Cometary Hydrogen and Hydroxyl Comas

Delsemme (*1*) has used ultraviolet data for the total brightness of Comet Tago-Sato-Kosaka in H and OH to derive the variation of the H_2O release rate with the comet's heliocentric distance r. From a brightness varying with the -6 power, he deduced a release rate $Q(H_2O)$ proportional to the -2 power of r. Crudely summarized, his argument states that the dissociation rate varies with the -2 power and the fluorescent excitation rate with a second -2 power, so that the evaporation rate also has a dependence to the -2 power.

The fallacy in this argument is revealed by a consideration of the scale lengths. Assuming that the evaporating H_2O photodissociates at the radius R_d and the resultant H ionizes at the distance R_+ from the comet's center, one can consider a number density

$$N(\mathrm{H}) = R^{-2}Q(\mathrm{H_2O})/V \quad \text{for } R_d < R < R_+$$

and $N(\mathrm{H}) = 0$ outside this range of radial distance R. This simple model may be used since $R_d \simeq r^2 \times 10^5$ km $\ll R_+ \simeq r^2 \times 10^8$ km (*2*), r being measured in astronomical units (A.U.). The expansion velocity V is assumed constant. The line-of-sight density at perpendicular distance p is then

$$\int N ds = 2\frac{Q}{V}p^{-1}\left(\cos^{-1}\frac{p}{R_+} - \cos^{-1}\frac{p}{R_d}\right) \quad (1)$$

if it is understood that the function $\cos^{-1}$ is zero for an argument $p/R > 1$. The total number in the head out to a distance $R_d < R_0 < R_+$ is

$$\mathcal{N}(\mathrm{H}) = \int_0^{R_0} 2\pi p dp \int N ds = 4\pi\frac{Q}{V}\left\{R_0\cos^{-1}\frac{R_0}{R_+} + R_+ - (R_+^2 - R_0^2)^{\frac{1}{2}} - R_d\right\}$$

This gives for $R_0^2 \ll R_+^2$

$$\mathcal{N}(\mathrm{H}) \simeq 4\pi\frac{Q}{V}\left(\frac{\pi}{2}R_0 - R_d\right) \quad (2)$$

but if the head is viewed to past R_+, the $\pi R_0/2$ of Eq. 2 is to be replaced by R_+. The measured value, $R_d = 2 \times 10^5$ km (*2*), which expresses the central deficit in $\mathcal{N}$, may well comprise optically thick (*3*) and extended source effects, as well as reflecting the finite dissociation time.

The consequences of Eq. 2 for the total brightness of the head are

1) There is little dependence on the dissociation rate for $R_d < R_0$. In any case, Delsemme's (*1*) inverse square dependence is invalid.

2) Since the mean ionization rate $1/\tau_i$ is proportional to the solar proton and radiation fluxes (both proportional to r^{-2}), the total brightness of the head, when the fluorescence factor is allowed for, is

$$B \propto r^{-2}QR_+/V = r^{-2}Q\tau_i \propto Q \quad (3)$$

3) In practical cases, the head is taken to extend out to $R_0 < R_+$, where the intensity falls to a value comparable to the background: $R_0 = p$ in Eq. 1, which implies for $\frac{1}{3}R_+ > R_0 > R_d$ that

$$(Q/V)R_0^{-1}r^{-2}\Delta^{-2} \simeq \text{constant} \quad (4)$$

Here Δ is the geocentric distance of the comet, which to a rough approximation might be taken as a constant during the observation period (*4*). Thus, by using Eq. 2, one obtains the brightness

$$B \propto r^{-2}(Q/V)^2r^{-2}\Delta^{-2} \propto r^{-4}Q^2/V^2 \quad (5)$$

Equation 3 is inappropriate for the reported sizes of the H head. Equation 5, with the observed sixth-power dependence implies that Q/V varies as $1/r$ over some 2 weeks while the comet was receding from 0·78 to 1·02 A.U. For OH, the analysis would be similar but the published data are more scanty. The scale radii are smaller—R_+ is probably around 10^7 km and R_d is 7×10^4 km (*5*). The fact that the sixth-power dependence is shown by OH too (*1*) indicates that neither of these scales enters importantly.

It should be pointed out that for the H head, distortion of the isophotes due to radiation pressure appears (*2*) to have the scale R_0 at the smallest r. Effects on the total brightness are perhaps ignorable because the limiting intensity was chosen appreciably larger than the background. I would comment further that the parameter "total brightness" B appears to be a useful average over the dynamic distortions of the profiles. But the limiting isophote should adequately exceed the background intensity (whose rather variable geocoronal part should be small). Useful additional information for rival theoreticians would be values of B out to several limiting isophotes.

Since the sixth-power variation leading to $Q/V \propto 1/r$ does not support the evaporating icy model of a comet (*6*), one should ask whether it provides definite evidence against. First, I would answer that it illustrates the dangers in using limited observations of one comet for a short time and a short part of its orbit. Comets are well known to have nonmonotonic behavior. Second, it is possible that a cloud of icy particles would decay appropriately slowly, over more than 10 days. Such a time lag is statistically indicated between cometary aphelia and the maximum brightness (*7*). Third, measurements of the H head from alternative OGO 5 Lyman-α profiles do not confirm a steady sixth-power variation. They give (*8*) a rather variable power index 8 ± 3 and imply that Q/V decreases with r at a somewhat variable rate for this comet (Bennett, 1969 i) (*9*).

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References and Notes

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3. L. Biermann, *JILA Rep. No. 93* (1968); A. D. Code, paper presented to Commission 15 at the International Astronomical Union meeting, Brighton, England (1970).
4. The observation period was 16 to 30 January 1970; Δ was beginning to increase appreciably toward the end of this period.
5. Private communication, A. D. Code to C. Arpigny. This value would be the real scale radius for H/OH production. If so, it can be checked [with Eq. 4 and the profiles of (2)] that by 1.02 A.U. R_d is still smaller by a factor of 5 than the radius of the 1-k*R* isophote. The optical depth in H is invariant, and neglect of the variation of R_d is quite tolerable.
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8. Private communication from V. Vanýsek, to whom thanks are due.
9. C. R. O'Dell informs me that he has independently carried out an analysis similar to the above (unpublished).
10. I acknowledge a fellowship from the Royal Society under its European program.

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Wallis (*1*) does not object to any of the physical mechanisms I have proposed (*2*), but only to the mathematical model of the cometary coma which I have implicitly used. Wallis's model and mine differ because they are not based on the same simplifying assumptions. As the two models disagree, I have worked out a more rigorous approach that will be submitted for publication soon. It proves that Wallis's model as well as mine were too simplified.

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