Book Reviews

Human Population Dynamics

The Growth and Structure of Human Populations. A Mathematical Investigation. ANSLEY J. COALE. Princeton University Press, Princeton, N.J., 1972. xviii, 228 pp., illus. $9.50.

Coale has written a fascinating and highly original book. Population mathematics has all too often been obsessed with the mathematics to the detriment of demographic content; however, Coale's virtually unique blend of technical ability and empirical "feel" for population dynamics allows him to avoid this common pitfall. Possibly the most adequate review of this book has already been written by Coale himself, whose final chapter constitutes a remarkably balanced assessment of the relative merits of the various topics covered. He is, however, too modest to attempt to place his work in a wider context.

The book seems (at least to this reviewer) to divide into four major parts, each of which is worthy of separate consideration. The cornerstone of the whole book is the late Alvaro Lopez's proofs of the theorem of "weak ergodicity." This well-known theorem states that the age structure of a population is dependent entirely on "recent" fertility and mortality and not at all on the age structure in the distant past. (In practice "recent" means in the previous 200 years or so.) As a result of this theorem it is possible to investigate the effects of various regimes of fertility and mortality on age structures without specifying everlasting regimes—all that is required is a knowledge of recent history.

The first part of the book reformulates the theory of stable populations (that is, populations in which the regime of fertility and mortality has been constant), and naturally draws heavily on the work of Lotka. This presentation of stable population theory is the clearest I have seen. The discussion is mathematically not entirely rigorous (which allows the reader not to lose sight of the population aspects), but is full of heuristic justification. Great emphasis is laid upon understanding how fertility and mortality affect the stable population characteristics. For example, the approximation

$$r \doteq \frac{\log [GRR \cdot p\ (\bar{m})]}{\bar{m} - \sigma^2\ (\log GRR/2\bar{m})}$$

allows examination of the effects on the rate of natural increase (r) of the level of fertility or gross reproduction rate (GRR), the mean age of fertility ($\bar{m}$), the variance of the fertility distribution (σ^2), and the level of mortality at the mean age of fertility [$p(\bar{m})$]. These effects can then be examined in terms of the partial derivatives of r. A thorough examination is also conducted of the differences in age structure of stable populations with differences in mortality and fertility, such as are feasible for human populations.

An approximation is obtained for the crude birthrate [$b = GRR \cdot c(\bar{m})$, where $c(\bar{m})$ is the proportion at the mean age of fertility in the stable population]; this allows a discussion of factors which affect the birthrate in stable populations, and particularly a study of the variation of $c(\bar{m})$ with different (empirically feasible) values of expectation of life at birth, levels of fertility, and mean ages of the fertility distribution. Another new finding is that the extreme proportion with a given characteristic occurs in that stable population with the same average age as the average age of people with (and thus also without) the characteristic. Thus, for example, the minimum death rate for a given mortality schedule within the universe of stable populations based on that schedule occurs when the mean age at death is the same as the mean age of the population.

The approach taken here is a novel and very valuable one, shifting the emphasis in the study of stable populations firmly in the direction of population dynamics rather than mathematics

The second part of the book consists of the first detailed study of the process of convergence of a population to stable form or, more precisely, convergence of the birth sequence to exponential form. This study has already appeared in virtually identical form elsewhere (at the Population Association of America's Annual Meeting in 1967 and subsequently in the *Journal of the American Statistical Association*, Vol. 63, No. 322) although, strangely, this fact is nowhere mentioned in the text. The analysis is in terms of finding the roots (r_i) of the characteristic equation

$$\int_0^\beta e^{-ra} p(a) m(a) da = 1$$

and then expressing the birth sequence as

$$B(t) = \sum_{i=0}^{\infty} Q_i e^{r_i t},$$

the well-known formulation due to Lotka. As is well known, the one real root (which has the largest modulus) eventually comes to dominate the birth series. Coale finds that only the complex root with largest modulus (and lowest frequency) has any further consequence for the birth sequence at durations after the institution of the regime of fixed fertility and mortality in excess of a few years more than the mean age of childbearing. During the course of his analysis he calculates, for one example of the net maternity function, the first 266 pairs of complex roots of the characteristic equation. For these calculations he uses a net maternity function by single years of age (that is, a step function). The resulting higher-order pairs of roots clearly reflect the distortions introduced by this step function, although the first dozen or so pairs of roots would appear little affected. The reviewer was disappointed that no attempt was made to look at the high-order roots with a more continuous net maternity function. (For example, short-term osculatory polynomials could have been fitted to, say, five single years of age and a curve for the central year of age thus approximated. This approach would have left the integrals needed to find the roots in an analytic form and thus avoided increasing computer time astronomically.) However, the high-order roots are clearly of little or no demographic importance, so the point is somewhat academic.

The treatment of the process of convergence is, nevertheless, impressively full, including a heuristic approach to

visualizing the nature of convergence with different forms of initial deviation from the stable population age structure.

The third part of the book is perhaps potentially the most valuable in practical terms. This constitutes a study first of the effects of fertility that changes at a constant rate with mortality remaining constant and second of changing mortality with fertility constant.

The analysis of fertility changes at a constant rate is an extension of some work in Coale and Zelnik's *New Estimates of Fertility and Population in the United States*. Approximate analytic expressions for the birth function and age structure at any time are developed for the cases where (i) fertility has been constant for a long time and then changes at a constant rate, (ii) fertility has been changing at a constant rate for a long time and then becomes constant, and (iii) fertility has been changing at a constant rate and then alters to a different constant rate of change. Such formulations can be used to evaluate the growth effects of a population with fertility continuously declining to replacement, and then with constant fertility at replacement level.

The treatment of declining mortality is less satisfactory. To obtain analytical approximations for the age structure, changes in mortality during early childhood have to be specified in such a way as to produce the same effects on the age structure as constantly declining fertility. Such a pattern of change would appear unlikely, at least in terms of the Coale and Demeny "West" model life tables. Even when such a pattern does occur the agreement between the approximate functions for age structure and the age structure derived by projection does not appear to be very good.

It is unfortunate that the estimation of the effects of declining mortality on an initially stable population should be the least adequately handled element of this book, because such analytic formulations are potentially the most useful in analysis of population data from developing countries. That Coale is aware of this need is clear—witness his work with Demeny in the United Nations Manual IV, "Methods of Estimating Basic Demographic Measures from Incomplete Data." It would appear, however, that the methods used to deal with changing mortality in that manual were those outlined in this book. Further work is undoubtedly required in this field, although Coale's is the best available so far.

The fourth part of the book is an analysis of cyclical fluctuations in fertility (around a constant value). This too has appeared elsewhere in almost identical form (*Demography*, Vol. 7, No. 1), and this fact is again not mentioned. First the effects of a simple, sinusoidal fluctuation of fertility are investigated. If the amplitude of the cyclical variation is small the birth sequence is an exponential multiplied by a cyclical function of the same frequency. However, in general, the amplitude will be different and there will be a shift in phase. A thorough study is made of the variations of amplitude and phase shift for different frequencies of variation in fertility. Not surprisingly, maximum amplification in birth cycles is achieved when the fertility fluctuations have a period of about the mean length of a generation—that is, when maximum fertility occurs when survivors from the previous maximum of births reach the childbearing ages and hence the fluctuations are amplified. Similarly, maximum attenuation of the birth cycle occurs when the period of fertility fluctuations is about twice the mean length of a generation—when peak fertility occurs while the survivors of the previous minimum of fertility are of childbearing age.

The analysis of a simple sinusoidal pattern of variation becomes too complicated to be of great use when the amplitude of the fluctuations becomes too large—harmonics are introduced. Finally a Fourier analysis of any time pattern of fertility is obtained, but this provides no simpler a formulation in the general case than a succession of projection matrices. Thus the attempt at Fourier analysis does not contribute to our understanding of population dynamics and therefore is of largely academic interest, as I think Coale would agree.

In conclusion, this book is an impressive contribution to the understanding of the dynamics of human populations. Most of it is entirely original to Coale. On the whole the presentation is near perfect. Although occasionally results are presented before their justification—which is irritating to the reader—the explanation is never more than a page or two after the result. The style is very clear, for which most social scientists will be grateful, especially when reading about amplifiers and electrical circuits. Although Coale regards the level of mathematics as unsatisfactory, it did not seem so to this reader; it is adequate for the demographic tasks at hand, and this is a book *about* demography *using* mathematics—for which we may be grateful!

Finally a word of appreciation for Erna Harm, who did the computer programming for most of the computations in the book—it is no mean task finding roots of integral equations or evaluating high-order Fourier series. The ultimate debt is to Coale for his remarkable book, which is perhaps the best single contribution to the field since the work of Lotka (and this despite Lopez's proof of weak ergodicity), although with its more heuristic approach—perhaps of necessity given Coale's self-claimed mathematical limitations—of a slightly different nature from Lotka's work.

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Parasites

Life Cycles of Coccidia of Domestic Animals. YEVGENIY M. KHEYSIN. Translated from the Russian edition (1967) by Frederick K. Plous, Jr. Kenneth S. Todd, Jr., Ed. University Park Press, Baltimore, 1972. xii, 264 pp., illus. $18.50.

The Coccidia are a large and important group of parasites, with complex life cycles, which cause serious disease in domestic and wild animals and, occasionally, in man. In 1970, toxoplasmosis, a disease of considerable importance in man, was found to have a developmental cycle of coccidian nature in the intestine of cats. Thus Coccidia are of interest to those in medicine as well as to those in veterinary medicine and to biologists.

The author of this monograph, who died in 1968, was one of the foremost investigators of Coccidia, particularly the life cycles and cytology of species occurring in rabbits. The four families of the suborder Eimeriidea are considered in detail in the book, with emphasis on the Eimeriidae, which include most of the species that cause disease. The morphology and physiology of the various stages in the life cycle, the development of these, the course of infection, the survival of the oocysts outside the bodies of the host, sporulation of the oocysts under various conditions, and factors affecting initiation of infection by Coccidia receive major attention. Much of the information presented concerns Coccidia of domestic animals, but data