

## Gravitation Theory: Empirical Status from Solar System Experiments

All observations to date are consistent with  
Einstein's general relativity theory of gravity.

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### Introduction

Newton's law of gravitation states that there is an attractive force between two masses  $m_1$  and  $m_2$  separated by a distance  $r_{12}$  which has the magnitude  $F = Gm_1m_2/r_{12}^2$ , where  $G$  is the gravitational constant. This force is supposed to instantaneously change its magnitude and direction if one of the masses changes position relative to the other.

However, special relativity requires that the speed of propagation of a causal influence should not exceed the speed of light. In addition, it requires that all laws of nature, including gravitational laws, should be Lorentz invariant; that is, constant motion relative to "space" should be unobservable or meaningless, and observations from various inertial frames should be consistent with each other and with the principle of the constancy of the speed of light. The gravitational interaction requires motional corrections from the static interaction and can not act instantaneously at a distance if it is to be Lorentz invariant.

Einstein's theory of gravity—general relativity (1, 2)—in addition to making gravitation compatible with special relativity, had consequences for space and time measurements even more revolu-

tionary than those in special relativity. In general relativity, the proximity of matter alters the geometry of extended rigid objects, affects the magnitude of ruler and clock measurements, and causes deflection of rays of light. The local equivalence of gravitational fields and accelerated coordinate systems was also incorporated in a fundamental way into general relativity by Einstein.

The minute gravitational effects which go beyond the predictions of Newton's theory are called post-Newtonian effects. Recent discoveries in astronomy increase our need to have confidence in post-Newtonian gravitational theory. In astrophysical phenomena such as quasistellar objects (3), pulsars (4), and possibly gravity waves (5), extremely strong (post-Newtonian) gravitational fields may play dominant roles. "Black holes," whose external structure and observational aspects are entirely determined by gravitation, have been proposed (6) and are being sought observationally. In all these areas of relativistic astrophysics, the complete post-Newtonian properties of gravity are being employed by theorists. So, although the post-Newtonian effects in the solar system are minute and of negligible practical significance, precise, noninferential knowledge of gravity in the solar system can be applied in the extreme conditions of the universe with increased confidence.

Technological developments have made possible a renaissance of experimental activity to explore the relativistic or post-Newtonian details of gravitation. The capability of ranging between the earth and other (natural and man-made) objects in the solar system with radio and laser radiation, and the development of sophisticated electronic and cryogenic systems that make it possible to detect very weak signals, have led to a variety of experiments in which minute gravitational effects are detected. Also—in keeping with the present spirit in physics of focusing on invariances, symmetries, and conservation laws of physical phenomena—experiments that reveal the symmetry properties of gravitational interactions (high-precision null experiments) are playing an important role in pinpointing nature's choice of a theory of gravitation.

General relativity was the first successful post-Newtonian theory of gravitation and still commands dominant support among physicists. It has so far met all empirical tests, and most agree that Einstein's theory is the simplest, most elegant, theoretical formulation of all the proposed theories. But, for this article, I will assume that we are not sure of the validity of general relativity precisely as formulated by Einstein. Instead, from a theoretical perspective in which a broad spectrum of theories of gravitation are accepted as possibly correct, I will discuss key foundational experiments specifying the necessary properties of possible theories, and then discriminating experiments specifying particular gravitation equations. The end of the journey will be an increased empirical confidence in general relativity (2).

### Foundational Experiments

Einstein was particularly influenced by the fact that different material bodies fell at identical rates in gravitational fields, a phenomenon first pondered by Galileo. In the gravitational equation of motion for a body

$$M_1 \mathbf{a} = M_0 \mathbf{g} \quad (1)$$

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the two masses  $M_I$  and  $M_G$  can conceptually be distinguished as the inertial mass and the gravitational mass, respectively. The inertial mass determines the resistance to acceleration,  $\mathbf{a}$ , of a body experiencing any force, while the gravitational mass determines the strength of the gravitational force on a body located in a gravitational field,  $\mathbf{g}$ . Identical acceleration rates in a gravitational field, then, means that the ratio  $M_G/M_I$  is the same for all materials (the conventional choice of physical units makes this ratio 1).

From special relativity it is known that the inertial mass is determined by the entire energy content of a body,  $E$ , divided by the square of the speed of light,  $c$ :

$$M_I = E/c^2 \quad (2)$$

For any body there are contributions to  $E$  from nuclear energy, electromagnetic energy, weak interaction energy, and kinetic energy, as well as the mass energy of any fundamental constituent particles.

The ratios of these various energies differ from one material to another; iron nuclei, for example, possess a larger fraction of nuclear binding energy than lithium nuclei, and iron atoms have a larger fraction of Coulomb binding energy than lithium atoms. Einstein concluded that the gravitational mass was also proportional to the total energy content of a body:

$$M_G = E/c^2 \quad (3)$$

If the ratio of gravitational to inertial mass is the same for all bodies, Einstein pointed out in his equivalence principle that physics would be the same in an accelerated coordinate system (imagine the interior of an accelerated elevator) as in a uniform gravitational field (the interior of an elevator at rest in a gravitational field). Extending this equivalence conceptually beyond experience, he concluded that because a horizontal light beam "looks" as if it is being deflected in the accelerated elevator, it really will be deflected in the gravitational field. Einstein also considered two clocks at different heights ( $\Delta h$ ) in an elevator accelerating upward at rate  $g$ . Pulses emitted by the upper clock are received at a time  $\Delta h/c$  later by the lower clock, which by then is traveling upward at a speed  $g\Delta h/c$  greater. A Doppler shift results in the frequency,  $\nu$ , of the received signal,

$$\Delta\nu/\nu = g\Delta h/c^2 \quad (4)$$

He then concluded by the principle of equivalence that clocks at different gravitational potentials run at different rates, and that this entails a shift toward the red of solar spectra. This "red shift" is given by Eq. 4.

Experiments at Harvard University by Pound and his collaborators in the early 1960's (7) confirmed this to an accuracy of 1 percent. Monochromatic gamma radiation emitted by iron nuclei was absorbed by other iron nuclei at a vertical separation of several tens of meters. By using Mössbauer techniques involving the recoilless emission and absorption of radiation, they measured the frequency shift; a change of 10 meters in height resulted in a fractional frequency shift of  $10^{-15}$ . Spectral lines received on the earth from the surface of a white dwarf star can be shifted toward the red by 1 part in  $10^5$ , but because of pressure shifts, pressure broadening, and rotational and Doppler shifts in the observed spectra, and the uncertainty in the gravitational potential at the surface of a white dwarf star, precise measurements of this gravitational shift are difficult, if not impossible. In the 1970's atomic clocks will be sent into space and compared with atomic clocks on the earth; this is expected to increase the accuracy with which the gravitational frequency shift can be measured by a factor of 100 or 1000.

Around the turn of the century Eötvös (8) made precise measurements for the ratio  $M_G/M_I$  for various materials, and showed that the masses are equal with the impressive accuracy of 3 parts in  $10^9$ . More recently, Dicke and his collaborators (9) found no inequality between  $M_G$  and  $M_I$  for various materials to 3 parts in  $10^{11}$ , and

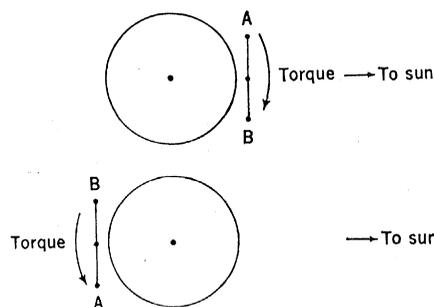


Fig. 1. A torsion bar is shown at times 12 hours apart, during which the earth rotates  $180^\circ$ . If material  $A$  has a larger ratio  $M_G/M_I$  than material  $B$ , the sense of a torque produced on the torsion bar by the sun reverses sign in the 12 hours. This is essentially the arrangement of Dicke and collaborators (9).

Braginsky and Panov (10) have improved this accuracy to 1 part in  $10^{12}$ . Figure 1 shows a torsion bar with different materials,  $A$  and  $B$ , which is monitored as the earth rotates. If  $A$  and  $B$  are accelerated toward the sun at different rates, a solar torque on the torsion bar reverses sign every 12 hours, producing an oscillating displacement. No such oscillating displacements have been seen within experimental accuracy. Contemporary experiments of the Eötvös type are sufficiently accurate to show that even the elusive weak-interaction energy properly participates in gravitational physics. The current-current weak-interaction hypothesis of Feynman and Gell-Mann (11) has been used to estimate the weak energy for a typical nucleus as  $10^{-8}$  of the total energy (12).

A most significant limitation, however, of these laboratory tests of the equivalence principle is that they do not exclude the possibility that the gravitational and inertial masses differ by the gravitational energy content of the body. The bodies typically used in the Eötvös-type experiments contain only about  $10^{-25}$  of their total energy as internal gravitational energy, and this is an experimentally undetectable amount. Efforts to extend the Eötvös-type experiments to celestial bodies are discussed later in this article.

In the late 19th century Mach (13) criticized the Newtonian concept that force was required to accelerate matter relative to space; only motion relative to other matter in the universe was significant or meaningful in the Machian view. If the inertia of matter was due to the other matter in the universe, it was plausible that matter could possess more inertia in the direction of the center of the galaxy, where substantial matter is located. Hughes *et al.* (14) and Drever (15) searched for such a mass anisotropy by examining the degeneracy of atomic energy levels due to the orientation of the angular momentum (an anisotropy of mass would break such degeneracies). They used nuclear magnetic resonance techniques, and found no anisotropy of the inertial mass to an accuracy of 1 part in  $10^{22}$ .

Experiments with high-energy accelerators are constantly testing the energy-momentum-velocity properties of extremely relativistic elementary particles, that is, particles with speeds near that of light. These properties are in accord with special relativity. Also, experiments on the electromagnetic inter-

actions of matter confirm special relativity as a valid principle for localized interacting matter. Without presenting the details of the argument, I will only state that the complete set of empirical facts above—(i) the “red shift” of clock rates in gravitational potentials, (ii) the equality of the gravitational and inertial masses for all laboratory bodies, (iii) the isotropy of the inertial mass, and (iv) the validity of special relativity for interacting laboratory matter—seems to be compatible with only a restricted class of gravitational theories. These are called metric theories of gravity (16, 17).

### Metric Theories of Gravity

Metric theories of gravity can best be described by listing their key features:

1) There exists at least one gravitational field in nature,  $g_{\mu\nu}$ , a second rank tensor field which is the metric field. There may exist other cosmological gravitational fields. Physicists are free to arbitrarily pick space-time coordinate systems without affecting physical predictions or the forms of physical laws (this is the general covariance of physical laws). There always exist local coordinate systems in which  $g_{\mu\nu}$  takes the Minkowski form of special relativity;

$$g_{\mu\nu} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \equiv \eta_{\mu\nu} \quad (5)$$

Globally, the metric field determines how the various local inertial frames of special relativity are related to each other (because of the presence of matter in the universe, there are no global inertial frames).

2) All laboratory forms of the energy of matter (such as mass, kinetic, nuclear, and electromagnetic energy) couple to the metric field in a specific universal manner. Gravity pulls on energy. The total energy density of a body,  $T_{00}$ , is only a single component of a 4 by 4 symmetric matrix of physical densities called the stress-energy tensor,  $T_{\mu\nu}$ ;

$$T_{\mu\nu} = \begin{bmatrix} T_{00} & T_{0x} & T_{0y} & T_{0z} \\ T_{x0} & T_{xx} & T_{xy} & T_{xz} \\ T_{y0} & T_{yx} & T_{yy} & T_{yz} \\ T_{z0} & T_{zx} & T_{zy} & T_{zz} \end{bmatrix}$$

where  $T_{0x}$ ,  $T_{0y}$ ,  $T_{0z}$  are the three vector components of momentum density, and  $T_{xx}$ ,  $T_{yy}$ , and so forth, represent stress densities in a body. The entire tensor  $T_{\mu\nu}$  participates in being the source of gravity (18). As a result of the universal

manner in which matter couples to the metric field, at each locality in space and time the metric field determines the spatial and temporal intervals measured by rulers and clocks constructed of ordinary matter (19); hence the name metric field and the traditional interpretation of  $g_{\mu\nu}$  as establishing a geometry for space-time.

Where the metric field takes the Minkowski form, the fundamental equations of matter are the conservation equations of special relativity

$$\partial T^{\mu\nu} / \partial x^\mu = 0 \quad (6)$$

where we use the convention of summing the common Greek indexes over the four coordinate dimensions:  $x^\mu = ct, x, y, z$  for  $\mu = 0, 1, 2, 3$ , respectively.

More explicitly, Eq. 6 can be given as

$$\partial T^{00} / \partial ct + \partial T^{0x} / \partial x + \partial T^{0y} / \partial y + \partial T^{0z} / \partial z = 0$$

and so forth. In the presence of a non-Minkowski metric field  $g_{\mu\nu}(\mathbf{r}, t)$ , matter obeys the generally covariant “divergence” equation:

$$\partial T^{\mu\nu} / \partial x^\mu + \Gamma_{\mu\lambda}^{\mu} T^{\lambda\nu} + \Gamma_{\mu\lambda}^{\nu} T^{\mu\lambda} = 0 \quad (7)$$

The  $\Gamma_{\mu\lambda}^{\nu}(\mathbf{r}, t)$  are the Christoffel symbols (playing the role of gravitational fields) formed from derivatives of the metric field. Equation 7 then gives the response of stress energy (matter) to the presence of the metric (gravitational) field.

3) There exist field equations for the metric field through which its value can be related to the local distribution of mass and energy, and possibly to various global features of the universe through boundary conditions.

4) Although other cosmological gravitational fields may exist, they can not couple to the energy in laboratory forms of matter, but can only couple to the metric field. Hence, matter does not respond to any other cosmological gravitational fields.

An important property of metric theories of gravity follows from these features. Specifying only the metric field in the solar system, one can find the equations of motion of matter. It is not necessary to know how the metric field was obtained in order to proceed phenomenologically to calculate matter's response to gravity.

The physics of almost any gravitational experiment in the solar system is therefore calculable solely from the metric field in the solar system. In the Appendix (presented for the interested

reader but not a necessary part of the body of this article) the most general gravitational metric field for a system of massive bodies is given. Any metric theory leads to a metric field of the form given in the Appendix, and many different theories may lead to the same metric field, to our approximation, and consequently make the same predictions for gravitational experiments. A goal of experiments is to specify uniquely the seven coefficients in this general metric and thereby point toward specific theories of gravity.

### Experiments to Differentiate among Metric Theories of Gravity

All metric theories of gravity comply with the results of the foundational experiments. Nonmetric theories, in about every case, violate one or more of these results (as yet there is no proof totally ruling out nonmetric theories of gravity, so some caution is advisable on this point). I now discuss a variety of experiments that differentiate among metric theories. The predicted magnitudes of the various experimental effects differ from theory to theory. In this discussion, measurable quantities will be given with coefficients  $\eta_i$ , which are dimensionless and of order 1, and theory dependent. All the  $\eta_i$  are related to the theoretically more fundamental coefficients that appear in the general metric field expression of the Appendix. Table 1 gives the values of the  $\eta_i$  for several representative contemporary theories of gravity, while Table 2 in the Appendix relates the  $\eta_i$  to the theoretical metric parameters.

*Light deflection and retardation experiments.* When electromagnetic radiation passes through a gravitational field it is deflected, as Einstein predicted according to the equivalence principle. However, the actual deflection is about twice what Einstein first predicted, the additional deflection being related to the fact that “straight physical rulers” are warped when placed in gravitational fields. (In traditional language the geometry of space becomes non-Euclidean in the proximity of matter.) A light beam passing the sun at a distance  $D$  is deflected by the angle

$$\theta = \eta_1 4GM_s / c^2 D \quad (8)$$

where  $G$  is the gravitational constant,  $c$  the speed of light,  $M_s$  the mass of the sun, and  $\eta_1$  the theory-dependent coefficient. When  $D$  is approximately equal

Table 1. Values of the theory-dependent coefficients for the experimental effects discussed in this article, for four representative theories of gravity. Einstein's general relativity predicts the greatest number of null experimental results. In the Brans-Dicke version of scalar-tensor theories (36)  $\omega$  is a positive coupling constant which in the limit  $\omega \rightarrow \infty$  brings their theory back to general relativity. The more general type of scalar-tensor theory (41) has a second coupling constant contribution to the  $\eta_2$  and  $\eta_5$  coefficients. Yilmaz's modified scalar theory (42) which possesses an a priori special inertial frame is a particular example of a class of such theories discussed in general by Ni (43). The vector-tensor theory (31) is a particular example of a class of theories in which the cosmological scalar field is replaced by a vector or second tensor field. In such theories special inertial frames related to the universal rest frame are not a priori but result from the field equations relating fields to the matter in the universe. In this particular theory  $\kappa$  is a cosmological parameter that changes in value as the universe expands.

| Theory                       | $\eta_1$                      | $\eta_2$                      | $\eta_3$                      | $\eta_4$                      | $\eta_5$             | $\eta_6$ | $\eta_7$ | $\eta_8$ | $\eta_9$ | $\eta_{10}$ |
|------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|----------------------|----------|----------|----------|----------|-------------|
| General relativity           | 1                             | 1                             | 1                             | 1                             | 0                    | 0        | 0        | 0        | 0        | 0           |
| Scalar-tensor                | $\frac{3+2\omega}{4+2\omega}$ | $\frac{4+3\omega}{6+3\omega}$ | $\frac{4+3\omega}{6+3\omega}$ | $\frac{3+2\omega}{4+2\omega}$ | $\frac{1}{2+\omega}$ | 0        | 0        | 0        | 0        | 0           |
| Scalar with "prior geometry" | 1                             | 1                             | 1                             | 0                             | -8                   | 0        | -8       | 0        | 0        | $\kappa$    |
| Vector-tensor                | 1                             | 1                             | 1                             | 1                             | $\kappa$             | $\kappa$ | 0        | 0        | $\kappa$ | 0           |

to the radius of the sun the angle is 1.75 arc seconds.

In 1964 Shapiro (20) pointed out that light is also slowed down when passing the sun (and measured by rulers and clocks far from the sun). The additional time for a round trip of an electromagnetic pulse traveling between the earth at a distance  $R_1$  from the sun and another body at  $R_2$  from the sun, the light passing the sun at a distance  $D$ , is about

$$\Delta t = \eta_1 (4GM_s/c^3) \ln(4R_1 R_2 / D^2) \quad (9)$$

For ranging to the planet Venus, Eq. 9 gives a maximum time delay of 230 microseconds. Historically, the deflection of light was measured by observing the shift in the apparent positions of stars near the sun during a solar eclipse. The size stability and calibration of photographic plates limited the accuracy of this technique, although effects of about the right size have been seen in almost all the experiments since the first during a 1919 eclipse. Experiments of higher accuracy are presently being performed by Shapiro and his collaborators (21) at Massachusetts Institute of Technology, Cambridge; they use radar ranging between the earth and the inner planets Mercury, Venus, and Mars during periods when the lines of sight to these planets pass close by the sun. At Jet Propulsion Laboratory, Pasadena, California, radio ranging to artificial satellites is being used to measure the time delay (22). The values of  $\eta_1$  given by the two groups are

$$\eta_1 = 1.015 \pm 0.05 \quad (21)$$

$$\eta_1 = 1.00 \pm 0.04 \quad (22)$$

The main difficulties in the way of improved experimental accuracy for  $\eta_1$  are: the fluctuating electron density in the solar corona, in which electromagnetic pulses (particularly the longer-wavelength radio waves) are randomly

retarded; the planetary surfaces whose topographies, until better known, introduce errors in ranging; random spacecraft accelerations because of gas leaks; and the varying drag experienced by artificial satellites due to interplanetary material or solar radiation pressure. By employing multiple-frequency radio ranging, through which solar corona effects can be eliminated; using laser ranging, which is unaffected by corona effects; mapping out planetary topographies; and developing a drag-free satellite, it seems possible that within the decade  $\eta_1$  could be measured with an accuracy of  $10^{-3}$ .

*Perihelion precession of planetary orbits.* According to the Newtonian theory of gravity, planetary orbits responding solely to the central gravitational field of the sun would describe fixed ellipses in inertial space with the sun at one focus (Kepler's first law). When we allow for the perturbations of the planets by each other, their elliptic orbits precess; the major axis (the line joining aphelion to perihelion) rotates slowly in orientation relative to the fixed stars. The precession for the earth is about 6000 arc seconds per century. Observations show precession rates for the inner planets, however, that exceed those predicted by Newtonian theory, particularly for the planet Mercury.

Metric theories of gravity predict additional post-Newtonian precession rates  $\Omega$  for bodies in the solar system; one effect for near-circular orbits gives

$$\Omega = \eta_2 3GM_s \omega / c^2 R \quad (10)$$

where  $R$  is the radius of the planetary orbit and  $\omega$  is the orbital angular frequency. The coefficient  $\eta_2$  was first measured by using several centuries of optical astronomical data; for the planet Mercury, where the effect is largest (43 arc seconds per century), good agreement with the predictions of Einstein's

general relativity was obtained. Shapiro *et al.* (23) used radio ranging data to Mercury to obtain the value

$$\eta_2 = 1.005 \pm 0.02$$

where a negligible solar gravitational quadrupole moment is assumed. Dicke and Goldenberg (24) observed a visual flattening of the sun, the equatorial dimension exceeding the polar dimension by 5 parts in  $10^5$ . If the sun's mass density is distorted in the same proportion, a Newtonian gravitational quadrupole moment results which produces an appreciable precession of Mercury's orbit and spoils the good agreement with general relativity. However, it has not been established that the sun's contours of constant brightness do coincide with contours of constant gravitational and centrifugal potential. Quadrupole contributions to perihelion precession scale as  $R^{-7/2}$ , whereas the relativistic effect of Eq. 10 scales as  $R^{-5/2}$ . Accurate measurements of several planetary orbits can distinguish between the effects. Continued radio ranging to the inner planets should resolve this question of a possible solar quadrupole moment in the next few years (23).

*Orbiting gyroscope.* In Newtonian physics a torque-free gyroscope will maintain a fixed spin axis in inertial space. In metric theories of gravity a gyroscope spin axis will slowly precess when in gravitational orbit around a massive body. In the geometrical language of gravity, the "parallel" transport of a vector (the spin axis) in the non-Euclidean geometry near a massive body does not leave the vector unchanged on traversing a closed path in space. In the language of field theory or potential theory, a post-Newtonian central gravitational field produces a torque on a moving gyroscope (analogous to the spin-orbit interaction of electromagnetism). A conceptually

fundamental prediction made by Lense and Thirring (25) can also be tested by a gyroscope orbiting the earth. According to these authors, a spinning massive body (the rotating earth) produces gravitational potentials of the motional type, which act on matter as if inertial space were being slowly dragged around the spinning body. The Lense-Thirring effect is a form of "gravitational magnetism," one spinning mass producing a torque on another. A gyroscope orbiting the earth should precess at the rate (26)

$$\dot{\mathbf{S}} = \boldsymbol{\Omega} \times \mathbf{S}$$

where

$$\boldsymbol{\Omega} = \eta_5 \frac{3}{2} \frac{GM_E}{c^2 R} \boldsymbol{\omega} + \eta_4 \frac{G}{c^2 R^3} \left( \frac{3\mathbf{R} \cdot \mathbf{J} \mathbf{R}}{R^2} - \mathbf{J} \right) \quad (11)$$

where  $M_E$  is the mass of the earth,  $\mathbf{J}$  its spin angular momentum vector,  $\mathbf{R}$  the position vector to the gyroscope from the center of the earth, and  $\boldsymbol{\omega}$  the angular velocity vector of the gyroscope's orbit. A superconducting gyroscope is being developed at Stanford University, Palo Alto, California, and the National Aeronautics and Space Administration, Huntsville, Alabama (27). The system is planned to have sufficient accuracy to measure both effects in Eq. 11, which have magnitudes of about 7 and 0.05 arc seconds per year, respectively, for low earth orbit. The two effects can be differentiated from each other experimentally by inclining the orbit of the satellite from the equatorial plane of the earth, although precise orbital tracking will be necessary in order to measure the smaller of these effects.

*Equivalence principle for massive bodies (lunar laser ranging).* The Eötös-type experiments have shown that laboratory bodies of different material accelerate at identical rates in a gravitational field. It is worth extending this test of Einstein's equivalence principle to celestial bodies, which contain an appreciable amount of the internal gravitational energy that is negligible in the laboratory bodies. Calculations in the framework of metric theories have shown that celestial bodies will, in general, violate the equivalence principle, their mass ratio,  $M_G/M_I$ , being given by a 3 by 3 matrix (16, 28) which differs from 1:

$$\left( \frac{M_G}{M_I} \right)_{\alpha\beta} = \left( 1 + \eta_5 \frac{U_G}{Mc^2} \right) \delta_{\alpha\beta} + \eta_6 \left( \frac{1}{3} J^{\alpha\beta} - J^{\alpha\gamma} J^{\gamma\beta} \right) \frac{1}{2Mc^2 I} \quad (12)$$

Table 2. Relationship between experimental and "parameterized post-Newtonian" (PPN) parameters. Each coefficient  $\eta_i$  of an experimental effect discussed in this article is equal to some combination of PPN metric parameters; so each experiment in some sense maps out or is sensitive to some combination of the gravitational potentials. A complete package of experimental results is needed to uniquely specify the PPN metric. A redundancy of experimental results (more experiments than PPN parameters) can be used to check the consistency of the assumptions in PPN metric theory; a breakdown of this phenomenological theory of gravitational theories would call for a more radical and innovative gravitational theory in order to agree with all experience.

| $\eta_i$    | Theoretical PPN metric parameters                                  |
|-------------|--|
| $\eta_1$    | $(1 + \gamma)/2$   |
| $\eta_2$    | $(2 + 2\gamma - \beta)/3$  |
| $\eta_3$    | $(1 + 2\gamma)/3$  |
| $\eta_4$    | $(4 + 4\gamma + \alpha_1)/8$                                       |
| $\eta_5$    | $(4\beta - 3 - \gamma - \alpha_1 + \alpha_2 - \rho_1) - 1/3\eta_6$ |
| $\eta_6$    | $(\alpha_2 + \rho_2 - \rho_1)$                                     |
| $\eta_7$    | $\alpha_1$   |
| $\eta_8$    | $\alpha_3$   |
| $\eta_9$    | $\alpha_2$   |
| $\eta_{10}$ | $\alpha_2$   |

The acceleration of a body then results from multiplying the gravitational vector  $\mathbf{g}$  by the matrix of Eq. 12:

$$\mathbf{a} = \left( \frac{M_G}{M_I} \right) \mathbf{g}$$

In Eq. 12,  $U_G$  is the internal gravitational potential energy of the celestial body,  $M$  its mass,  $J$  its spin angular momentum,  $I$  its moment of inertia, and  $\delta_{\alpha\beta}$  is 1 for indexes  $\alpha = \beta$  and 0 for  $\alpha \neq \beta$ . General relativity is almost unique among metric theories in predicting zero magnitude for both  $\eta_5$  and  $\eta_6$  (29). If we apply Eq. 12 to the earth and the moon, a pair of massive bodies being accelerated toward the sun, and neglect the  $\eta_6$  term, which is presently unmeasurable, an oscillation in the earth-moon range results (30):

$$X(t) = -\frac{3}{2} \eta_5 \left( \frac{\omega'}{\omega} \right) \left( \frac{U_G}{Mc^2} \right)_s R_s \cos(\omega - \omega')t \quad (13)$$

with time  $t$  being measured from the new-moon lunar phase. In Eq. 13  $\omega'$  is the angular frequency of the earth around the sun,  $\omega$  is the angular frequency of the moon about the earth, and  $R_s$  is the distance from the earth to the sun. The amplitude of this oscillation is only about  $20\eta_5$  meters, but the laser ranging presently taking place between the earth and reflectors placed on the moon by Apollo astronauts appears to have the capability of eventually detecting an effect of this size. Range data will be analyzed by

Fourier techniques and the effect will be sought as a residual amplitude left after Newtonian effects of the same frequency are eliminated. If an ultimate accuracy of 10 centimeters, which was suggested by the experimenters, is reached, then  $\eta_5$  can be measured to an accuracy of 1 part in 200.

### Experiments to Test Machian Concepts

In the general form of the metric field  $g_{\mu\nu}$  (given in the Appendix) there are gravitational potentials dependent on the velocity of the inertial coordinate frame with respect to the universal rest frame, although a theory like general relativity produces no potentials of this type. Such potentials are called Machian because their source must be related in some manner to the entire matter distribution of the universe. For experiments in the solar system this Machian velocity has a magnitude of approximately 200 kilometers per second, the speed of the sun in our galaxy, if it is assumed that the galaxy is approximately at rest in the universe. It has been shown (31) that these Machian potentials are present in metric theories which contain a first or second rank tensor field or an a priori special inertial frame in addition to the metric field. Several experiments put limitations on the strengths of the Machian potentials in nature.

*Contributions to planetary perihelion precession.* Two additional contributions to the precession of planetary orbits are produced by the Machian potentials (32);

$$\delta\Omega = \eta_7 g_s \frac{W'}{4ec^2} - \eta_8 \frac{W'}{V_0} \left( \frac{U_G}{Mc^2} \right) \frac{\omega_s}{2e} \quad (14)$$

where  $g_s$  is the sun's gravitational field at the planetary orbit,  $W'$  is the component of the sun's velocity through the universe lying in the plane of the planetary orbit and perpendicular to its major axis,  $e$  is the orbital eccentricity,  $V_0$  is the orbital speed, and  $\omega_s$  is the sun's rotational angular frequency. For the planet Mercury the terms in Eq. 14 have magnitudes of about 30 and  $10^5$  arc seconds per century, respectively. Since the precession rate of Mercury is in agreement with theory to an accuracy of about 0.5 arc second per century, empirical limits for  $\eta_7$  and  $\eta_8$  can be made;

$$\begin{aligned} |\eta_7| &\approx 0.02 \\ |\eta_8| &\approx 10^{-5} \end{aligned}$$

These limits must be relaxed by a factor of about 5 if it is assumed that  $\eta_2$  differs from 1 by about 5 percent.

*Earth gravimeter variations.* The Machian potentials rescale the strength of the earth's central gravitational force, producing effects dependent on the speed of the earth through the universe and also on the angle between the direction of the gravitational force and the direction of motion of the earth through the universe. A gravimeter, which measures the strength of the earth's gravity, changes its orientation daily because of the rotation of the earth and changes its speed annually because of the orbital motion of the earth around the sun. This produces several possible oscillatory effects in gravimeter readings, the dominant term being (32, 33)

$$\Delta g/g = \eta_0 (\mathbf{W} \cdot \hat{\mathbf{R}})^2 / 2c^2$$

where  $\mathbf{R}$  is the unit radial vector from the center of the earth to the site of the gravimeter. This term has a magnitude of about  $3 \times 10^{-8}$ . The present theory of the response of the earth to the Newtonian tidal deformation forces is consistent with the world gravimeter data to about an order of magnitude less, giving the approximate limit;

$$|\eta_0| \lesssim 0.1$$

None of the present gravimeter data was designed to look for these relativistic effects, so the present limit on  $\eta_0$  is tentative; future gravimeter experiments to specifically search for Machian gravitational contributions are now being planned.

*Earth rotation rate variations.* If the gravitational self-attraction of the earth changes strength with a 12-month period (because of changing speed relative to the universe) the earth will "breathe," that is, expand and contract as a result of the variation in its gravitational compression. With this breathing the moment of inertia of the earth changes, and its rotation rate also changes because its angular momentum is conserved. The possible directional dependence of the earth's self-attraction resulting from a Machian potential produces a permanent quadrupolar deformation of the planet, which is aligned with its velocity through the universe, the deformation also changing magnitude with an annual period. Metric theories predict a rotation anomaly of (32, 34)

$$\Delta t = \eta_{10} \int \left( \frac{(\mathbf{V}_0 \mathbf{W})_{\perp} - \frac{1}{3} \mathbf{V}_0 \cdot \mathbf{W}}{c^2} \right) dt \quad (15)$$

where  $\mathbf{V}_0(t)$  is the velocity of the earth relative to the sun,  $\mathbf{W}$  is the velocity of the sun through the universe, and  $\perp$  means components perpendicular to the equatorial plane of the earth. A 3-millisecond amplitude from Eq. 15 seems about the maximum compatible with existing data, since most of the anomaly in the earth's rotation rate is accountable in terms of seasonal variations in atmospheric winds (35). A limit on  $\eta_{10}$  then results:

$$|\eta_{10}| \lesssim 0.1$$

*Evolving gravitational constant.* It has frequently been speculated that Newton's gravitational "constant" may, in fact, not be a constant. The empirical observation that

$$GM_u/c^2 R_u \sim 1$$

where  $M_u$  is the mass of the universe and  $R_u$  its radius, has led some cosmologists to suggest that this is a physical equation which determines  $G$  in some gravitational theory, and that as the radius of the universe expands  $G$  would change accordingly. A scalar-tensor gravitational theory was formulated, in part with this motivation by Brans and Dicke (36). The goal of deriving  $G$  from cosmological field equations motivates continued efforts to find new gravitational theories as alternatives to general relativity (37). In theories with a changing  $G$ , planetary orbits will adjust their size in response:

$$\dot{R}/R = -\dot{G}/G$$

and the orbital frequency will also vary:

$$\dot{\omega}/\omega = 2 \dot{G}/G$$

(The dot over a letter denotes the time derivative.) From the radio ranging data on the inner planets and the orbital motion of the earth, Shapiro *et al.* (38) have found no evidence for a variation in  $G$  and put an upper limit on its rate of change:

$$|G/G| \lesssim 4 \times 10^{-10} \text{ year}^{-1}$$

Improvements in these ranging experiments in the next few years may decrease this limit by an order of magnitude. In most cosmological models,  $G/G$  is expected to be comparable to the rate of change of the radius of the universe. This rate is given by Hubble's constant, which relates the galactic recession velocity (inferred from the frequency shift of spectral radiation) to distance;

$$\dot{R}_u/R_u = H_0$$

A reevaluation of  $H_0$  (39) gives it the new empirical value of  $5 \times 10^{-11} \text{ year}^{-1}$ . A theoretical calculation of  $G/G$  in a metric theory requires more than the metric field in the solar system vicinity, as  $G/G$  is determined partly by a cosmological solution of the field equations of the theory.

## Gravity Waves

One consequence of special relativity is that physical influences propagate no faster than the speed of light. If an interaction between matter is transmitted by a field instead of being direct action at a distance, the differential equations of the field usually allow free wave-like solutions as well as static solutions "attached" to sources. Consider schematically the partial differential equation for a typical  $1/R$  field

$$-\nabla^2 F(\mathbf{r}, t) = S(\mathbf{r}, t) \quad (16)$$

where  $F(\mathbf{r})$  is some field and  $S(\mathbf{r})$  is its source. The Laplacian differential operator  $\nabla^2$  is not Lorentz invariant; making the differential operator invariant generally leads to a field equation of the form

$$\left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) \Gamma(\mathbf{r}, t) = S(\mathbf{r}, t) \quad (17)$$

which approximates Eq. 16 in the static limit, but also possesses homogeneous wave-like solutions. Thus, the discovery of gravity waves would not be primarily a test of a particular gravitational theory, but rather a test of the validity of special relativity in the gravitational domain. Only detailed experimental information about the polarization and interaction properties of the wave can differentiate among contemporary gravitational theories.

J. Weber of the University of Maryland, College Park, has probably detected gravity waves (40). Several hundred times a year two massive aluminum cylinders separated by 1600 km are found to be coincidentally mechanically excited appreciably above the thermal noise level, the vibrations of a normal mode of the cylinders being detected by attached piezoelectric strain gauges. The cylinders, fortunately, have a directional dependence to their efficiency (or cross section) for absorbing gravity waves, maximum absorption occurring when the waves approach perpendicularly to the cylinder axis. A periodicity is seen in the event counting rate, the period being the earth's rotation rate with respect to inertial space (the sidereal day),

rather than the rotation rate relative to the sun (the solar day). The peaking of the events is consistent with and suggests the source (or sources) of the waves being located at the center of our galaxy.

Theoreticians are hard put to construct mechanisms which can produce frequent wave pulses of the magnitude detected by Weber. As a measure of the problem, if the gravity wave energy received at the earth is emitted in all directions from the center of the galaxy, then at least several hundred solar masses are converted into gravity wave energy per year in several hundred individual pulse-like events. A number of other gravity wave detectors are being constructed and operated around the world, some incorporating technological developments such as cryogenic cooling to reduce thermal noise, and optimum signal extraction from noise. Work in this area in the next few years is aimed at obtaining a better idea of the pulse shapes, power spectrum, and speed of propagation of the gravity waves. The last is expected to be the speed of light.

### Summary

I have reviewed the historical and contemporary experiments that guide us in choosing a post-Newtonian, relativistic gravitational theory. The foundation experiments essentially constrain gravitation theory to be a metric theory in which matter couples solely to one gravitational field, the metric field, although other cosmological gravitational fields may exist. The metric field for any metric theory can be specified (for the solar system, for our present purposes) by a series of potential terms with several parameters. A variety of experiments specify (or put limits on) the numerical values of the seven parameters in the post-Newtonian metric field, and other such experiments have been planned. The empirical results, to date, yield values of the parameters that are consistent with the predictions of Einstein's general relativity.

### Appendix

An expression for the gravitational metric field of sufficient generality to be valid for any metric theory of gravity (the viable class of theories) is given here. A physical interpretation of the various potentials in the metric field is also given.

Far away from localized matter (for example, the solar system) the metric field approaches the Minkowski form of special relativity, given by Eq. 5. In the vicinity of matter, however,  $g_{\mu\nu}$  has small corrections;

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}(\mathbf{r}, t) \quad \text{with } |h_{\mu\nu}| \ll 1 \quad (\text{A1})$$

The term  $h_{\mu\nu}(\mathbf{r}, t)$  can be expressed as a series of potential terms, the various terms multiplied by dimensionless coefficients (Greek letters  $\gamma$ ,  $\beta$ ,  $\alpha_i$ ,  $\rho_i$ ) that vary in magnitude from one theory to another. The expansion parameter for the series of potential terms is  $1/c$ , and solar system experiments during the next decade or so will be capable of measuring effects resulting from (i) the  $h_{00}$  potential to order  $1/c^4$ , (ii) the  $h_{0k}$  potentials to order  $1/c^3$ , and (iii) the  $h_{kk'}$  potentials to order  $1/c^2$ . (Here,  $k$  is a subscript ranging over the three spatial coordinates.)

Will and Nordtvedt (31) found the complete "parameterized post-Newtonian" (PPN) metric field expansion by seeking the most general combination of gravitational potentials which was form-invariant (maintained the same functional form in terms of physical variables) under a Lorentz transformation. This invariance of the metric under a Lorentz transformation is equivalent to the physical requirement that all distant observers in various inertial frames agree on the gravitational physics of a local system (for example, the solar system) after making the necessary adjustments among their rulers and clocks according to the rules of special relativity.

Consider a local system of mass elements  $M_i$ , located at positions  $\mathbf{r}_i$ , and having velocities  $\mathbf{v}_i$ ; the inertial frame in which the system is described is moving at velocity  $\mathbf{W}$  with respect to a "preferred inertial frame" (the universal rest frame). In terms of the above physical variables, the PPN metric at field point  $\mathbf{r}$  is given below.

The complete form of the Newtonian-type potential  $g_{00}$  is

$$\begin{aligned} g_{00} = & 1 - 2\psi + \\ & 2\beta\psi^2 + (4\beta - 2 - \rho_2) \sum_{i \neq j} \frac{G^2}{c^4} \frac{M_i}{|\mathbf{r} - \mathbf{r}_i|} \frac{M_j}{|\mathbf{r}_i - \mathbf{r}_j|} + \\ & \rho_1 \sum_i \frac{G}{c^4} \frac{M_i}{|\mathbf{r} - \mathbf{r}_i|^3} (\mathbf{v}_i \cdot \mathbf{r} - \mathbf{r}_i)^2 - (2\gamma + 1 + \rho_1 + \alpha_3) \sum_i \frac{G}{c^4} \frac{M_i}{|\mathbf{r} - \mathbf{r}_i|} v_i^2 + \\ & (\alpha_1 - \alpha_2 - \alpha_3) \psi \frac{W^2}{c^2} + \alpha_2 \sum_i \frac{G}{c^4} \frac{M_i}{|\mathbf{r} - \mathbf{r}_i|^3} (\mathbf{r} - \mathbf{r}_i \cdot \mathbf{W})^2 + \\ & (\alpha_1 - 2\alpha_3) \sum_i \frac{G}{c^4} \frac{M_i}{|\mathbf{r} - \mathbf{r}_i|} \mathbf{W} \cdot \mathbf{v}_i + \dots \quad (\text{A2}) \end{aligned}$$

In metric theories of gravity there are gravitational vector potentials of the form

$$\begin{aligned} (g_{0z}, g_{0y}, g_{0x}) \equiv \mathbf{h} = & \\ (2\gamma + 2 + \frac{\alpha_1}{2}) \sum_i \frac{G}{c^3} \frac{M_i}{|\mathbf{r} - \mathbf{r}_i|} \mathbf{v}_i - & \\ (\frac{1 - \rho_1 + \alpha_2}{2}) \nabla \sum_i \frac{G}{c^3} M_i \mathbf{v}_i \cdot \nabla |\mathbf{r} - \mathbf{r}_i| + & \\ \frac{\alpha_1}{2} \psi \frac{\mathbf{W}}{c} - \alpha_2 \nabla \mathbf{W} \cdot \nabla \sum_i \frac{G}{c^3} M_i |\mathbf{r} - \mathbf{r}_i| + \dots & \quad (\text{A3}) \end{aligned}$$

and there are also "space-space" gravitational potentials of the form

$$g_{kk'} = - (1 + 2\gamma\psi) \delta_{kk'} + \dots \quad (\text{A4})$$

In Eqs. A2 to A4 the Newtonian potential appears in several places

$$\psi = \frac{1}{c^2} \sum_i \frac{GM_i}{|\mathbf{r} - \mathbf{r}_i|}$$

These potentials, which collectively make up  $g_{\mu\nu}$ , govern the response of all forms of matter and energy to gravitational fields through Eq. 7.

Each of the various gravitational potentials in Eqs. A2 to A4 has a physical interpretation. The first line of Eq. A2 is the Newtonian approximation to metrical gravitational theories. The second line gives two types of nonlinear potentials proportional to the squares of the strengths of the source masses. This is unique to gravitation, where gravity begets more gravity, whereas in electromagnetic theory all fields are linear in the source strengths,  $e_i$ . The third line of Eq. A2 gives motional corrections to Newtonian static gravity, analogous to and of the form of the motional corrections to the Coulomb interaction of charges in electromagnetic theory. The last two lines of Eq. A2 gives possible Machian-type potentials related to motion relative to the mass of the universe. Equation A3 gives the gravitational vector potentials which produce "gravitational magnetism" effects, the first two lines giving the regular vector potentials and the third line the Machian vector poten-

tials. Equation A4 gives the spatial potentials unique to the tensor metric field and not present in the electromagnetic field.

It is the goal of experiments to determine the magnitudes of the dimensionless coefficients  $\gamma$ ,  $\beta$ ,  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$ ,  $\rho_1$ ,  $\rho_2$ , which vary from one metric theory to another. Table 2 relates these PPN metric coefficients to the  $\eta_1$  parameters used in the body of this article to scale the various experimental effects.

#### References and Notes

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2. By general relativity I mean Einstein's equations for the gravitational field and its coupling to matter. It is unnecessary to, and I particularly do not in this paper, adopt the common interpretation of general relativity in which there is geometrical "curvature" of space and time. Any gravitational theory is complete if it specifies the motion of matter and energy, and thereby uniquely predicts outcomes for all properly formulated experimental questions. The hypothesis that space and time ontologically possess geometrical structure and that this alleged structure is non-Euclidean (or Euclidean) is superfluous.
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$$M_G c^2 = \int T_{00} dv$$

But because of an accident of nature that any stable, free body has the properties

$$\int T_{xx} dV = \int T_{yy} dV = \int T_{zz} dV = 0$$

it is possible to have another definition of gravitational mass;

$$M_G c^2 = \int (T_{00} - T_{xx} - T_{yy} - T_{zz}) dV$$

The Lorentz-invariant combination  $(T_{00} - T_{xx} - T_{yy} - T_{zz})$ , called simply the scalar  $T$ , vanishes for pulses of light energy. Therefore, the latter choice of a definition of gravitational mass will generally alter the gravitational physics of light rays. Theories with scalar gravitational fields (36) are based, at least in part, on this definition of gravitational mass.

19. That the metric field determines the spatio-temporal intervals spanned by rulers and clocks is considered by many as a postulate of metric theories of gravity. It is not; it is a derived property of the theory, which can be shown explicitly by examining the equations of matter when coupled to the metric field in the universal manner and constructing the rulers and clocks of this matter.
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## Genetic Control of Insect Populations

A wide variety of documented genetic methods should be considered for regulation of pest populations.

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Genetic control of population size has been applied most extensively to insect species, both in theory and in fact. There are two reasons for this—the long tradition of insect genetics, in

which chromosome manipulation has become a refined science, and the long tradition of economic entomology, which developed from the need to control insects that carry diseases or com-

pete with man for his food. In principle, though, the rules for genetic control can be applied to any eukaryotic species from rusts to rabbits that undergoes union of gametes during reproduction. Different problems are presented by prokaryotic organisms such as bacteria and viruses, by mitotic cell populations such as tumors, and by azygous species such as thelytokous mites, where females produce females from unfertilized eggs.

The conditions that lead to genetic collapse and extinction of a population were described by Wallace and Dobzhansky in 1959 (1). They considered the simplest cases—induced recessive lethal mutations and dominant lethal mutations—and formulated the dictum that only an overwhelming degree of dominant lethality could cause extinction. More insidious genetic mechanisms that could cause population