Reports

Large-Scale Variations in the Obliquity of Mars

Abstract. *Large-scale variations in the obliquity of the planet Mars are produced by a coupling between the motion of its orbit plane due to the gravitational perturbations of the other planets and the precession of its spin axis which results from the solar torque exerted on the equatorial bulge of the planet. The obliquity oscillates on a time scale of approximately 1.2×10^5 years. The amplitude of this oscillation itself varies periodically on a time scale of 1.2×10^6 years. The present-day obliquity is approximately 25.1 degrees. The maximum possible variation is from about 14.9 to 35.5 degrees. Significant climatic effects must be associated with the phenomenon.*

Since Leighton and Murray's article (*1*) on carbon dioxide and other volatiles on Mars, there has been considerable interest in acquiring an understanding of the atmospheric history and behavior of volatiles on Mars. Climatic changes stemming from alterations in the orbit of Mars are of prime importance to the atmospheric problem. Murray *et al.* (*2*) presented modulations in the yearly polar insolation produced by variations in the orbital eccentricity, e, of Mars as a result of the gravitational perturbations of the other planets. The average yearly insolation at the poles is equal to $(S/\pi) \times \sin\theta/(1-e^2)^{1/2}$, where S is the solar constant at the distance of Mars (1.52 A.U.) and θ is the planet's obliquity (the angle between the spin axis and the normal to the orbit plane). Changes in the polar insolation thus depend on the second power of the eccentricity. Although the eccentricity varies from .004 to .141 (*3*), the resulting alteration in the average polar insolation is only of the order of 1 percent. However, the polar insolation would be more sensitive to changes in the obliquity, were such changes to occur. Recent dynamical calculations indicate that, indeed, the obliquity of Mars has undergone large periodic variations—sometimes changing by as much as $\sim 17°$ in $\sim 6 \times 10^4$ years. The climatic effects of this phenomenon must be significant, and it would not be surprising if they were intimately associated with the shaping of many of the surface features.

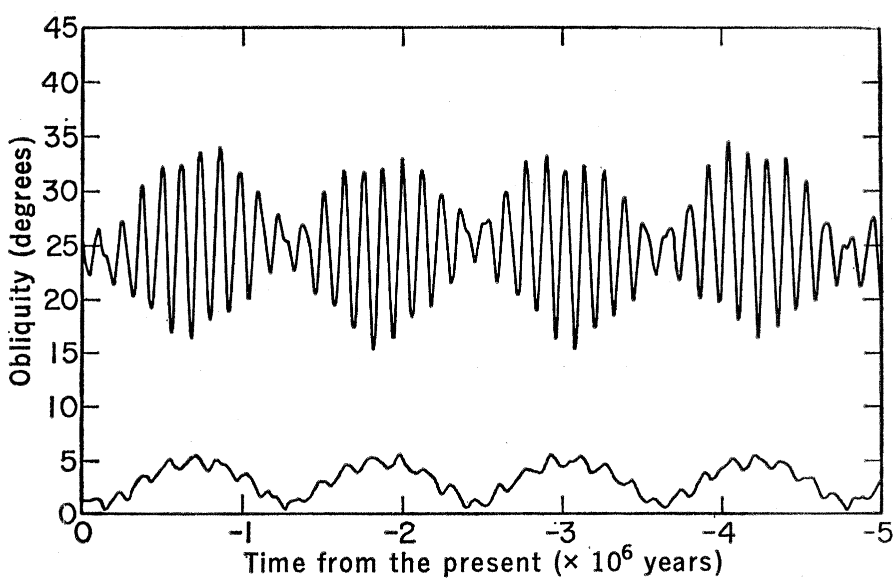

Fig. 1. Variations in the obliquity and orbital inclination of Mars for the past 5×10^6 years.

Secular changes in the orbits of the planets due to their mutual gravitational perturbations were calculated to lowest order in the disturbing function by Brouwer and van Woerkom (*4*). These included changes in the orbital inclination, i, and the longitude of the ascending node, Ω, for each planet, as well as the changes in the eccentricity and argument of the perihelion employed by Murray *et al.* (*2*). In deriving the secular changes, Brouwer and van Woerkom used the auxiliary variables $p = \sin i \cos\Omega$ and $q = \sin i \sin\Omega$ for each planet in order to avoid complications that arise with vanishingly small inclinations. The motions of all the planets are coupled to each other and the problem reduces to a system of eight linear differential equations (Pluto was excluded in these calculations) which, when solved simultaneously, yield eight eigenvectors and eigenfrequencies, ω_k. The motion of any planet is given by expressions of the form

$$p = \sum_k N_k \cos(\omega_k t + \delta_k) \qquad (1)$$

$$q = \sum_k N_k \sin(\omega_k t + \delta_k) \qquad (2)$$

where the amplitudes, N_k, and phase constants, δ_k, are found from the values of p and q in the epoch 1900, and t is time. The longitudes and inclinations are referred to the ecliptic of 1950. The evolution of the orbit plane is most easily understood when its position is expressed relative to the invariable plane of the solar system. When this is done, one eigenvector (with $\omega = 0$) vanishes. Table 1 is a list of the amplitudes and phase constants for the remaining seven terms for Mars, along with the eigenfrequencies and their corresponding periods (P_k). The inclination to the invariable plane can be obtained from the relation

$$\sin i = (p^2 + q^2)^{1/2} \qquad (3)$$

This quantity is the lower curve in Fig. 1, plotted for 5×10^6 years back in time. Two basic periodicities can be seen in the plot: a rapid oscillation of small amplitude ($\sim 1°$) superimposed on a slower one of large amplitude ($\sim 5°$). The first has a period of approximately 1.6×10^5 years, the second a period of the order of 1.2×10^6 years. From Table 1, we see that the solution is dominated by the third, fourth, and fifth terms. The large-amplitude changes result from a beat between the two largest terms and have a frequency corresponding to $(\omega_3 - \omega_4)$. The small-amplitude modulations are due to the fifth term and have a frequency given by $(\omega_5 - \omega_4)$. The theoretical maximum of $\sin i$ is found from $\sum_k |N_k| = 0.102$ (*3*), which corresponds to an inclination of 5.87°. The largest and smallest values obtained in Fig. 1 are 5.60° and 0.23°, respectively. Theoretically the inclination can become zero, although this does not happen for the time interval shown. The rotation of the ascending node is also dominated by the third and fourth terms so that the period for a complete cycle is of the order of 7×10^4 years, comparable to P_3 and P_4. The nodes move in the opposite sense to the orbital motion.

Both a changing inclination and a rotating node are capable of producing a variation in the obliquity. If the spin axis of Mars were fixed in inertial space, the time evolution of the obliquity could be obtained from the motion of the orbit normal. However, the problem is complicated by the fact that the spin axis of Mars is also moving in inertial space as a result of the solar torque exerted on the equatorial bulge of the planet. This motion must be taken into account before the actual change in the obliquity can be ascertained. In fact, it has been shown (*5*) that if the precession frequency of the spin axis is much larger than the relevant frequencies describing the motion of the orbit normal, the obliquity will remain essentially constant. In such a case, the spin axis has the ability to "follow" the moving orbit normal. It is this effect which partially suppresses variations in the earth's obliquity during the motion of its orbit plane. However, for Mars the situation is considerably different.

The equations governing the motion of the spin axis observed from a noninertial reference frame attached to the moving orbit plane (with the x-axis in the instantaneous direction of the ascending node on the invariable plane and the z-axis in the direction of the normal to the orbit plane) take the form,

$$\frac{d\theta}{dt} = -\sin i \cos\phi \left(\frac{d\Omega}{dt}\right) + \sin\phi \left(\frac{di}{dt}\right) \qquad (4)$$

$$\frac{d\phi}{dt} = -\alpha\cos\theta - \cos i \left(\frac{d\Omega}{dt}\right) + \sin i \sin\phi \cot\theta \left(\frac{d\Omega}{dt}\right) + \cos\phi \cot\theta \left(\frac{di}{dt}\right) \qquad (5)$$

Table 1. Frequencies (ω_k), corresponding periods (P_k), phase constants (δ_k), amplitudes for the inclination (N_k), and amplitudes for the obliquity (N_k') of Mars.

k	ω_k (arc second/ year)	P_k (years)	δ_k (degrees)	N_k	N_k'	N_k'/N_k
1	− 5.202	249,200	272.06	0.00180	− 0.00422	− 2.342
2	− 6.571	197,200	210.06	0.00180	− 0.01393	− 7.740
3	− 18.744	69,100	147.39	− 0.03589	− 0.05940	1.655
4	− 17.633	73,500	188.92	0.05025	0.08678	1.727
5	− 25.734	50,400	19.58	0.00965	0.01356	1.405
6	− 2.903	449,500	207.48	− 0.00126	0.00081	− 0.643
7	− 0.678	1,912,900	95.01	− 0.00123	0.00012	− 0.101

The azimuthal angle of the spin axis, ϕ, is measured from the instantaneous position of the ascending node. From measurements of the spin axis orientation (*6*), one can deduce a current value of $\phi(t=0) = \Phi = 356.3°$, and a present value of the obliquity of Mars of $\theta(t=0) = \Theta = 25.1°$. The quantity α is a measure of the frequency of spin axis precession. A recent determination of the oblateness of Mars yields a precession period, $P = 2\pi/\alpha\cos\Theta$, of 175,000 years (*7*), from which $\alpha\cos\Theta =$ 7.42 seconds of arc per year. To solve for the obliquity to first order in the orbital inclination we first solve the zero-order equations with $i = 0$ to obtain

$$\theta = \Theta \qquad (6)$$

$$\phi = \Phi - \alpha t\cos\Theta - \Omega + \Omega_0 \qquad (7)$$

where Ω_0 is the current location of the ascending node. These are then substituted into the right-hand side of Eq. 4. With the aid of Eqs. 1 and 2, the resulting first-order equation can be solved, yielding

$$\theta = \Theta - \sum_k \left(\frac{\omega_k N_k}{\omega_k + \alpha\cos\Theta}\right)[\sin(\omega_k t + \alpha t\cos\Theta + \delta_k - \Phi - \Omega_0) - \sin(\delta_k - \Phi - \Omega_0)] \qquad (8)$$

This expression well illustrates the effect of the spin precession on the variation in the obliquity. If $\alpha \rightarrow 0$, one obtains a first-order expression for the variation in the obliquity of a spin axis fixed in inertial space, which can also be computed directly from the relation $\cos\theta = (\hat{s} \cdot \hat{n})$, where $\hat{s}$ is a unit vector in the direction of the spin axis and $\hat{n}$ is the unit orbit normal. On the other hand, if $\alpha \rightarrow \infty$, $\theta \rightarrow \Theta$ and the obliquity remains constant in spite of the movement of the orbit normal. However, Eq. 8 also illustrates another possible behavior. Since $\omega_k < 0$, if for any k, $|\omega_k + \alpha\cos\Theta| < 1$, a resonance-type phenomenon occurs and the variation in the obliquity due to the kth eigenvector is enhanced. Table 1 shows $N_k' = \omega_k N_k/(\omega_k + \alpha\cos\Theta)$ for Mars. In all but the last two terms the amplitudes are enlarged. Hence, rather than suppressing obliquity changes, the precession of the spin axis magnifies them.

Equation 8 is plotted in the upper portion of Fig. 1. The dominance of the third and fourth terms can be recognized. There are rapid oscillations with a period of the order of 1.2×10^5 years as a result of the frequencies $(\omega_3 + \alpha\cos\Theta)$ and $(\omega_4 + \alpha\cos\Theta)$ of the largest terms. The amplitude of these oscillations varies periodically on a larger time scale of approximately 1.2×10^6 years, which corresponds to half a cycle of the beat frequency $\frac{1}{2}(\omega_3 - \omega_4)$. There is a significant correlation between the two plots of Fig. 1.

The variation in the obliquity is quite dramatic. From Eq. 8 we see that the maximum amplitude of the variation is $\Sigma|N_k'| = 0.179$, which is equivalent to $\sim 10.3°$. The maximum total variation is twice this value, or $\sim 20.6°$. The oscillations occur about the value,

$$\Theta + \sum_k N_k' \sin(\delta_k - \Phi - \Omega_0) \qquad (9)$$

which is equal to 25.2°. Hence, Mars is very close to this position now. During its dynamical history, its obliquity has varied from $\sim 14.9°$ to $\sim 35.5°$.

Although Eq. 8 is only a first-order solution, we have numerically integrated Eqs. 4 and 5 and the agreement with Fig. 1 is quite good. We have also obtained an analytic solution correct to second order in the orbital inclination (*8*). However, insight into the physical nature of this phenomenon is furnished by the first-order solution presented here.

Recent Mariner 9 orbital photographs have revealed several remarkable features on the surface of Mars, including regions composed of unusual layered structures (designated laminated terrain), a vast system of canyons and channels, and a persistent residual south polar cap thought to be composed of water ice (*9*). It is an exciting prospect that the origin of some of these features may be linked to periodic climatic changes associated with the oscillating obliquity. No complete understanding of the history and behavior of volatiles on Mars is possible until this effect is taken into account (*10*).

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References and Notes

1. R. B. Leighton and B. C. Murray, *Science* **153**, 136 (1966).
2. B. C. Murray, W. R. Ward, S. C. Yeung, *ibid.* **180**, 638 (1973).
3. D. Brouwer and G. M. Clemence, *Planets and Satellites* (Univ. of Chicago Press, Chicago, 1961), p. 50.
4. D. Brouwer and A. J. J. van Woerkom, *Astron. Pap. Amer. Ephemeris Naut. Alm.* **13** (part 2), 81 (1950).
5. W. R. Ward, in preparation.
6. J. Lorell *et al.*, *Science* **175**, 317 (1972).
7. ———, *Icarus*, in press.
8. W. R. Ward, in preparation.
9. B. C. Murray *et al.*, *Icarus* **17**, 328 (1972).
10. W. R. Ward, B. C. Murray, M. C. Malin, in preparation.
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An Equatorial Jet in the Indian Ocean

Abstract. *At the surface of the Indian Ocean along the equator a narrow, jet-like current flows eastward at high speed during both transition periods between the two monsoons. The formation of the jet is accompanied by thermocline uplifting at the western origin of the jet and by sinking at its eastern terminus. This demonstrates that a time-variable current can have profound effects in changing the mass structure in the ocean.*

A narrow, high-speed surface jet flows along the equator from west to east across the entire Indian Ocean during the transition periods between the two monsoon seasons. This jet is apparent in the monthly maps of surface currents published by the hydrographic offices of several countries (*1–3*), but as yet it has not been reported in the literature. This is probably because emphasis has been placed mostly on contrasting the circulation at the time of full development of the two monsoons.

The extent of this equatorial jet is most strikingly seen in a plot of the resulting current vectors for all the 1-degree squares in which speed exceeds 20 miles per day (43 cm/sec) (Fig. 1). The jet appears in April and May, and again in September and October, at which times the countercurrent shifts from one hemisphere to the other. This jet also coincides with the occurrence of strong westerly winds along the equator (*2–4*), which leads to the conclusion that the jet is driven by the wind. The jet is narrow—about 500 km wide—and is symmetrical to the equator. It is strongest between 60°E and 90°E, where surface speeds often exceed 30 miles per day (64 cm/sec), and maximum values of 100 miles per day (215 cm/sec) have been reported (*2*).

The only direct current measurements available from which conclusions about the depth of this jet may be

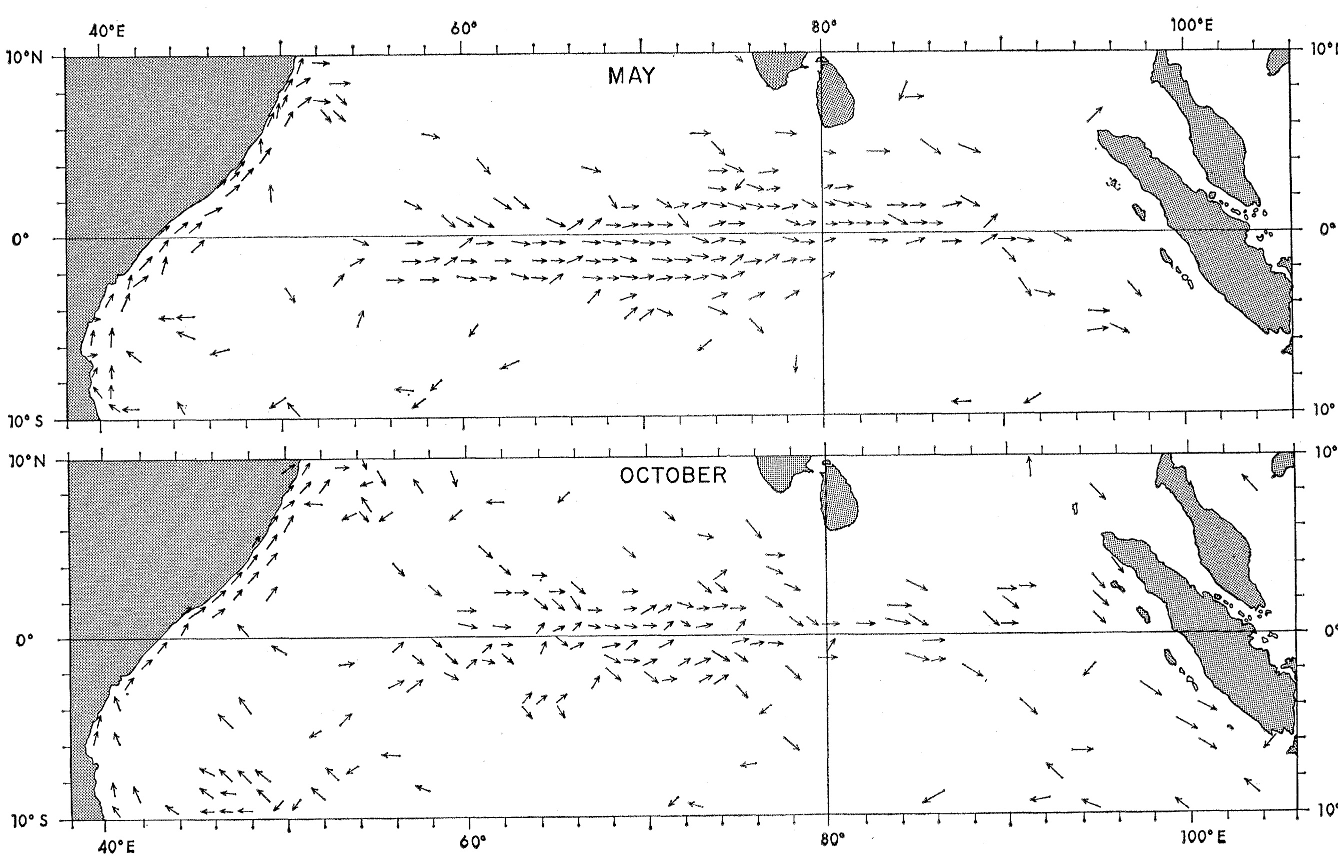

Fig. 1. The equatorial jet in the Indian Ocean in May and October shown by surface current vectors for all 1-degree squares where the speed exceeds 20 miles per day (43 cm/sec), according to data in (*1*).