plasma production and weak atmosphere-magnetosphere coupling. The comparison indicates that the theory can account for the observed corotation lag, provided that the plasma mass production rate during the Voyager 1 encounter was rather larger than expected, namely ~ 10^{30} atomic mass units per second.*

Jupiter's magnetosphere contains prodigious sources of plasma, the most conspicuous being the innermost Galilean satellite Io (*1*). The magnetospheric plasma tends to corotate with the Jovian ionosphere, to which it is magnetically connected (barring large magnetic-field-aligned potential drops); thus the production and outward transport of magnetospheric plasma requires a net torque to transfer angular momentum outward from Jupiter's atmosphere to the magne-

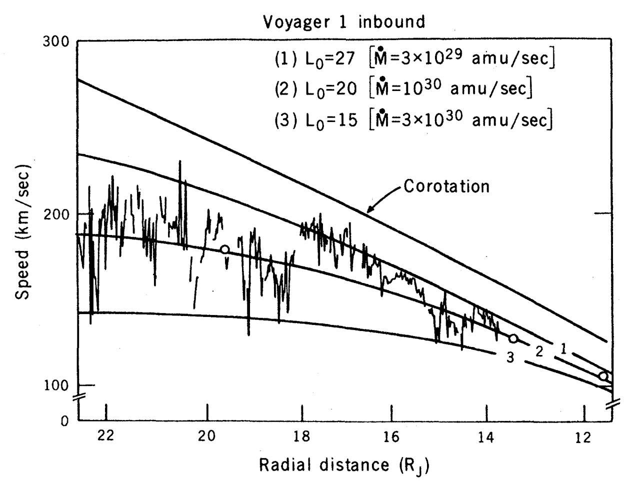

Fig. 1. Plasma rotational flow velocity as a function of radial distance in Jupiter's magnetosphere. The data (noisy curve plus circles) are reproduced from (*5*). The theoretical curves are derived from (*3*), with appropriate allowance made for the small but nonzero angle between the corotation direction and the viewing axis of the Voyager 1 instrument.

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