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14 August 1972; revised 25 September 1972

Orbit-Orbit Resonance Capture in the Solar System

Abstract. *A realistic model involving mutual gravitation and tidal dissipation for the first time provides a detailed explanation for satellite orbit-orbit resonance capture. Although applying directly only to Saturn's satellites Titan and Hyperion, the model reveals general principles of resonance capture, evolution, and stability which seem applicable to other orbit-orbit resonances in the solar system.*

Many pairs of natural satellites have orbits that are locked in stable resonances: Their gravitational interaction maintains the low-order commensurability of their orbital periods. The relatively large number of such commensurabilities in the solar system has led many to believe that satellites have evolved into resonances from orbits with originally incommensurate periods (*1*). But the mechanism of capture into resonance has never been described. In this report, we discuss a simple theoretical model which demonstrates that tidal evolution can cause certain satellite pairs to be inevitably captured into orbit-orbit resonance.

The model was inspired by Saturn's satellites Titan and Hyperion. The average ratio of their respective orbital periods, measured relative to Hyperion's major axis, is 3/4. The longitude of their conjunction librates about Hyperion's longitude of apocenter (see Fig. 1). Conventionally, this resonance is described by using a "resonance variable," ϕ, defined as $4\lambda_2 - 3\lambda_1 - \tilde{\omega}_2$, where λ is the mean longitude and $\tilde{\omega}$ the longitude of pericenter, with the subscripts 1 and 2 referring to Titan and Hyperion, respectively. In this case, ϕ librates about 180° with an amplitude of 36° and a period of 640 days. A simple model can be useful because of Titan's larger mass (10^3 times that of Hyperion) and smaller orbital eccentricity (0.03 compared to Hyperion's 0.10). The slight asymmetry of Titan's orbit, represented by its eccentricity, has no effect on the qualitative aspects of the resonance capture mechanism.

Our model consists of a large planet of mass M and two satellites in coplanar orbits. The inner satellite, of mass m_1, has a circular orbit with a period constantly increasing due to tidal friction (*1*). The outer satellite, of mass m_2, is too small to perturb m_1 or to raise a significant tide on M ($M \gg m_1 \gg m_2$). The eccentricity of the outer orbit, e_2, is taken to be small enough to permit a valid first-order analysis.

An analytical approach is more fruitful than a numerical one because it leads to a simple physical interpretation. The analysis is outlined below with the emphasis placed on the qualitative mechanism of capture (*2*).

The behavior of the resonance variable, ϕ, may be deduced from Lagrange's equations for the variation of orbital elements. The disturbing function that represents the gravitational effects of m_1 on m_2 in these equations has a well-known trigonometric series expansion (*3*). Using this expansion and ignoring terms with short-period arguments, we find

$$\frac{d\phi}{dt} = 4n - 3 - \frac{1}{2e_2}\left(\frac{m_1}{M}\right)F(n)\cos\phi \quad (1)$$

where t is the time (in units of the instantaneous period of m_1), n (which is less than 1) is the ratio of the satellites' orbital periods measured relative to an inertial reference frame, and the function $F(n)$ is a dimensionless coefficient of order unity found from the expanded disturbing function. Similarly, the other variables, n and e_2, satisfy:

$$\frac{de_2}{dt} = -\frac{1}{2}\left(\frac{m_1}{M}\right)F(n)\sin\phi \quad (2)$$

$$\frac{dn}{dt} = 6e_2\left(\frac{m_1}{M}\right)nF(n)\sin\phi - n\beta \quad (3)$$

where $\beta < 0$ represents the rate of increase in the period of m_1 due to tidal friction.

If n were constant, the solutions of Eqs. 1 and 2 would yield circular trajectories in the e_2, ϕ polar coordinate plane (see Fig. 2). Circle B in this figure represents libration of ϕ about 180°; circle A depicts circulation through 360°. The centers of both circles have ordinate values $e_2\sin\phi$ equal to 0, whereas the abscissas $e_2\cos\phi$ are approximately proportional to $(n - 3/4)^{-1}$.

Now consider circulation with n less than 3/4 but gradually increasing due to tidal evolution, and with e_2 much less than 1. The center of the corresponding circular trajectory in Fig. 2 will then move leftward, toward a state of libration. As this evolution continues beyond circle B, e_2 increases, causing the first term on the right side of Eq. 3 to become significant. During the course of a single libration, n therefore varies considerably so that the instantaneous center of curvature of the trajectory moves back and forth on the axis, yielding "bean-shaped" trajectories such as C and D in Fig. 2. For trajectory D, the value of n actually becomes greater than 3/4, causing the center of curvature to pass to a positive abscissa for part of the libration cycle.

The gradual evolution of e_2, ϕ trajectories from A to D describes a solution of Eqs. 1 to 3. This evolution begins with a typical nonresonant system in which ϕ circulates, n is not yet near a ratio of small integers, and e_2 is small. The system then evolves to a state similar to the present Titan-Hyperion configuration in which ϕ librates, n oscillates about 3/4, and e_2 is relatively large. Moreover, this mechanism permits capture of ϕ into libration only about 180°. Libration about 0° is possible for small values of e_2 and is represented by the circular trajectory E in Fig. 2. But tidal evolution destroys such a libration, since as n increases the center of the circle drifts leftward until a state of circulation is reached.

Although here we have presented only a heuristic discussion, a rigorous approach has confirmed these results. Specifically, an analytical solution was found for the trajectories for β equal to 0, and the tidal evolution was then incorporated by a technique of variation of parameters (*2*).

To further clarify the physical basis of the capture mechanism and the maintenance of the resulting resonance, we may discuss this model from a different point of view. We first consider qualitatively the stability of the resonance for β equal to 0 and for e_2 sufficiently high that the longitude of Hyperion's major axis is essentially constant (its variation is inversely proportional to e_2). If n were exactly 3/4 and if conjunction of the two satellites occurred at Hyperion's apocenter, then the longitude of conjunction would be invariant. Suppose, however, that conjunction takes place after apocenter passage but before pericenter passage. Since the orbits are then converging at conjunction, the closest approach of the satellites takes place afterward. Titan, having a greater angular velocity, will be ahead of Hyperion at closest approach and will therefore add energy to Hyperion's orbit, thus increasing the latter's period. The next conjunction will therefore occur closer to Hyperion's apocenter. Similarly, if conjunction occurs before apocenter, as in Fig. 1, it is also driven toward apocenter. Thus, conjunction at Hyperion's apocenter passage is a stable configuration and is, in fact, the only stable situation if e_2 is sufficiently large, as in trajectory *D* of Fig. 2 (*4*).

For e_2 very small, the mechanism is more intricate because Hyperion's major axis is more readily rotated by Titan. Here Titan's gravitational effect on Hyperion is approximated accurately by a radial impulse applied at conjunction. (There are tangential forces as well, but with both orbits nearly circular these forces reverse at conjunction so that their net effect is less important.) The effect of such an impulse on the longitude of pericenter and on e_2 varies as $\cos\theta$ and $-\sin\theta$, respectively, where θ is the longitude of conjunction minus that of pericenter. Applied at apocenter, such an impulse causes the major axis to regress; at pericenter, to advance. The maximum decrease in e_2 is produced when conjunction occurs 90° before apocenter; the maximum increase for conjunction 90° after apocenter.

If conjunction occurs at apocenter with a libration amplitude of zero, then, since the radial impulse causes apocenter to regress, n must be less than 3/4 to insure that the conjunction longitude regresses to keep pace with apocenter. Such a situation is represented in Fig. 2 by a circle of radius zero centered on the left half of the abscissa.

If the conjunction longitude does not regress quite as fast as apocenter, and if conjunction initially occurs soon after Hyperion's apocenter passage, then the radial impulse will cause e_2 to increase and the regression of the major axis to decrease correspondingly. Conjunction can then overtake apocenter. Similarly, if conjunction initially occurs before apocenter, its longitude will move toward apocenter. If conjunction occurs initially just before or after Hyperion's pericenter passage, the conjunction longitude will tend toward pericenter. Thus, for e_2 much less than 1 and β equal to 0, the longitude of conjunction is stable at either apocenter or pericenter. Libration in these cases involves considerable variation in e_2, but little in n, as described by trajectories *B* and *E* in Fig. 2.

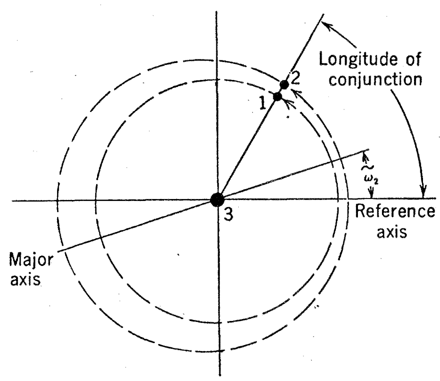

Fig. 1. Two satellites, 1 and 2, are shown in conjunction relative to a central body, 3. The dashed lines depict the satellites' orbits, and $\tilde{\omega}_2$ represents the longitude of pericenter of the outer satellite.

The gradual increase of n due to tidal friction (β less than 0) tends to advance the longitude of conjunction, or at least to slow its regression. When libration takes place near apocenter, the average longitude of conjunction is displaced to an orientation where the radial impulse increases e_2. Thus, the tide causes a gradual increase in e_2, reflected by the evolution from *A* to *D* in Fig. 2.

But how does tidal friction lead to the critical capture into libration of a conjunction longitude originally circulating relative to Hyperion's apocenter? In circulation with n less than 3/4 and e_2 much less than 1, the radial impulse causes alternate increases and decreases in e_2. These variations are apparent in trajectory *A* of Fig. 2. As n increases toward 3/4, circulation slows and e_2 thereby achieves higher maxima and lower minima, just as characterized by trajectory *A* (Fig. 2) drifting leftward. Eventually, the conjunction longitude circulates so slowly that the minimum value of e_2 can nearly reach zero, allowing apocenter to regress so rapidly that it is able to overtake the conjunction longitude. After apocenter regresses past the conjunction longitude its rate of regression decreases, allowing conjunction to overtake apocenter and the stability mechanism for low e_2 to maintain the resonance. This capture process is illustrated in Fig. 2, where the critical passage from circulation (*A*) to libration (*B*) requires the minimum e_2 to reach zero instantaneously (*5*).

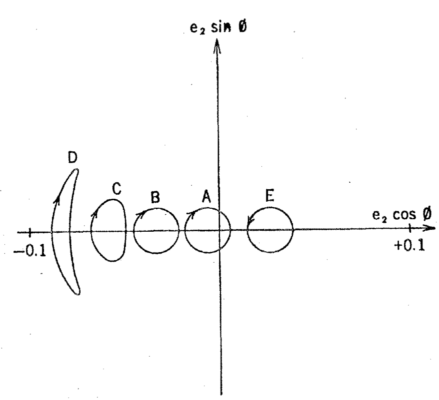

Fig. 2. Various trajectories in e_2, ϕ polar coordinates for $\beta = 0$ (see text).

The simplifying characteristics of the Titan-Hyperion case do not apply in detail to other resonances in the solar system, but our model nonetheless helps to understand them. For example, in the case of Saturn's satellites Enceladus and Dione, the outer one is only about 15 times as massive as the inner and the eccentricities are comparable. But if their orbital evolution were governed by a mechanism qualitatively similar to that of our model, these satellites could only have evolved into their present type of resonance with the conjunction longitude librating about the inner satellite's pericenter, as observed. The resonance of Saturn's satellites Mimas and Tethys, which involves orbital inclination rather than eccentricity, could have evolved in an analogous way, with the inclinations increasing secularly due to tidal friction (*6*) just as e_2 does in our model. The commensurabilities among Jupiter's Galilean satellites could also have evolved in a sequence of resonance captures of the type described by our model.

Thus, our model of orbit-orbit resonance capture lends credence to the hypothesis that satellite commensurabilities evolved since the formation of the solar system, that is, since the planetary environment became substantially the same as it is today. However, tidal dissipation, which is essential to this model of satellite evolution, is not applicable to planetary orbit-orbit resonances, such as the orbital resonance of Neptune and Pluto or the many known asteroidal cases. Unless these other resonances were formed by chance, we must suspect that the key to their evolution is a dissipative mechanism not present in the solar system today. It might have involved a viscous medium in the early solar nebula or the influence of a passing star. A study of evolutionary models with such alternate mechanisms might provide useful clues about conditions in the young solar system.

R. J. GREENBERG
C. C. COUNSELMAN, III
I. I. SHAPIRO
Department of Earth and Planetary Sciences, Massachusetts Institute of Technology, Cambridge 02139

References and Notes

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2. A complete discussion is given by R. J. Greenberg, thesis, Massachusetts Institute of Technology (September 1972).
3. D. Brouwer and G. Clemence, *Methods of Celestial Mechanics* (Academic Press, New York, 1951), p. 492.
4. This qualitative aspect of the stability for relatively high values of e_2 was described by Goldreich (*1*) and by C. J. Cohen and E. C. Hubbard [*Astron. J.* **70**, 10 (1965)].
5. The time scale for the evolution of the Titan-Hyperion resonance presents certain problems; these are discussed in (*2*).
6. R. R. Allan, in *Symposia Matematica* (Istituto Nazionale di Alta Matematica, Città Universitaria, Rome, 1970), vol. 3, p. 75. Here the conjunction longitude librates about the average longitude of the satellites' ascending nodes on Saturn's equatorial plane, so the resonance involves the inclinations of both satellites.

30 June 1972

Polystyrene Spherules in Coastal Waters

Abstract. *Polystyrene spherules averaging 0.5 millimeter in diameter (range 0.1 to 2 millimeters) are abundant in the coastal waters of southern New England. Two types are present, a crystalline (clear) form and a white, opaque form with pigmentation resulting from a diene rubber. The spherules have bacteria on their surfaces and contain polychlorinated biphenyls, apparently absorbed from ambient seawater, in a concentration of 5 parts per million. White, opaque spherules are selectively consumed by 8 species of fish out of 14 species examined, and a chaetognath. Ingestion of the plastic may lead to intestinal blockage in smaller fish.*

Polystyrene spherules are widespread in the coastal waters of southern New England. We first observed spherical plastic particles in plankton tows in January 1971 while sampling to determine the effects of a nuclear power station on the ecology of Niantic Bay (northeastern Long Island Sound). The particles, although usually present in zooplankton samples throughout the year, were not investigated in detail until February 1972. The spherules are markedly different in size, shape, distribution, and chemical composition from the plastics on the Sargasso Sea surface (*1*).

Infrared spectrophotometry of the particles indicated that they were polystyrene plastic. Two types are present in seawater, in approximately equal proportions. One is a clear or crystalline polystyrene, and the other is a white, opaque form with pigmentation due to the presence of a diene rubber compound in the plastic, as indicated by infrared spectrophotometry and confirmed by a representative of the plastics industry (*2*). Both forms are virtually perfect spheres and average about 0.5 mm in diameter, ranging from 0.1 to 2 mm. They contain various sizes and numbers of gaseous voids. Thus, they are found at the sea surface, in the water column, and presumably in the sediments since polystyrene is of a greater density than seawater.

The spherules are present in coastal waters from western Long Island Sound to Vineyard Sound (Table 1), and may be more widespread since their total range was not delimited. All samples were collected with oblique plankton tows by using the National Academy of Sciences (NAS) reference net (0.5 m in diameter at the mouth; mesh size 333 μm) equipped with a flowmeter (*3*). The highest concentrations observed were in the Niantic Bay area with an average of about 1 spherule per cubic meter for 72 samples taken on six dates between February and May 1972. Concentrations up to 14 m^{-3} were observed in this area. At other stations sampled in February to March 1972 the average concentrations were as follows: Long Island Sound (stations 22 to 27), 0.07 m^{-3}; east of Block Island (stations 13 to 18), 0.03 m^{-3}; Great Salt Pond on Block Island and west to Long Island Sound (stations 19 to 21), 0.02 m^{-3}.

Bacteria and polychlorinated biphenyls (PCB's) are present on surfaces of the plastic particles. Freshly collected spherules from Niantic Bay were transferred through four washings of sterile seawater and plated onto A-C seawater medium (*4*), where rod-shaped gram-negative bacteria were observed after incubation. An extraction of the surface of the spherules from Niantic Bay with hexane showed that they contained PCB's (Aroclor 1254) in a concentration of 5 parts per million. Since PCB's are not added in the manufacture of polystyrene (*2*), it is probable that the source was ambient seawater.

Table 1. Sample location, date, volume filtered, and concentration of plastic spherules in coastal water. Stations 1 to 12 were in an area of about 10 km^2; the averages and ranges of the spherule concentrations at these 12 stations are presented.

Station	Location	Date (1972)	Volume filtered (m^3)	Spherules per cubic meter	
				Avg.	Range
		Niantic Bay			
1–12	41°18′N, 72°10′W	1 February	475	0.75	0.39–1.94
1–12	41°18′N, 72°10′W	17 February	140	2.58	0.62–14.1
1–12	41°18′N, 72°10′W	16 March	513	0.79	0.00–2.52
1–12	41°18′N, 72°10′W	7 April	603	0.13	0.00–0.51
1–12	41°18′N, 72°10′W	25 May	387	0.61	0.03–2.44
		Buzzards Bay			
13	41°34′N, 70°43′W	9 March	59	0.03	
14	41°34′N, 70°43′W	9 March	50	0.02	
		Vineyard Sound			
15	41°30′N, 70°39′W	10 March	48	0.02	
16	41°30′N, 70°39′W	10 March	31	0.00	
		Rhode Island Sound			
17	41°20′N, 71°03′W	24 March	108	0.10	
18	41°13.5′N, 71°18′W	25 March	76	0.00	
		Great Salt Pond			
19	41°09′N, 71°33′W	25 March	94	0.04	
		Block Island Sound			
20	41°12′N, 71°44′W	25 March	191	0.01	
21	41°12′N, 72°00′W	25 March	104	0.01	
		Long Island Sound			
22	41°10′N, 72°20′W	25 March	280	0.10	
23	41°09′N, 72°36′W	25 March	122	0.05	
24	41°08′N, 72°52′W	25 March	48	0.10	
25	41°16′N, 72°01′W	23 March	109	0.05	
26	41°17′N, 72°03′W	23 March	125	0.07	
27	41°17′N, 71°59′W	23 March	151	0.04	