Athey implies that we can rely on failure brought about by the lack of genuine learning in the "real" world, after college, to motivate students to work honestly at earning their degrees. Unfortunately this does not square with what cheaters perceive are the rewards to be gained from cheating, nor with the experiences we all have had of seeing both the ignorant and the unethical go on to attain quite considerable success.

In any event, let's not wait for future employers (or "life") to teach our students an ethics lesson; it is part of our responsibility. So let's do our share, recognizing that it is regrettably much easier said than done.

AMITAI ETZIONI

Department of Sociology, Columbia University, New York 10027, and Center for Policy Research, Inc., 475 Riverside Drive, New York 10027

Understanding Mathematical Proofs: Conceptual Barriers

The article by Gina Bari Kolata "Mathematical proofs: The genesis of reasonable doubt" (Research News, 4 June, p. 989) is in some respects misleading. Since it has already inspired a *New York Times* editorial on the so-called "Crisis in mathematics" (*1*), a reply seems warranted. Has mathematics in fact become so complex that proofs are often too long and involved to be properly understood by human beings?

The specific case alluded to—recent work in "homotopy theory" by E. Thomas and R. Zahler—can be dismissed. Like other scientists, mathematicians sometimes make errors and disagree; the issue in question has now been resolved (in favor of Thomas and Zahler) (Letters, 9 July, p. 98), and the amount of time required was not especially long.

Long proofs are hardly an innovation in mathematics. It is easy to find examples from the 19th century. In mathematical astronomy, Delaunay's theory of the moon's motion contained many enormous equations (some fill whole folio pages). It used to be said that one could check such an equation by measuring it—if over 18 inches long, it must be wrong. Shanks spent years calculating pi to 707 decimal places; after the advent of computers it was found that his last 200 digits were wrong.

But the intellectual barriers to be surmounted are more often conceptual than computational. Hawkins (*2*) has written an enlightening account of the struggles of some very distinguished 19th-century mathematicians with the "easy" concepts of continuity and differentiability. There were quite a few blunders. Extremely bright people went astray, not because the proofs were excessively long, but because, even though the concepts were correctly defined on a formal or verbal level, their ramifications were not yet understood on an intuitive level. The "standard" examples and counterexamples with which we now stimulate and guide our imaginations had yet to be discovered. One hundred years later these concepts and theorems cause no trouble at all; they form part of every course in advanced calculus.

Perhaps the most famous "monster proof" in recent mathematics is the theorem of Feit and Thompson (*3*), which settled a fundamental problem about the structure of finite groups. Their proof fills an entire issue of a journal. Yet this work has been assimilated without intellectual indigestion; on the contrary, the new ideas and techniques it introduced have caused group theory to flourish.

The point is simply this: a human mathematician does not attain an understanding of a proof merely by checking that all the individual steps have been strung together according to the rules. On the contrary, such detailed mechanical plodding is neither necessary nor sufficient. What is crucial is to see through the technicalities to grasp the underlying ideas and intuitions, which often can be expressed concisely and even pictorially. Once the gestalt is perceived, the competent technician can fill in as much formal detail as needed. Jacob Bronowski (*4*), speaking of the work of John von Neumann, has put it most beautifully: "What is running through the page is a clear intellectual line like a tune, and all the heavy weight of equations is simply the orchestration down in the bass."

PAUL R. CHERNOFF

Department of Mathematics, University of California, Berkeley 94720

References

1. *New York Times*, 2 June 1976, p. A34.
2. T. Hawkins, *Lebesgue's Theory of Integration: Its Origins and Development* (Univ. of Wisconsin Press, Madison, 1970).
3. W. Feit and J. Thompson, *Proc. Natl. Acad. Sci. U.S.A.* **48**, 968 (1962); *Pac. J. Math.* **13**, 775 (1963).
4. J. Bronowski, *The Ascent of Man* (Little, Brown, Boston, 1973), p. 433.

Erratum: In the letter "Kepone chronology" by Rudolph J. Jaeger (9 July, p. 94, column 3, paragraph 3, line 9), the airborne Kepone concentration in the Life Science Products plant measured by state of Virginia officials in July 1975 was erroneously given as 3 mg/cm^3. The correct concentration was 3 mg/m^3. A portion of a sentence in the preceding paragraph was also erroneously omitted (line 14). The sentence should have read, "A chronicity factor, calculated from these data, is the ratio of the single LD_{50} value *divided by the LD_{50} value* in repeatedly dosed animals."