movement conflicts with the immobility or very slow movement generally attributed to hot spots.

Ninetyeast Ridge is only one of many similar ridges that could have been formed by hot spots. But data from other submarine ridges are scarce, so discussions of their origins are liable to remain speculative for some time. The convincing amount of data from the Hawaiian-Emperor chain required 10 years to gather, and then only with much persistence and a little luck. Researchers would like to have more samples gathered along Ninetyeast Ridge, but no major effort there or on any other ridge is anticipated in the forseeable future.

—RICHARD A. KERR

The Fields Medals (II): Solving Geometry Problems with Algebra

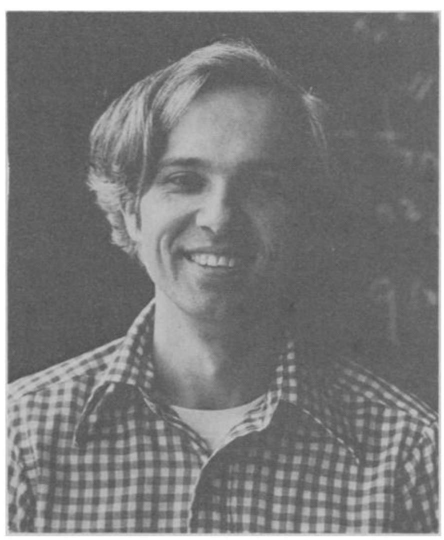
Daniel Quillen was awarded the Fields Medal for his fundamental work in algebra, notably algebraic K-theory, and in topology. Quillen was born on 22 June 1940 in Orange, New Jersey. His father is a high school physics teacher, originally trained as a chemical engineer. Quillen went from the Newark Academy to Harvard, where he earned a B.A. in 1961 and a Ph.D. in 1964. His thesis, on formal aspects of the theory of partial differential equations, was written under the direction of Raoul Bott. Bott, having also been the University of Michigan thesis adviser of 1966 Fields Medalist Stephen Smale, joins his Harvard colleague Oscar Zariski as the second person to count two Fields Medalists among his students.

When Quillen received his Ph.D. at the age of 24, he and his wife Jean, a violist, were already caring for two of their five children. His precocity as a mathematician and as a father perhaps influenced the early graying of his hair, but it has not altered his boyish look or his easy and modest manner. He has a somewhat retiring life-style, appearing rarely in public, and then almost invariably with some extraordinary new theorem or idea in hand. Quillen went from Harvard down the Charles to Massachusetts Institute of Technology, where he has since remained on the faculty. He spent three of the intervening academic years away: 1968 and 1969 as a Sloan Fellow in France; 1969 and 1970 as a visiting member of the Princeton Institute for Advanced Study; and 1973 and 1974 as a Guggenheim Fellow in France.

Quillen's dissertation on partial differential equations was soon absorbed into the working repertoire of that classical field. His interests quickly shifted, however, into mathematical domains of more recent vintage, which are more difficult to describe and motivate to an audience that includes nonmathematicians.

Topology is what underlies the various modern fields of geometry. The geometric objects of its study are called topological spaces. By the middle 1930's an arsenal of rather sophisticated algebraic techniques had evolved for solving the geometric problems of topology. The basic theme was to associate with a topological space X a type of algebraic object $H(X)$, called an Abelian group. The point is that whereas the geometric object is continuous and potentially quite complicated, the Abelian group is discrete and often effectively computable. Finally, the passage $X \rightarrow H(X)$ from the "hard geometry" to the "easy algebra" retains enough significant information to solve important geometric problems.

In order that these algebraic techniques not remain a special craft, the private reserve of a few virtuosos, it was necessary to put them in a broad, coherent, and supple conceptual setting. This was accomplished in the 1940's and 1950's through the efforts of many mathematicians, notably Samuel Eilenberg at Columbia University, Saunders MacLane of the University of Chicago, the late Norman Steenrod, and Henri Cartan of the University of Paris. The result of this enterprise of simplification, unification, and axiomatization was a methodological instrument that could be applied far beyond the setting of its birth; we call it homological algebra. It intruded naturally and fruitfully into every major area of mathematics, and eventually required, in turn, a still more abstract and general conceptualization of its own techniques.



Daniel Quillen [Photo by Margo Woodruff]

This led to what we now call the theory of categories and functors.

Quillen is one of the greatest masters of these homological and categorical techniques. He has used them with stunning originality to treat a variety of problems. He has even invested them with a kind of naïve geometric character which helps remove the shroud of ponderous formalism that had estranged many earlier mathematicians.

One of Quillen's early applications of these methods was to construct a "good" cohomology theory for commutative rings. The latter are algebraic systems in which the familiar kind of addition and multiplication can be performed. The basic examples are number rings, such as the ring of integers, and function rings, which consist of appropriate classes of functions on some geometric space X. In the latter setting Quillen's cohomology, constructed independently by Michel André of the University of Switzerland, is used to study "deformations of structure" on X.

A 1958 Fields Medal was awarded to René Thom for his invention of an extraordinary cohomology theory in topology, called cobordism theory. Quillen showed how the methods of an apparently unrelated field, the theory of formal groups, could be naturally and effectively introduced for cobordism calculations.

A celebrated conjecture in homotopy theory, made by the English mathematician J. F. Adams, had resisted the efforts of numerous mathematicians. Quillen showed how some rather exotic new methods developed by Michael Artin of the Massachusetts Institute of Technology and Barry Mazur of Harvard in the quite distinct field of algebraic geometry could be used to transform the Adams conjecture into a more plausible one in the latter context. The latter conjecture was subsequently proved by Quillen's student Eric Friedlander of the University of Illinois. Meanwhile Quillen developed a totally different but equally original method for proving the Adams conjecture, appealing this time to another remote field, the modular representa-

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tion theory of finite groups, a subject developed by the late R. Brauer, which few topologists before Quillen knew or had occasion to consider. This method had a decisive impact on Quillen's later work, on group cohomology, and especially in algebraic K-theory.

Group cohomology is a special but important chapter of homological algebra. Quillen, in two fundamental papers, gave a definitive structure theorem for certain structures known as the mod p cohomology rings of finite groups, and answered several open questions in the process. He proved this purely algebraic result about finite groups by vastly generalizing it to a statement about topological spaces in which a group operates, thereby affording a flexibility of reasoning not otherwise available. This was an exemplary case of Hilbert's principle, "simplify by generalizing."

The work principally cited in awarding Quillen the Fields Medal is in algebraic K-theory. Two of the Fields Medalists at the 1966 congress in Moscow were Michael Atiyah of Oxford University and Alexandre Grothendieck of the Institut des Hautes Etudes Scientifiques in France. In evolving ideas on algebraic K-theory Quillen was strongly influenced by Grothendieck, during his 1968–1969 stay in France, and by Atiyah during his 1969–1970 year in Princeton.

Grothendieck had orchestrated a monumental refounding of the whole field of algebraic geometry, systematically incorporating the powerful homological and categorical techniques and refining them considerably for the purpose. One of the first stages of Grothendieck's program to be consummated was his proof of a significant generalization of the so-called Riemann-Roch-Hirzebruch theorem. This proof abounded in original and seminal ideas. His formulation of the theorem featured an Abelian group $K(X)$ associated in a geometrically natural way to an "algebraic variety" X. Atiyah and Fritz Hirzebruch of the University of Bonn saw that Grothendieck's construction could be fruitfully employed in topology, whence their creation of topological K-theory. Atiyah's Fields Medal was awarded for the numerous and important applications to geometry and analysis of this theory.

Algebraic K-theory is, similarly, an extension of Grothendieck's construction, this time to the algebraic category of (commutative) rings. When A is a suitable ring of functions on an algebraic variety or topological space X, then $K(A)$ coincides with the $K(X)$ of Grothendieck or of Atiyah and Hirzebruch, respectively. Topological K-theory made $K(X)$ the initial term of a whole sequence of Abelian groups $K_0(X)$, $K_1(X)$, $K_2(X)$, ... It was natural to seek an algebraic analog $K_n(A)$, $n = 0,1,2, \ldots$ of this sequence, although there was no clearly satisfactory way to construct it. Early, somewhat ad hoc constructions of K_1 and K_2 by Bass and Milnor, respectively, led to interesting contacts with and applications to topology, group theory, and number theory. It was thus clear that a reasonable algebraic K-theory would contain a good deal of subtle information of very classical interest. Of course, "reasonable" means both that the theory exhibits the desired formal properties and that it comes equipped with effective tools of computation.

A number of mathematicians proposed constructions of such higher algebraic K-groups $K_n(A)$. However, for not a single nontrivial ring A could they be computed for all n. Further, it was not clear how all these constructions were related to each other. At this point, in 1972, Quillen entered the scene. Inspired by the ideas in his proof of the Adams conjecture, he was led to a very natural and elegant definition of higher K-groups, for which, in some special but very interesting cases, he could supply complete computations. This construction appeals to topology, more specifically homotopy theory, and it has since found wider application by topologists and algebraists. After a flurry of research it was established that Quillen's theory gave the same K-groups as the other proposed definitions.

However, the criterion of effective computability in interesting cases was still not met. A number of theorems proved for K_0,K_1, and K_2 remained open questions for K_n ($n > 2$). The difficulty resided in the fact that many of the known theorems relied on the original Grothendieck style of construction, whereas the higher algebraic K-groups were constructed in an essentially different manner, not susceptible to the same type of reasoning and manipulation.

Quillen completely resolved this difficulty. Again he borrowed techniques from homotopy theory, and in a completely novel way. The paper in which this so-called Q-construction occurs is essentially without mathematical precursors. Reading it for the first time is like landing on a new and friendly mathematical planet. One meets there not only new theorems and new methods, but new mathematical creatures and a complete paradigm of gestures for dealing with them. Higher algebraic K-theory is effectively built there from first principles and, in 63 pages, reaches a state of maturity that one normally expects from the efforts of several mathematicians over several years.

What is the mathematical interest of algebraic K-theory? Since its inception it has figured in an essential way in some area of geometric and differential topology, where there is a systematic method for reducing certain geometric problems to calculations in algebraic K-theory.

The higher K-groups of number rings such as the integers appear to have unexpectedly deep connections with algebraic number theory, involving so-called higher reciprocity laws and zeta functions. The precise form of this relationship is the subject of some spectacular conjectures of Stephen Lichtenbaum of Cornell University, which have been partly confirmed by Armand Borel of the Institute for Advanced Study in Princeton, John Coates of the University of Paris, and Lichtenbaum. A premise of the Lichtenbaum conjectures is that the K-groups of number rings are finitely generated—that is, they are described by a finite number of integer parameters. This was verified by Quillen with a beautiful geometric argument.

Further applications of algebraic K-theory to the subtle questions in intersection theory in algebraic geometry have been made by Spencer Bloch of the University of Chicago. The basis for these applications is a result conjectured by Stephen Gersten of the University of Utah and proved by Quillen.

On top of this, Quillen recently proved a well-known conjecture, made in 1955 by Jean-Piere Serre of the College de France, which had resisted the efforts of numerous mathematicians for more than 20 years. The conjecture was independently proved by the young Russian mathematician Andre Suslin of the University of Leningrad. Roughly speaking, it asserts that, in an analog of a vector space where the scalars are allowed to be polynomials rather than numbers, one can introduce a global coordinate system of the usual kind.

Mathematical talent tends to express itself either in problem-solving or in theory-building. It is with rare cases like Quillen that one has the satisfaction of seeing hard, concrete problems solved with general ideas of great force and scope and by the unification of methods from diverse fields of mathematics. Quillen has had a deep impact on the perceptions and the very thinking habits of a whole generation of young algebraists and topologists. One studies his work not only to be informed, but to be edified.

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