Book Reviews

Resource Management

Mathematical Bioeconomics. The Optimal Management of Renewable Resources. COLIN W. CLARK. Wiley-Interscience, New York, 1976. xiv, 352 pp., illus. $21.95. Pure and Applied Mathematics.

Applied mathematics has two closely related missions. First, it attempts to provide a basis for the detailed design and day-to-day management of human affairs, ranging from architecture and engineering to communications and economic planning. Second, it endeavors to create a bridge between pure and applied science, illuminating the principles that underlie the accumulated experience of practical affairs.

Colin Clark's aptly if prosaically titled book provides a brilliant example of applied mathematics engaged in the creation of theory for an applied science. In general, although there are significant exceptions, reliable day-to-day management models cannot be produced until the task of theory building is well developed. This is certainly true in the case of renewable resources, such as fish, game, and forests, where a fuller understanding of the dynamics of the interaction of ecological and economic processes is required before tactical mathematical models will be of any use. *Mathematical Bioeconomics* tackles this problem, concentrating on the harvesting of fish and whales, an enterprise in which, as the state of world fish stocks attests, both theory and practice are still at a primitive level.

Fully realistic fishery models, when they are eventually produced, will have to be able to cope with the complexities of age structure in fish populations, with the spatial heterogeneity of the marine environment, and with the problem of multiple-species and whole-ecosystem harvesting. These topics are addressed by Clark in the last three chapters of the book, and he indicates lines of research through which the very formidable obstacles, in particular the difficulty of devising appropriate optimization techniques, may be overcome. The largest part of the book, however, is devoted to showing how the important policy and strategy issues in fishery management can be powerfully illuminated by building on relatively simple population models, beginning with the classical use by Graham, Schaefer, and Gordon of the logistic equation.

Many of Clark's results are presented for the first time or are of recent publication. He extends Gordon's analysis of the open-access fishery to include supply and demand and demonstrates how the intersection of shifting demand curves with the sharply backward bending supply curves that is characteristic of such fisheries produces dramatic instabilities in harvesting. Interestingly, when discounting is taken into account the sole-owner fishery, hitherto regarded as a more desirable situation, can exhibit similar-shaped supply curves and their attendant instabilities.

The capital-theoretic aspects of the subject, which have been long neglected, are given full treatment. The owner of a fishery or a forest aims to maximize the present value of the net revenues it is expected to yield in the future, as he would with any other form of capital. In the case of renewable resources, however, the problem has a unique twist in that the commercial discount rate δ is paired with the renewing rate of the resource, that is, the natural growth rate of the harvested population, r. Clark defines a bionomic ratio δ/r; for $r > \delta$ the traditional concept of maximum sustainable yield provides a reasonable approximation to a long-term conservation strategy, but where r is low relative to δ, as with most whale stocks, overfishing is an inevitable consequence. In the latter event there is no mathematically derived optimum harvesting policy that satisfies both economic and conservationist objectives. Applied mathematics has no further insights to offer and fishery policy is set in effect by a social discount rate, arrived at through the actions of opposing pressure groups in society.

Written by an applied mathematician primarily for applied mathematicians, the book also demonstrates a further benefit of the subject, that is, that it makes it possible to teach fundamental mathematical properties and theorems by fleshing them out in a context of practical importance and thereby adding to their intrinsic interest. A solution to one-dimensional linear control problems based on elementary methods is presented in an early chapter, and later both the continuous and discrete forms of the maximum principle for nonlinear systems are clearly expounded. A chapter is also devoted to the currently fascinating topic of the phase plane analysis of dynamical systems. This, incidentally, complies neatly with May's recent plea (*Nature* [*London*] **261**, 459 [1976]) for more exposure of students of all disciplines to the nonlinear intricacies of behavior displayed by systems of simple differential equations.

At least two-thirds of the book has also a great deal to offer both the biologist and the economist concerned with fishery and other harvesting problems. Only elementary calculus is assumed at the outset, and economic concepts are explained with a lucidity that is all too rare in economics texts, although to get the most benefit the reader should have some facility in mathematics and some familiarity with ecological and economic concepts. In 1968, with his book *Ecology and Resource Management*, Watt laid the foundations of graduate training in the quantitative aspects of resource management. Since that time there has been a growth in the mathematical training of ecologists at the undergraduate level, but this has been rather uneven, partly because of uncertainties about the kind of mathematics that is likely to prove useful. Clark's book provides a much clearer guide to what is required, and there is now an excellent chance that we will see an expansion of graduate training in this field.

The fundamental theoretical points the book develops are illustrated by practical examples from fishery and forestry management, but the examples are fewer than I would have liked and are rather unevenly distributed. A final chapter bringing all the analytical insights developed in the book to bear on a selected single fishery would be a desirable addition for the second edition.

GORDON R. CONWAY
Centre for Environmental Technology and Department of Zoology and Applied Entomology, Imperial College of Science and Technology, London, England