Reports

Quarks with Unit Charge: A Search for Anomalous Hydrogen

Abstract. *Quarks of charge +1 and other anomalous hydrogen have been sought by using the 88-inch cyclotron at Berkeley as a high-energy mass spectrometer, with natural hydrogen and deuterium as the sources of ions. No quarks were observed, and limits were placed on their ratio to protons on the earth that vary from* $< 2 \times 10^{-19}$ *for high masses (3 to 8.2 atomic mass units) to* 10^{-13} *for the lowest masses (< 1/3 atomic mass unit).*

Quarks, the components of the nucleon proposed by Gell-Mann (*1*) and Zweig (*2*), have been the subject of extensive searches by both elementary-particle and cosmic-ray physicists (*3*). In most of these experiments the quarks' fractional charge (1/3 or 2/3 that of the proton) was the distinctive signature that would have indicated their presence. But fractional charge is not required by all quark models; in particular, theories proposed by Lee (*4*) and by Han and Nambu (*5*) have quarks of integral charge ($Z = 1$). These theories are more complex, however, in that they require more than the three quarks of Gell-Mann and Zweig; the Han-Nambu theory, for example, has nine. When the classical quark theory was improved by the addition of the new quantum number "color," the number of fractionally charged quarks had to be increased from three to nine (*6*). The integrally charged quarks of Han and Nambu, on the other hand, accepted color in a very natural way and did not require an increase in number (*7*). It seems reasonable to speculate that the quark searches failed because they assumed the wrong signature; perhaps quarks have integral charge.

Experimental limits on the existence of quarks with integral charge are not nearly as good as those for quarks with fractional charge. Experiments at accelerators would have observed $Z = 1$ quarks only if they were produced with a cross section at least 10^{-3} of that for proton-antiproton pair production and if their mass were no more than 4 atomic mass units (amu) (*8*, *9*). If quarks are stable against decay, as they are in several versions of the theories, then a search in terrestrial material is potentially more sensitive than a search at accelerators since the natural production of quarks had literally aeons of time to take place. Quarks with $Z = +1$ would be equivalent chemically to a new isotope of hydrogen, and would be found on the earth mixed with that element. With this in mind, we decided to search in terrestrial hydrogen for $Z = +1$ quarks, using the same technique that was used in the discovery of stable (and natural) ^{3}He by Alvarez and Cornog (*10*); that is, we used a cyclotron as a high-energy mass spectrometer. After we began this work, we found that Zeldovich and co-workers (*8*, *11*) had shown that if such particles existed they would be found in natural hydrogen not only from cosmic-ray production, but also as remnants of the big bang, in much the same way that protons and the cosmic microwave radiation are thought to be remnants. The ratio q/p of quarks to protons predicted by Zeldovich, Okun, and Pikelner depends somewhat on the assumed values for the quark mass and the cross sections, but is of the order of $q/p = 10^{-10}$ to 10^{-12}. They also calculated that q/p from cosmic-ray production is $\simeq 10^{-17}$.

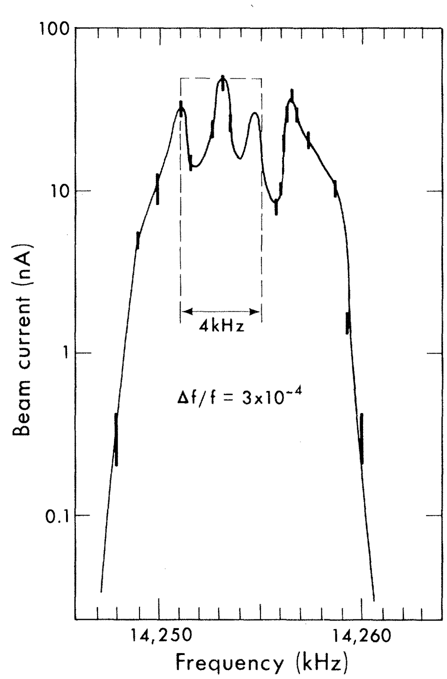

Fig. 1. Beam current as a function of frequency for D^+ ions, as measured by a Faraday cup. The smooth curve is a guide to the eye. The effective width of a "flattop" curve with the same area is 4 khz, as shown, giving a fractional width $\Delta f/f$ of 3×10^{-4}. For these measurements, we reduced the beam current to prevent heat damage to the cyclotron deflector electrodes. The fine structure at the top of the resonance curve reflects changes in the number of turns made by the beam in the cyclotron before extraction.

The most sensitive search before ours was by Alvager and Naumann (*12*), who introduced deuterium into a mass separator and scanned in the charge-to-mass ratio (Z/m) to look for new isotopes of hydrogen. Ions with $Z > 1$ were eliminated by sending the accelerated beam through a nickel foil, which stopped molecules and particles of high Z. Unfortunately, this technique works only for high values of mass, where background from scattered beams is low; for quark masses $m_q > 6$ amu, they were able to set the limit $q/p < 3 \times 10^{-18}$. For the low-mass region, the best limit was obtained by Kukavadze *et al.* (*13*), who used a mass spectrometer to find $q/p < 10^{-8}$ for $m_q > 2$ amu. Thus, previous experiments do not rule out $Z = +1$ quarks with $m_q < 6$ amu at the levels predicted by Zeldovich and Okun.

We used the 88-inch sector-focused cyclotron at the Lawrence Berkeley Laboratory for our search. Both hydrogen and deuterium gases were introduced as sample materials into the filament ion source. Only ions with the appropriate value of Z/m, given by the cyclotron resonance equation, were accelerated. In their original discovery of stable ^{3}He, Alvarez and Cornog scanned in Z/m by varying the cyclotron's magnetic field; in our search we kept the magnetic field constant and scanned in frequency.

For the 88-inch cyclotron to be "in tune"—that is, for it to accelerate particles into our detectors—several criteria must be met. The magnetic field profile (the field as a function of radius) must be appropriate to compensate the relativistic increase in mass as the particle is accelerated, and the voltages applied to the electrostatic deflector and dee must be at the correct values to assure efficient beam extraction. To eliminate the need to change the magnetic field profile during a scan, we operated the cyclotron in the third harmonic mode, where the particle revolution frequency is one-third of the oscillator frequency and the maximum particle velocity is only $0.1c$, where c is the speed of light. During scans we changed the deflector and dee voltages in proportion to the change in frequency. The ultimate proof that we were in tune in our scans came from the observation of expected beams; for example, when we set the cyclotron to accelerate $Z/m = 1/2$ (D^+) and scanned to $Z/m = 1/3$, we found that DH^+ molecular ions were being efficiently accelerated and extracted.

The emerging beam from the cyclotron was focused onto two silicon detectors: a thin one (20 to 60 μm) to measure the ionization rate dE/dx, followed by a thick one (1 mm) to measure the total energy. By requiring a coincident signal between the two detectors, and that the signals in them correspond to a single $Z = 1$ particle with the correct energy, we were able to discriminate against heavier particles, molecules, and background radiation from the cyclotron. Except when the cyclotron was tuned near the resonant frequency for known species (integral values of Z and m) the number of background counts was essentially zero; during one run we operated for over an hour with no background counts. The rapid falloff of the tails of the cyclotron resonance allowed us to search quite close to values of Z/m for which known particles exist. Figure 1 shows the beam current plotted against frequency for a D^+ beam. By the time we had moved off a main peak by about 1 percent, the rate of particles emerging from the cyclotron was reduced to less than one per minute.

The advantage of the cyclotron over an ordinary mass spectrometer does not come from its high resolution (see Fig. 1) or from its beam current (10 to 50 μa), but from the high energy of the emerging beam: several million electron volts per nucleon. This high energy allowed us to send the beam into the particle identification detectors, and to get useful information on a particle-by-particle basis. The cyclotron has, of course, one additional advantage. Had we discovered quarks in natural material, we would have had a high-energy beam of them, and study of their properties would have been straightforward.

For quark masses above 2 amu, we used deuterium gas in the cyclotron ion source. The deuterium had been separated from water at the Savannah River Heavy Water Plant, using the GS process followed by vacuum distillation and electrolysis (*14*). The efficiency for each of these concentration processes increases as the mass of the isotope increases; thus for masses just above 2 amu quarks would have been concentrated by *at least* a factor of 6600, the ratio of ^1H to ^2H in river water. For masses >> 2 amu we know that the concentration of quarks could not have been increased by more than a factor of $5 \times 6600 = 33 \times 10^3$, since 20 percent of the original deuterium in the water was recovered, and obviously no more than 100 percent of the original quarks could have been recovered. Kaufman and Libby (*15*) have shown that for electrolytic separation, this transition to maximum concentrations takes place below a mass of 3 amu. Thus, the sensitivity of a search in deuterium is 6600 times greater than that of a search in hydrogen for quark masses near 2 amu, and 33,000 times greater for quark masses above 3 amu. In a typical run a 13-μa deuterium beam was observed as we tuned through the $^2H^+$ frequency, corresponding to 8×10^{13} ions per second. The deuterium beam was measured with a Faraday cup, preceded by collimators which simulated the acceptance geometry of the silicon detectors. Our typical dwell time for a particular value of Z/m as we manually scanned in frequency was 2 seconds, during which we would have recorded as significant the observation of even a single particle with the proper signature in the dE/dx and total E detectors; no quarks were observed. Thus the ratio of quarks to deuterons was less than 6×10^{-15}. The ratio of quarks to ^1H is then less than 10^{-18} near mass 2 amu, and less than 2×10^{-19} for masses ≥ 3 amu.

For quark masses in the range 1 to 2 amu we used ordinary hydrogen in the source rather than deuterium; our sensitivity was corresponding 6600 times poorer. We repeated the scan in the mass range 1.5 to 2 amu with deuterium, thinking that it would give us some additional sensitivity in the range just below 2 amu. We saw no quark events in this scan. It is difficult to estimate the sensitivity of this scan without studying the detailed physical chemistry of the isotope separation process; thus we claim no limit obtained from this scan. For the mass range 0 to 1, we looked for quark-hydrogn molecules Hq, assuming that their chemistry would be similar to that of H_2. The mass range was covered in two runs; for quark masses of 1/3 to 1 amu the H_2^+ current we obtained was 1.7 μa, and for the mass range 0 to 1/3 amu the H_2^+ current was 0.16 μa. The corresponding sensitivities are in the range $q/H = 10^{-14}$ to 10^{-13}.

We have observed no quarks of $Z = +1$, nor other anomalous isotopes of hydrogen. The limits from our search and

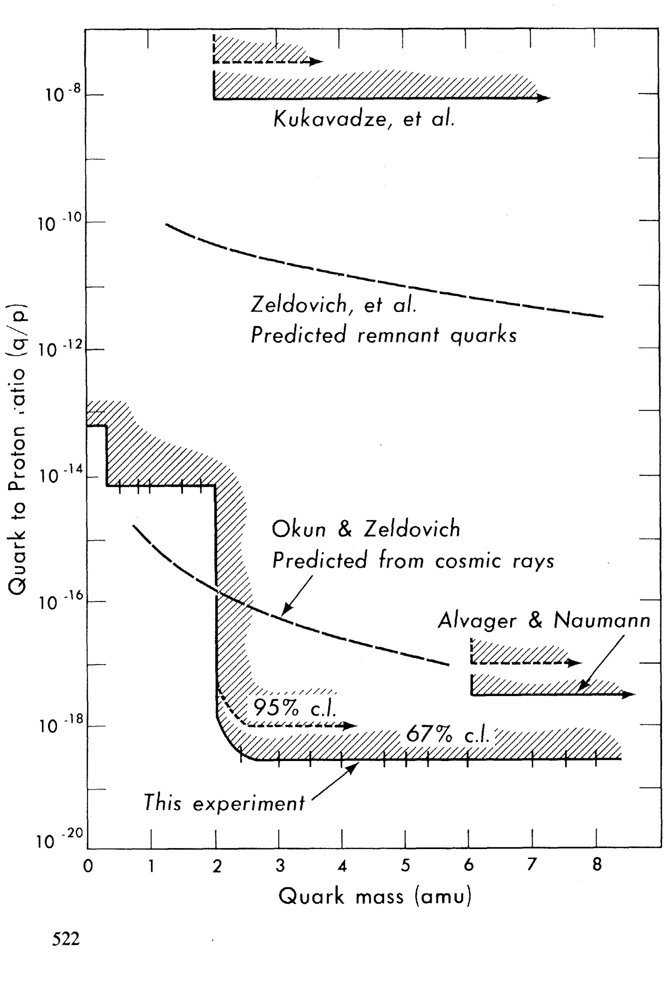

Fig. 2. Quark-to-proton ratio as a function of quark mass. Shown are the experimental limits, as well as predictions of the levels expected from cosmic-ray and big bang production if $Z = +1$ quarks exist. The predictions were scaled with mass to take into account changing cross sections and cosmic-ray fluxes. The short vertical bars indicate the narrow regions (generally less than 1 percent wide) where we could not search because of the existence of beams of common ions with integral values of Z and m. The experimental limits plotted as solid lines are for the 67 percent confidence level, corresponding to approximately 1 standard deviation. Each limit must be raised by a factor of 3 (as indicated with dashed lines) for a 95 percent confidence level.

previous experimental results are plotted in Fig. 2. These limits correspond to 1 standard deviation, that is, a confidence level of 67 percent. For 95 percent confidence levels, each limit must be adjusted upward by a factor of 3, as indicated on the plot. In addition, we have taken the calculations of Zeldovich *et al.* (*8*) and, after scaling them appropriately with energy, plotted the predicted q/p ratios. For several regions corresponding to integral values of Z and m, the intensity from known ions (such as $^2H_2^+$ and $^{14}N^{2+}$) was sufficiently high to require the removal of the silicon detectors from the beam. As a result there are regions, generally less than 1 percent wide in mass, in which we were unable to search for quarks. These regions are indicated in Fig. 2 by short vertical bars. The existence of these beams served the useful purpose of proving that the cyclotron was still in tune and that if there were quarks at concentrations greater than the limits we have placed, they would have been observed.

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Territorial Division: The Least-Time Constraint Behind the Formation of Subnational Boundaries

Abstract. *Nations usually locate their smaller administrative subdivisions in regions of highest population density. This report derives a precise form of the size-density relationship from the general assumption that social structures evolve in such a way as to minimize the total time expended by society in their operation. The result is confirmed empirically.*

All modern societies are subdivided into sets of primary political divisions (for example, states, counties, departments). Where societies exhibit internal variation in population density, the smaller territorial units tend to be located in the more densely settled regions (*1*). This negative relationship between size and density can be derived from the general assumption that social structures evolve under the constraint of minimizing the total societal time expended in their operation.

Territorial subdivision results from the necessity for people to travel between dispersed residences and some central place (for example, a county seat) under limiting conditions of time (the 24-hour day) and time-saving technology (the average velocity of the means of transportation). If territorial divisions are too large, portions of the population will not be able to interact with a center. If divisions are too small, the cost of maintaining the centers would be unnecessarily high, assuming there were enough local resources to maintain them at all. The theoretical derivation will develop equivalencies between these opposing cost factors and societal time expenditure, determine the condition under which total time expenditure would be a minimum, and show that the negative size-density relationship follows from this condition.

Imagine an undifferentiated plane which is to be divided into territorial units, each containing a center designed to serve the population associated with it. Select an imaginary unit and call its area A and its population P. Now let S represent the average travel distance to the center, given the distribution of the population within the unit. This average distance, divided by the velocity of the means of transportation v, gives us the average travel time expended by the population in using the center.

Maintenance of the center and provision of its services to the population will require a further time expenditure, both in direct man-hours of work and in the form of indirect costs paid by the population to support such work. If we let h represent the time cost of maintaining the center, divide this cost by the total population, and add the result to the term S/v, we obtain the expression

$$T = S/v + h/P \qquad (1)$$

where T is the average societal time expended in using and maintaining the service center of the territorial unit.

Since our task is to find the area which will minimize average time expenditure, we must introduce A in both right-hand terms of Eq. 1. Simple dimensional analysis (*2*) suggests that the average distance S will be proportional to the square root of the area A, regardless of the shape of the territorial unit. Thus, with g as the constant of proportionality, we have the substitution $S = g\sqrt{A}$ for the first term. The constant has been evaluated for certain regular polygons which occur frequently in the study of spatial relationships (*3*); its exact value will not be essential in the present derivation. The definition of density, D, as population per unit area ($D = P/A$) permits substitution of AD for P in the second term of Eq. 1 to yield

$$T = g\sqrt{A}/v + h/AD \qquad (2)$$

from which we obtain the derivative

$$dT/dA = g/(2v\sqrt{A}) - h/A^2D \qquad (3)$$

which, set equal to zero and solved for A, gives us

$$A = (2vh/gD)^{2/3} \qquad (4)$$

as the condition under which T will be a minimum (the second derivative of Eq. 2 can be shown to be greater than zero).

Holding v, h, and g constant, we can obtain a linear form of Eq. 4, relating areal size to density,

$$\log A = K - 2/3(\log D) \qquad (5)$$

where K is the log of $2vh/g$ to the two-thirds power. Equation 5 readily lends itself to empirical test with a least-squares estimator to determine the slope relating log-size to log-density.

Such an analysis has been carried out for 98 modern nations (*4*). While the slopes for individual nations vary somewhat around the expected $-2/3$ value (and in some cases the number of subdivi-