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SCIENCE

Isospin in Nuclei

Isospin has been reborn as an important and useful quantum number for all nuclei.

D. Robson

Historically the development of the concept of isospin began with the introduction of the term by Heisenberg (*1*) in 1932. Heisenberg pointed out that the neutron n and the proton p are similar in all respects except for their electromagnetic properties. In particular, Heisenberg suggested that the two types of "nucleons" could be described in an elegant mathematical way by the introduction of two-component spinor wave functions in charge space. The uncharged neutron and the proton with charge e are described by first moving the charge origin to $e/2$ and then assigning the new charge coordinates $e/2$ and $-e/2$ to each particle, much in the same way that ordinary Pauli spin has two projections, that is, $\pm \frac{1}{2}\hbar$. The merit of the concept lies in the fact (*2*) that the short-range or strong nuclear interaction between nucleons is essentially charge-independent; that is, the nuclear interactions between a pair of protons, a neutron and a proton, or a pair of neutrons are very much alike. The equality of the nuclear interaction specifically between a pair of protons and a pair of neutrons was postulated by Heisenberg (*1*) and nowadays is described as the charge symmetry of the nuclear interaction.

Since 1932 the isospin concept has been applied extensively in particle physics and nuclear physics. In the case of particle physics, the isospin concept has been extended to more general approximate symmetries which successfully relate the properties of several elementary particles. Isospin in nuclei, where many neutrons and protons are involved, still proves to be a rather elusive concept even for the trained nuclear physicist. Part of this difficulty is due to the fact that we cannot measure isospin directly but must resort to deducing the isospin properties from measurements of other physical quantities. A second difficulty is the oversimplified picture of isospin, often propagated in introductory expositions of the concept, in which isospin is regarded as being completely analogous to ordinary spin and to arise of necessity from the symmetric nature of nuclear forces. Although this approach is essentially correct, it unfortunately suggests that isospin is a fundamental entity and that a proper understanding of the concept will lead by itself to an unlocking of the secrets of nuclei. This is not the function of isospin in nuclei.

The primary role of isospin in nuclei is to provide connections between more fundamental entities such as cross sections or energy levels. The set of energy levels related by the isospin concept is termed an isospin multiplet. This view was succinctly expressed by Wigner (*3*) who compared the role of isospin in nuclei to the role of thermodynamics in classical physics. As Wigner pointed out, thermodynamics gives relations between apparently disjointed quantities such as pressure and heat capacity, yet thermodynamics is in no way a substitute for either the kinetic theory of gases or the theory of crystal lattices. Similarly isospin relates quantities in different nuclei but does not provide a theory for describing any one of the quantities by itself. Indeed, in 1957 Wigner (*3*) with considerable foresight suggested that the principal function of the isospin concept in nuclei may well become that of enabling us to obtain the value of a physical quantity, which is more difficult to measure, from a quantity which is easier to measure or which has already been measured. The accomplishments of succeeding years have admirably shown the correctness of this forecast.

The major advance in the subject since 1957 is the remarkably successful usage of isospin in much heavier nuclei than anyone had thought could be possible. Indeed, Wigner was pessimistic about the use of isospin in heavier nuclei. Contrary to some of his contemporaries, Wigner did not think that isospin would be inaccurate for heavier nuclei but rather that isospin would become uninteresting because of the inaccessibility of all but one member of each isospin multiplet. The inaccessibility problem was removed by Anderson *et al.* (*4*) in 1961 when other members of isospin multiplets in heavy nuclei were actually found. These findings were clarified by a later series of experiments initiated by Fox *et al.* (*5*) in 1963 which showed that isospin is perhaps a more accurate and a more useful concept in heavy nuclei than it is in relatively light nuclei. Some of the experimental results could have been foreseen because the theoretical work of French and MacFarlane (*6*) was already available but apparently not understood (or be-

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lieved) by their nuclear colleagues at that time.

My aim in much of this article is to describe and make clear the spectacular extension of the isospin concept to heavier nuclei. In an effort to provide a lucid explanation of these ideas, I must attempt to cope with a maxim known to most authors of semitechnical papers, namely, that truth and clarity are somehow inversely related. Hopefully those readers who are experts will see that I am sacrificing truth for clarity while those readers who find the following discussion obscure will be grateful because they are being given the truth!

Isospin Relations

The concept of isospin in nuclei is partially understandable in terms of simple billiard ball physics, although, of course, we must eventually appeal to quantum mechanics for a more complete description. We can imagine a system involving two types of balls, some of which are colored red and the remainder white. If we were to scatter these balls in pairs, we would expect the classical laws to be obeyed regardless of whether the pairs are red or white or a mixture of red and white. The system of balls behaves as if it is color-blind, at least insofar as the scattering properties of the balls are concerned. Carrying the analogy further would lead us to conclude that all systems involving a fixed total number of A balls will behave in an identical fashion independently of the fraction that is red. In particular, if we stack our A balls in a container, we can expect the balls to fill to the same level no matter how many are red. Furthermore, if we jiggle the container up and down, the topmost balls will jump up and down independently of their color.

If we now convert our billiard balls into neutrons and protons and our jumping ball system into a nuclear level scheme, then, by analogy, we would expect all nuclei with A nucleons to have the same spectrum of energy levels; for example, for $A = 6$ our analogy might be to fill a container with balls such that only one ball can be at any one level as shown in Fig. 1a. The positions of the balls in each container have been chosen at random, and indeed any other pattern of coloring could equally well have been chosen. We may represent excited states by putting balls into higher positions. As long as we are color-blind, the possible spectra of all seven systems in Fig. 1a are predicted to be identical.

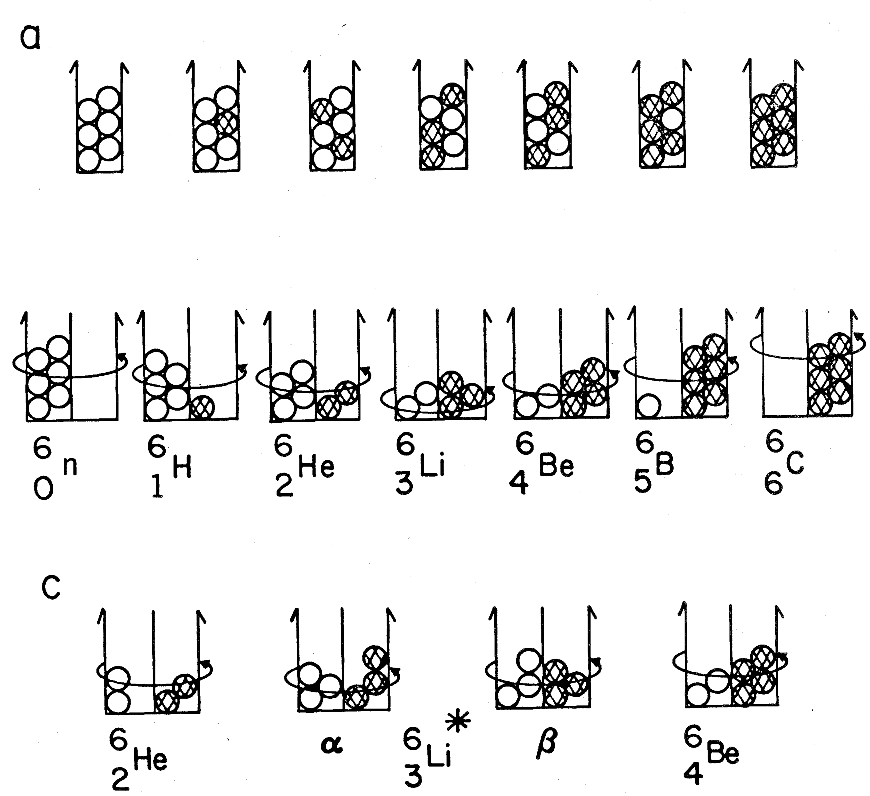

Fig. 1. Schematic models for mass-6 nuclei: (a) model ignoring the distinguishability of color; (b) and (c) model allowing for the difference in color. Arrows indicate the direction of rotation of the container.

Figure 2 is a schematic plot of the energy levels of nuclei with a mass A of 6 which clearly shows that the above picture is not correct. Although we have "satisfied" the Pauli principle by choosing a narrow container such that no two balls can be at the same level, we do not have a complete analogy with nuclei. According to quantum statistics for fermions, only completely identical particles *must* satisfy the Pauli principle. This means that our model above is incomplete since we have *required* that the Pauli principle hold *between* neutrons and protons (rather than for neutrons and protons as separate sets of identical particles) without introducing any concept which distinguishes between them. The Pauli principle is not color-blind. To correct this situation we now imagine a container that is just twice the size of the earlier container with a partition down the center which serves to separate the colors. In order to maintain our state of color blindness, we imagine the two sides of the container to be superposed so that the particular side containing a particular color is unobservable. One way to achieve this is to rotate the container very rapidly so that only the level of filling and the number of balls at each level is significant to the viewer. The rotation is presumed to make colored and white balls look the same color. The lowest states of our $A = 6$ nuclei now look quite different, as shown in Fig. 1b.

In this situation we see that, in general, only those pairs of nuclei having the same number of *oppositely* colored balls have the same energy levels. Such pairs of nuclei (mirrors) do indeed have essentially the same spectrum and are a manifestation of the charge symmetry of nuclear forces. On the other hand, the previous similarity between all mass-A nuclei has disappeared, and, according to the present model, the energy level spectra of the above systems are different. However, there are excited states of each system which still look identical, for example, $^{6}_{3}Li^{*}$ (excited $^{6}_{3}Li$) and $^{6}_{2}He$ or $^{6}_{4}Be$, as indicated in Fig. 1c.

From this result we may conclude that part of the spectrum of $^{6}_{3}Li$ looks like the spectrum of $^{6}_{2}He$ or $^{6}_{4}Be$. We may also conclude that part of the

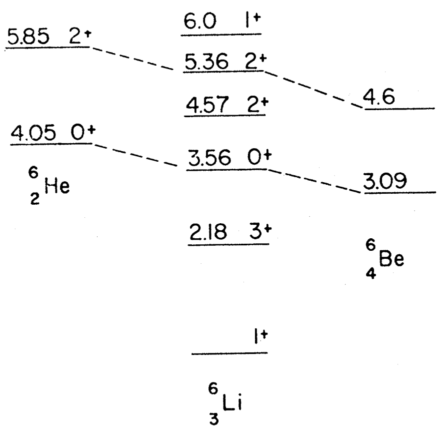

Fig. 2. Schematic energy-level diagram observed for mass-6 nuclei. Numbers on the left side are the energies of excitation above the ground state in million electron volts; numbers and signs on the right side are the spin and parity eigenvalues, respectively.

spectrum of ${}^6_3$Li has no counterpart in any other mass-6 system because it involves stacking arrangements that are not possible for any other nucleus. The energy level diagrams for mass 6 shown in Fig. 2 clearly show this feature.

Note that nucleon emission from the above configurations is always more complicated for ${}^6_3$Li than it is for ${}^6_2$He or its mirror ${}^6_4$Be. In particular, if we take off the topmost ball only, we have

$${}^6_2\text{He} \rightarrow {}^1_0\text{n} + {}^5_2\text{He or } {}^6_4\text{Be} \rightarrow {}^1_1\text{p} + {}^5_3\text{Li}$$

as single-channel decays (that is, the nucleus can break up in only one particular way), but

$${}^6_3\text{Li}^* \rightarrow {}^1_0\text{n} + {}^5_3\text{Li or } {}^1_1\text{p} + {}^5_2\text{He}$$

is a two-channel decay. This complication for nuclei with a minimum $|N-Z|$ (N is the neutron number, and Z is the proton number) indicates that we do not have a "nuclear democracy" for mass-6 systems. Clearly the nuclei with the largest value of $|N-Z|$ (keeping $N+Z=A$ fixed) are the simplest since the number of stacking arrangements is considerably reduced when all the balls are truly identical. As $|N-Z|$ becomes smaller, the number of stacking arrangements grows. In particular, ${}^6_3$Li contains all the stacking arrangements possible for all the other mass-6 nuclei. In this sense the fact that stable light nuclei have $N \simeq Z$ means that nature is far more complicated than it would have been if the ratio of Coulomb to nuclear forces had been much different.

Ladder Operators $\mathbf{T}_{\pm}$

According to the above picture, the stacking arrangements (energy levels) in ${}^6_3$Li that are equivalent to those in ${}^6_4$Be and ${}^6_2$He are those related by simply changing the color of one ball. To achieve this in a quantitative way, physicists use charge-exchange operators $t_{\pm}$ with well-defined properties for free particles, as shown in Fig. 3a. We see that t_+ turns red balls (protons) into white balls (neutrons), and t_- has the reverse effect. The zero values are a consequence of the fact that there are only two fundamental entities in our nuclear model. These properties of $t_{\pm}$ are more restricted in nuclei and in our schematic model because the balls are constrained to certain positions in the container. The meaning of $t_{\pm}$ applied to a particular ball in our model situation is to move it from one side of the container to the other side. If the position the ball is being moved to is already occupied (Fig. 3b), then the color change cannot be made because we allow only one ball of a given color at any one position.

The state of color blindness or indistinguishability appropriate to equivalent stacking arrangements requires that the operation of color changing be symmetric in all of the balls, that is,

$$T_{\pm} = \sum_{i=i}^{A} t_{\pm}^{(i)}$$

in which $i = 1, 2 \ldots A$ labels the various balls of both types in any chosen manner. The effect of $T_{\pm}$ on the ${}^6_3$Li* states in Fig. 1c is very simple because only the two topmost balls can have their color changed. A little thought in fact yields the results

$$T_+\alpha \rightarrow {}^6_2\text{He},\ T_+\beta \rightarrow {}^6_4\text{Be}$$

$$T_-\alpha \rightarrow {}^6_4\text{Be},\ T_-\beta \rightarrow {}^6_2\text{He}$$

where α and β are the two states of ${}^6_3$Li* shown in Fig. 1c.

This apparently nice relationship leaves us with a problem because both states in ${}^6_3$Li* appear to be related to ${}^6_2$He, ${}^6_4$Be by the $T_{\pm}$ operations. In actuality there is only one energy level in ${}^6_3$Li* which is related by $T_{\pm}$ to the ground states of ${}^6_2$He and ${}^6_4$Be. One obtains the correct result by superimposing the arrangements α and β in a manner which maintains *maximum* symmetry between the colors for the combined system, that is, $(\alpha + \beta)$. The

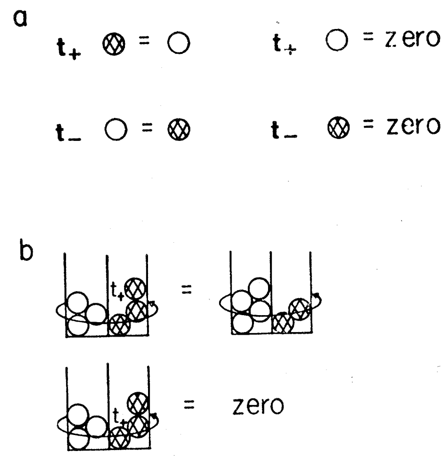

Fig. 3. (a) Ladder operator effects for free particles. (b) Ladder operator effects for bound particles.

alternative (orthogonal) superposition $(\alpha - \beta)$ is the second distinct stacking but it does not have the same color symmetry. We now have the results

$$T_+\ {}^6_3\text{Li}^*\ (\alpha + \beta) \rightarrow {}^6_2\text{He}$$

$$T_-\ {}^6_3\text{Li}^*\ (\alpha + \beta) \rightarrow {}^6_4\text{Be}$$

$$T_{\pm}\ {}^6_3\text{Li}^*\ (\alpha - \beta) \rightarrow 0$$

Clearly $(\alpha + \beta)$ is the equivalent stacking arrangement in ${}^6_3$Li* to those in ${}^6_2$He, ${}^6_4$Be, and $(\alpha - \beta)$ is an arrangement only contained in ${}^6_3$Li*. The states $(\alpha + \beta)$ are usually termed analogue states, and states like $(\alpha - \beta)$ in their color symmetry properties are often given the somewhat misleading name of antianalogue states.

The above discussion can, of course, be stated much more rigorously in terms of wave functions. The net result is to describe $(\alpha + \beta)$ states as $T = 1$ (isospin triplet) states with three projections $T_Z = -1$, 0, and $+1$ corresponding to ${}^6_4$Be, ${}^6_3$Li*, and ${}^6_2$He, respectively. The $(\alpha + \beta)$ states are unique and are designated by $T = 0$ (isospin singlet). The quantum number T is constructed to be the important ingredient in the eigenvalue $T(T + 1)$ of an operator $\mathbf{T} \cdot \mathbf{T}$, where $\mathbf{T}$ is an isospin vector operator with three spherical compounds T_Z, T_+, and T_-. Clearly the isospin quantum number T is analogous to the conventional total spin quantum number S in atomic physics and tells us the charge symmetry of the state. The third component T_Z is related to the neutron excess and therefore the total charge in the nucleus by

$$T_z = (N-Z)/2$$

The value of T_Z for a fixed number of particles $(N+Z)$ simply tells us which nucleus a given state is in. Since T_Z is definite for a given nucleus, we are dealing with a completely polarized system in isospin space. Unlike Pauli spin where the polar axis is often chosen for convenience, isospin has a polar axis always along the direction of the neutron excess.

Generalized Pauli Principle

The requirement that a state of definite isospin T (projection T_Z) satisfy the generalized Pauli principle is that the wave function describing the state be antisymmetric under the interchange of all coordinates (including charge as a coordinate) for any pair of nucleons. The generalized Pauli principle is necessary if we are to relate the states in different nuclei by ladder operations $T_\pm$ [such a set of $(2T+1)$ states is called an isospin multiplet]. However, we can only relate a state with N neutrons and Z protons to a state with $N-1$ neutrons and $Z+1$ protons and then to a state with $N-2$ neutrons and $Z+2$ protons, and so forth, if all the states are antisymmetric in all nucleons. This follows because a state with $Z+1$ protons *must* (independently of isospin considerations) be antisymmetric in all $Z+1$ protons, whereas the states with Z protons only normally need to be antisymmetric in Z protons. Since the ladder operators $T_\pm$ are symmetric in all nucleons, the ladder relations between the states of the multiplets can work only if we impose antisymmetry between all the neutrons and all the protons for all possible partitions of the A nucleons into N neutrons and Z protons, keeping in mind the restraint that T can never be less than its projection T_Z.

Electromagnetic Effects

The corresponding states in nuclei with a mass of 6 (which we saw earlier appear to be related by ladder operations $T_\pm$) are not degenerate in energy. The nuclei with more protons have higher energies because protons can be distinguished from neutrons by means of their repulsive electromagnetic interactions and, of course, the neutron-proton mass difference (δ). In lowest order perturbation theory this means a shift in the energy ΔE by the expectation value (denoted by the symbols $< >$ below) of the additional charge-dependent interaction H^c which breaks the symmetry of the isospin multiplets, that is,

$$\Delta E\ (T,T_z) \simeq \langle TT_z|H^c|TT_z\rangle$$

which from relatively general considerations (*7*) (based, however, on the assumption of two-body forces) can be shown to be of the form

$$\Delta E(T,T_z) = a + bT_z + cT_z^2$$

in which *a*, *b*, and *c* depend, of course, on T.

This "multiplet mass equation" is easily understood for the specific case of a Coulomb interaction e^2/r_{ij} which interacts pairwise between all protons i and j separated by a distance

$$r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$$

The number of interactions is clearly $Z(Z-1)$ which, if we use the relation

$$Z = \frac{1}{2}(N+Z) - T_z$$

leads to the quadratic form above for a fixed value of $(N+Z)$. The relative energy between two adjacent members of a multiplet is in fact loosely referred to by nuclear physicists as the "Coulomb displacement energy" (E_c) minus δ, that is,

$$E_c - \delta = \Delta E(T,T_z - 1) - \Delta E(T,T_z) = c - b - 2cT_z$$

The quadratic equation has now been tested for several multiplets in nuclei with $T = {}^3/_2$ and $N+Z < 40$ and appears to be an excellent description of the data except for the anomalous case of the mass-9 multiplet where it appears that a term proportional to $T_Z{}^3$ has to be invoked. Such a term, however, has a very small coefficient and appears to be related to the existence of higher order electromagnetic effects. The problem with the multiplet equation is that it involves three unknowns, *a*, *b*, and *c*, so that it is not useful unless there are at least four members of the multiplet available.

Recently calculations of the Coulomb displacement energies have been made, and serious discrepancies appear to exist, particularly (*8*) for mass 3, mass 17, and mass 41—all of which are situations where the nuclear structure is relatively well understood. The most reliable situation is in mass 3 where theoretical predictions and experimental results differ by as much as 120 kiloelectron volts. Short of relatively untenable or radically new theories for the structure of ${}^3_2$He and ${}^3_1$H, it appears that we have to invoke a breakdown in the charge symmetry of nucleon-nucleon strong interaction; that is, the neutron-neutron nuclear force is not quite the same as the proton-proton nuclear force. Direct measurements of the neutron-neutron cross section near zero energy appear to be necessary. In spite of the obvious difficulties associated with performing such experiments, there are serious proposals now appearing which suggest that an accurate measurement of the neutron-neutron scattering cross section will be possible in the not too distant future.

Isospin in Heavy Nuclei

The previous discussion has hopefully laid the foundations that enable us to consider systems for which N and Z are quite large. It is well known that, as one progresses from ${}^{40}_{20}$Ca to ${}^{238}_{92}$U, stable nuclei have more and more excess neutrons; that is, $N-Z$ increases from near zero to values like 50. This fact by itself has little to do with isospin but is simply a matter of maintaining an optimum balance between the electromagnetic repulsion and the nuclear attraction as the nuclear container is filled with neutrons and protons according to the dictates of the normal (as opposed to the generalized) Pauli principle.

Since $T_Z = (N-Z)/2$ becomes large, so does the minimum value of T. The importance of this increase was emphasized in an important theoretical paper by Lane and Soper (*9*). They pointed out that the large neutron excess itself has absolutely pure isospin (since no protons are involved) and therefore strongly dilutes the isospin "impurity" of the remaining part of the system with $N=Z$. Isospin impurity in the $N=Z$ core arises because of electromagnetic or charge-dependent interactions which do not conserve isospin. As we saw earlier, the Coulomb interaction H^c gives rise to energy shifts by way of expectation values or diagonal matrix elements. The same interaction also has matrix elements which connect states of different isospin, and consequently the physical system can include states which are not quite pure in the value of T.

The total isospin of the system is obtained by adding (vectorially) the isospin of the $N=Z$ core to the isospin of the neutron excess. If the core is presumed to be mainly isospin zero and to have an isospin impurity of one, then the addition of the neutron excess isospin of $(N-Z)/2$ produces total isospin values of $(N-Z)/2$ or $(N-Z+2)/2$, the latter arising entirely from the impurity in the core. However, because isospin adds vectorially, the impurity in the core can be added to the isospin of the neutron excess to make either of the total values of isospin. The probability of forming the value of $(N-Z+2)/2$ is less than the probability of forming the value of $(N-Z)/2$ by a factor of $2/(N-Z+2)$, and this is the "dilution" factor discussed by Lane and Soper.

The reader may verify such a dilution by drawing semiclassical vector triangles corresponding to the addition of two vectors $\mathbf{T}_1 + \mathbf{T}_2$ to form a resultant $\mathbf{T}$, with the sides of the triangle being of length $[T_1(T_1+1)]^{1/2}$, $[T_2(T_2+1)]^{1/2}$, and $[T(T+1)]^{1/2}$, respectively. In constructing such triangles it should be kept in mind that both the neutron excess and the total isospin vectors have the same projection $T_Z = (N-Z)/2$ so that an impurity vector of length $2^{1/2} = [(1+1)]^{1/2}$ simply is not long enough to connect the two long vectors with the same projection. Of course, this semiclassical argument would yield no probability for making a T value of $(N-Z+2)/2$ and corresponds to the semiclassical limit of $2/(N-Z+2)$ when $N-Z \rightarrow \infty$. The dilution of the core impurity by the factor $2(N-Z+2)^{-1}$ leads to the possibility of relatively pure isospin for heavy nuclei.

Analogue States in Heavy Nuclei

The crucial demonstration of isospin multiplets in heavy nuclei was provided by the experimental work of Anderson *et al.* (*4*) at the Lawrence Radiation Laboratory, University of California, Livermore. Nuclei like $^{89}_{39}Y$ were bombarded with protons, and the neutrons emitted (by the nuclear interactions in the target) were studied as a function of the energy of emission. Anderson *et al.* found that there was a relatively sharp peak in the yield of neutrons at a neutron energy which differed from the energy of the incident proton by the Coulomb displacement energy.

The first detailed explanation of the phenomenon was made by Lane (*10*), who invoked one of the components of the known nuclear interaction—the

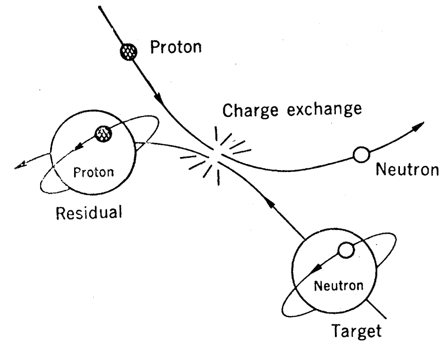

Fig. 4. Diagram illustrating a charge-exchange reaction in the center of the mass coordinate system.

Heisenberg interaction. This interaction involves the isospin vectors for each interacting nucleon and has the simple isospin-conserving form:

$$V_H = V_H(r_{ij})\mathbf{t}^{(i)} \cdot \mathbf{t}^{(j)}$$

The results of the Livermore experiments could then be attributed to the Heisenberg interactions of the incoming proton with the excess neutrons according to which, by means of the ladder products $t_+^{(p)}\ t_-^{(n)}$ contained in $\mathbf{t}^{(p)} \cdot \mathbf{t}^{(n)}$, the incident proton "deposits" its charge onto a neutron in the target nucleus (see Fig. 4). This "charge-exchange" process is possible when the energy of the residual nucleus is such that the proton formed out of the original neutron can behave in an analogous fashion, that is, when the state of the residual nucleus is a member of the same isospin multiplet as the target nucleus so that they are related by the ladder operator

$$T = \sum_j t_-^{(j)}$$

The point is that for analogue states each of the excess neutrons contributes an amplitude of the same phase, and constructive interference occurs to provide a large probability for the charge-exchange process. For such transitions Lane suggested that, instead of summing over the interactions between the incident proton and the excess neutrons, one could use an average interaction $U(r_p)$ of the "optical model" form

$$U(r_p) = U_0(r_p) + U_1(r_p)\mathbf{t}^{(p)} \cdot \mathbf{T}$$

where the isospin-dependent nature of the second term on the right side involves a charge-exchange factor $t_+^{(p)}$-T_- which automatically produces an "analogue state" by means of the ladder operator T_- and an outgoing neutron by means of $t_+^{(p)}$. The remainder of $U(r_p)$ is presumed to give rise to elastic scattering of the incident proton so that the Lane potential provides an interaction which describes the scattering of nucleons to all possible states of the target isospin multiplet. This model provides a good description of such phenomena.

Analogue Resonances in Heavy Nuclei

A very important development in the theory of heavy nuclei was the observation by Fox *et al.* (*5*) that states such as those excited by charge exchange could be excited as a compound nucleus resonance. Instead of converting a neutron into a proton by charge exchange, one simply bombards a nucleus with protons at just the right energy for the proton to "go into orbit" around the nucleus. This proton orbit behaves in an analogous fashion to the lowest orbits available to a neutron when it is added to the target nucleus, except that the neutron will normally be bound to the nucleus whereas the proton is free to escape.

It is reasonable to wonder how a free proton (free in the sense that it can escape) can behave in any way like a bound neutron. First, the proton is free and the neutron is bound because the Coulomb displacement energy increases linearly with the charge Z of the system whereas the binding energy of the last neutron in a stable nucleus is roughly constant and clearly not related to Z in a linear fashion. For analogue resonances it is easy to derive the relation

$$E_p = |E_c| - |B_n|$$

where E_p is the proton energy of escape, E_c is the Coulomb displacement energy, and B_n is the binding energy of the last neutron in the system formed by adding a neutron to the same target nucleus. Clearly E_p becomes positive as soon as $|E_c|$ exceeds $|B_n|$, which it *always* does for nuclei with masses greater than $A \sim 60$. Coulomb displacement energies are quite large in heavy nuclei, being about 20 million electron volts for nuclei heavier than lead. For many nuclei the analogue resonances occur at energies E_p less than the Coulomb potential barrier so that the incident proton must tunnel in and out of the nucleus.

Second, the proton "inside" the nucleus experiences to a good approximation a constant Coulomb repulsion

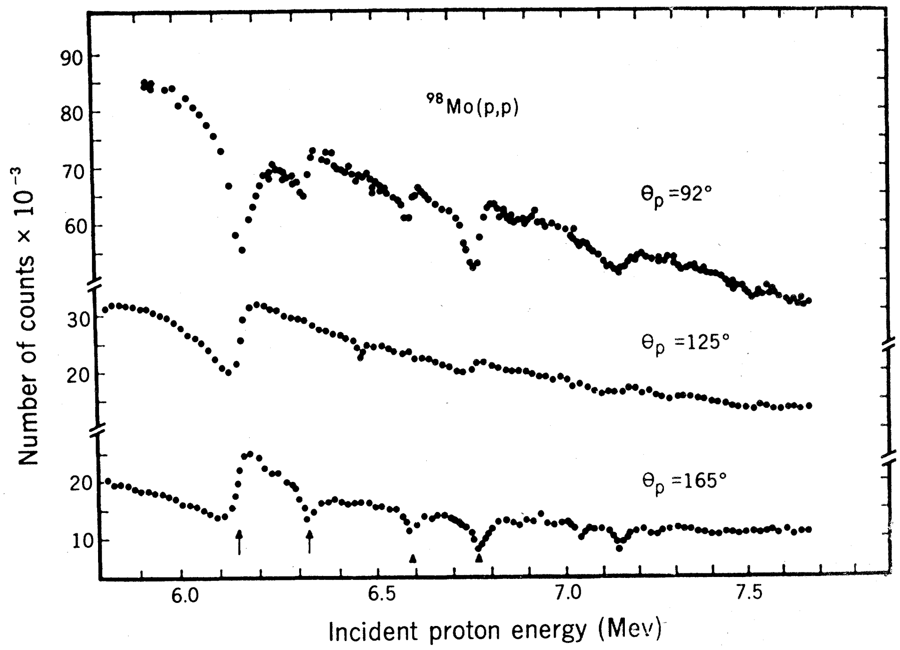

Fig. 5. Elastic scattering cross section for protons incident on ^{98}Mo as a function of the bombarding energy at various scattering angles θ_p.

which has the effect only of changing the energy of the system but not the wave function. Consequently, the proton wave function in the region where it experiences strong interactions may be almost the same as a neutron wave function except for the total shift E_c in its energy.

The foregoing discussion does not bring out the real significance of the proton resonances which are observed. A typical cross-section (*11*) measurement for ^{98}Mo + p is shown in Fig. 5. Two things are important:

1) Several resonances are observed, each resonance corresponding (by means of T_-) to known states of the nucleus ^{99}Mo and each resonance having a characteristic shape in virtue of its interference with the Rutherford scattering amplitude. The magnitude of the deviations arising from interference would perhaps even have perturbed Rutherford a little!

2) The resonances can decay by different emissions from the way they are formed. In particular, inelastic proton emission occurs in many resonances. It is the flexibility of the resonance mechanism as compared to the charge-exchange mechanism which allowed nuclear physicists to really exploit the isospin concept in heavy nuclei and reminds us again of Wigner's point (*3*) about isospin: "The principal function of the isospin concept in nuclei may well become that of enabling us to obtain the value of a physical quantity, which is difficult or impossible to measure, from a quantity which is easier to measure."

The point is that the compound resonance is the analogue of a neutron bound state, and the formation of such a bound state is limited to the addition of a neutron to a stable target (since as yet excited states of target nuclei are not feasible targets for experimentation). Since neutron emission is by definition forbidden from a bound neutron state whereas proton emission is allowed from the analogue resonance, Wigner's point is clear-cut.

Sequential and Transfer Reactions

The relationship between the charge-exchange mechanism of Anderson *et al.* (*4*) and the analogue resonance mechanism of Fox *et al.* (*5*) has been established by means of more recent experiments (*12*) in which an analogue resonance is excited via a (p,n) charge-exchange reaction and its decay is observed by proton ($\bar{p}$) emission. Such experiments are not hard if one does not try to observe the neutron as well, and they are becoming more popular as a method of observing isospin-dependent phenomena. Other reactions such as inelastic electron scattering (e,e'$\bar{p}$), pion-induced charge exchange (π^-, $\pi^0\bar{p}$), and proton transfer reactions (d,n$\bar{p}$) or (^{3}He,d$\bar{p}$) are being used to examine other interesting properties.

The particular case of T(^{3}He,d)R reactions is interpreted as a proton being transferred or "stripped" from the incident ^{3}He to the target nucleus T to form the residual nucleus R. This mechanism is conceptually the same as that of Fox *et al.* (*5*) but allows the proton to be transferred at "negative" energies into bound proton orbitals. In this way the location of proton orbitals belonging to different values of the isospin of the residual nucleus can be studied (*13*). In principle, this technique provides a measurement of the nuclear "symmetry energy" corresponding to the energy difference between states of different isospin containing the same proton orbitals. A few results are now available which agree qualitatively with theoretical expectations. Similar information has also been obtained (*14*) by the removal of a neutron from orbits also filled by protons with the use of "pickup" reactions like T(p,d)R. The important ingredient is that a "neutron hole" corresponding to the lack of a neutron in a given orbit has the same isospin quantum numbers t,t_z as a proton in the same nucleus.

It is interesting to try and locate the next members of isospin multiplets, for example, the so-called double analogue state given by $(T_-)^2$ operating on the nucleus with $T = T_Z$. Reactions like T(p,^{3}H)R have been used in light nuclei to form states in the residual nucleus with isospin $T = T_Z + 2$ and projection T_Z. In nuclei with masses heavier than $A \sim 70$, the use of such "two-neutron pickup" reactions has not yet led to the identification of any double analogues. The related reaction T(^{3}He,n)R involving "two-proton stripping" from ^{3}He also could be used to excite double analogues, but no success has been achieved as yet. Consequently, the use of isospin so far in heavy nuclei has been limited to just two members of a given multiplet. The existence of other members is expected but they remain to be properly identified in the future.

Summary

The major feature of isospin in nuclei that I have discussed here is its application to all nuclei. The rebirth of this quantum number in nuclear physics occurred in the early 1960's and was initiated almost entirely by the important work of Anderson *et al.* (*4*) and Fox *et al.* (*5*). There is still great interest in the use of isospin in its fullest sense as predicted by Wigner (*3*), and indeed isospin concepts have been largely responsible for demonstrating that nuclei in the doubly "magic number" region of ^{208}Pb are remarkably in agreement with shell model theory.

The early experiments have also initiated a whole new set of more sophisticated experiments (some of which I have briefly alluded to above) which promise to keep many physicists busy for a long time to come. A particularly interesting series of experiments are those being performed (*15*) at Duke University with high-resolution proton beams. This work shows the highly detailed nature of analogue resonances, that is, as coherent superpositions of many complicated compound states yielding a beautifully modulated wave train, the modulation being observed only in conventional experiments with poor-resolution proton beams. Similarly, nuclear theorists have been led to vastly improve their interpretation of, and computational techniques for, both nuclear reactions and nuclear structure in order to meet the more stringent tests provided by such experiments.

Perhaps a lesson can be learned from the historical development of the isospin concept. In the past the belief that $\mathbf{T} \cdot \mathbf{T}$ would not significantly commute with the dynamical Hamiltonian so that isospin would not be conserved sufficiently well enough certainly delayed the nuclear travels of isospin into the realm of heavy nuclei. Hopefully the same mistake will not occur in the future for other approximate symmetries of nature.

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Behavioral Implications of the Human XYY Genotype

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The significance of the XYY chromosome pattern in the human male is still the subject of considerable controversy. Despite the apparent high prevalence in some studies of socially deviant individuals, negative reports have also appeared, and instances of apparently well-adjusted XYY individuals have been published. Some have questioned the existence of any link whatsoever with antisocial behavior.

Three of the outstanding questions concerning the XYY human genotype are thus: (i) Is an XYY male significantly more likely than an XY male to be found in settings for antisocial deviant individuals? (ii) If not, then how has the present controversy arisen; but if so, then what is the nature and extent of the association between the XYY genotype and the tendency to such placement? (iii) What is the magnitude of the risk for an XYY individual, specifically a newborn, of eventually manifesting antisocial behavior compared to that for an XY individual born to similar circumstances?

Evidence for an Association with Deviant Behavior

Delinquent individuals may be segregated by society into a number of possible settings, not all of which may be appropriate for the behavior displayed, but which at least remove the individual from the community at large. For the discussion below I define as "mental" a setting for individuals who are retarded, disturbed, psychotic, alcoholic, or epileptic but which is not otherwise characterized; as "penal" a setting where there is some stated or implicit restriction on freedom because of punitive or security requirements; and as "mental-penal" a setting which meets both criteria. Examples of penal settings are general prisons and schools for juvenile delinquents and the like. Examples of mental-penal settings are hospitals for criminally insane and security wings in hospitals for the retarded. The populations of all three groups may overlap because admissions criteria are loosely applied, but in the initial analysis I consider these as separate types of settings. Furthermore, for the purposes of this review the terms "deviant antisocial behavior" or "deviance" are defined as that behavior which leads to or increases the likelihood of placement in a mental-penal or penal setting in a particular jurisdiction. This is not to imply that all of those in such settings have been placed there appropriately, or conversely.

Whereas a large number of institutionalized populations in different countries have been studied, only in Scotland has there been also a considerable study of "normal" groups for comparison (see Tables 1 and 2). Jacobs and her coworkers group detected 5 XYY's in 3500 consecutive male infants or 0.14 percent and no XYY's in 2040 "normal" adult males studied for a variety of reasons (*1, 2*). In contrast, in their original study in the wing for mentally retarded men in Carstairs maximum security hospital, 7 of 197 or 3.6 percent were XYY (*3*). Subsequent data from the same institution (*2, 4*) and one other mental-penal setting in Scotland (*2*) also yielded rates higher than the observed rate for Scottish newborns

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