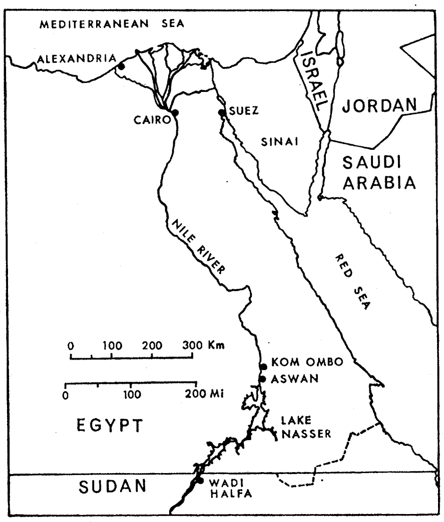

Fig. 1. Map of northeastern Africa and adjacent Asia Minor showing the location of Kom Ombo and other localities mentioned.

those of Dorcas gazelle, hippopotamus, and Nubian ass next most numerous in descending order of frequency. All other taxa were sparsely or uniquely represented.

Vertebrate remains have previously been reported from the Kom Ombo Plain by Vignard (*3*), Gaillard (*4*), Sandford (*5*), Reed (*6*), Reed and Turnbull (*7*), Oakley (*8*), and Smith (*9*). In summary these authors recorded catfish, man, spotted hyaena (*4*), Nubian ass, hippopotamus, long-horned and short-horned cattle, long-horned and short-horned African buffalo, bubal hartebeest, and Dorcas gazelle. The record of horse [*Equus caballus* (*4*)] may be discredited (*7*) and that of short-horned cattle (*Bos brachyceros*) or Cape buffalo (*Syncerus caffer*) is dubious.

The mammalian elements derive from both adult and immature individuals when there are sufficiently large samples for both age categories to be represented. Since gazelle in Arabia drop their young in either January or late July to August and oryx bear their calves between May and December (*10*), this suggests that hartebeest may have behaved similarly at the same latitude. Thus, it appears that the Kom Ombo Plain was occupied at least during those months of the year or shortly thereafter. The abundance of water birds present in Egypt only during the northern winter indicates that, at least at Gebel Silsila III during Sebekian times and probably at other sites also, the plain was occupied by man during the winter months of January to March. It is tentatively concluded, therefore, that the Kom Ombo Plain was occupied throughout most of the year by at least part of the population during Late Pleistocene times. The presence of catfish, some of large size, and of hippopotamus and wild cattle suggests that these may have been hunted when the river level was low and the animals drawn to the available forage and restricted water holes.

The general faunal aspect is that of the savanna or orchard bushveld biome with grazing plains or grassland species in contact with species more intimately dependent on riverine woodland and permanent water. Dorcas gazelle and hartebeest are possible indicators of dry savanna conditions, at least at some period of the year, while wild cattle and hippopotamus indicate the presence of permanent bodies of water.

The presence of the Egyptian bandicoot or pest rat indicates a wetter climatic regime than obtains in the region today (*11*) and supports its identification by Robinson (*12*) from Dabarosa West near Wadi Halfa. The avifauna from Gebel Silsila III also suggests a wetter regime, at least during the winter, with large, shallow meanders and side channels in which catfish would live. The restriction of abundant remains of avifauna to the upper levels of Gebel Silsila III is puzzling, and has been attributed to differential preservation at other sites along the Nile River, presumably where conditions did not allow the smaller elements of birds or rodents to be preserved. Alternatively, there may have been some difference in the mode of life of the Sebekian inhabitants that allowed them to use this source of protein more extensively than did those with other industries (*13*).

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References and Notes

1. Field expenses were provided by the government of Canada through the National Museum of Canada, Ottawa, as a contribution to the UNESCO campaign to save the antiquities of Nubia, with other assistance from the University of Toronto.
2. K. B. Butzer and C. L. Hansen, *Desert and River in Nubia* (Univ. of Wisconsin Press, Madison, 1968), pp. 1–562.
3. E. Vignard, *Bull. Inst. Fr. Archaeol. Orient. Cairo* **22**, 1 (1923).
5. K. S. Sandford, *Pleistocene Surv. Egypt & West Asia* **3**, 1 (1934).
5. K. S. Sandford, *Pleist. Surv. Egypt & West Asia* **3**, 1 (1934).
6. C. A. Reed, *Man* **65**, 101 (1965).
7. ——— and P. F. Turnbull, *in Palaeoecology of Africa and the Surrounding Islands and Antarctica*, E. M. van Zinderen Bakker, Ed. (Balkema, Cape Town, 1969), vol. 4, p. 55.
8. K. P. Oakley, *Man* **65**, 104 (1965).
9. P. E. L. Smith, *Quaternaria* **9**, 141 (1967); *La Préhistorie; Problèmes et Tendances* (Editions CNRS, Paris, 1968), p. 391.
10. D. L. Harrison, *The Mammals of Arabia* (Benn, London, 1968), vol. 2, pp. 193–381.
11. H. Hoogstraal, *J. Egypt. Public Health Ass.* **38**, 1 (1963).
12. P. Robinson, *Science* **154**, 264 (1966).
13. A detailed description of the specimens, discussion of the taxa, ecological consideration of each taxon and as a fauna, and the relationships with other vertebrate faunas of the same age known from neighboring regions will be published as Contribution 82 of the Royal Ontario Museum.

10 March 1972; revised 24 April 1972

Birch's Law: Why Is It So Good?

Abstract. *Birch's law arises in the physics of solids as a linear approximation, in a certain range of density, of a power law. For a change of chemical composition within the same crystal structure, the velocity-density relation is constant with a slope of nearly —0.5 in the first-order approximation.*

Birch's law is a velocity-density relation among oxides and silicates, important in the study of the elasticity and constitution of the interiors of the earth and other planets (*1*). The law is based on experimental observations. It states that the velocity V of elastic waves in oxides and silicates is mainly a function of density ρ and mean atomic weight $\bar{m}$:

$$V = a(\bar{m}) + b\rho \qquad (1)$$

where a and b are constants, frequently referred to as Birch's constants, and the mean atomic weight is defined as the product of the molecular weights divided by the number of atoms in 1 mole of the substance. Although Birch's law is used in most geophysical theories and models of the elasticity of the earth, the validity of Eq. 1 over the entire range of density distributions in the earth's interior has not been analyzed. Furthermore, the physics behind Birch's law is not yet generally understood (*2*). In this report I examine how Birch's law arises in the physics of solids.

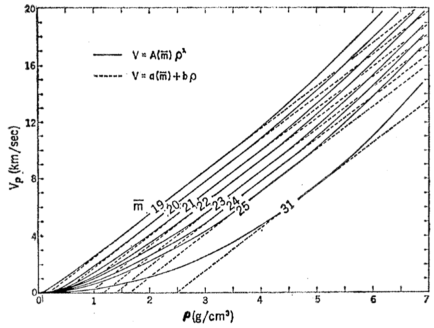


Fig. 1. Comparison of the relation among compressional wave velocity (V_p), density (ρ), and mean atomic weight ($\bar{m}$) due to Birch's law (Eq. 1) with that resulting from the power law (Eq. 4). It can be seen that Birch's law is a linearization, over a certain range of density, of the power law.

Consider the nondispersive relation among the frequency ω, the sound velocity V, and the wave number k,

$$\omega = V\, k \qquad (2)$$

Taking the partial derivative of Eq. 2 and evaluating it term by term, we find that

$$\left(\frac{\partial \ln V}{\partial \ln \rho}\right)_{\bar{m}} = \gamma - 1/3 \qquad (3)$$

where γ is Grüneisen's parameter (3). Since for most crystalline solids γ is of the order of 1 to 2, Eq. 3 implies that sound velocities increase with increasing density of the solid. When Eq. 3 is integrated, the velocity-density relation takes the form

$$V = A(\bar{m})\rho^{\lambda} \qquad (4)$$

where A is constant, a function of $\bar{m}$, and $\lambda = \gamma - 1/3$. Equation 4 itself is then a velocity-density relation for solids. As we shall see below, Birch's law, Eq. 1, is a linearization of Eq. 4 over a certain range of density ρ.

Let us examine the two functions given by Eqs. 1 and 4. For $\rho = \rho_0$, where ρ_0 is the reference density point, we have

$$a + b\rho_0 = A\rho_0^{\lambda} \qquad (5)$$

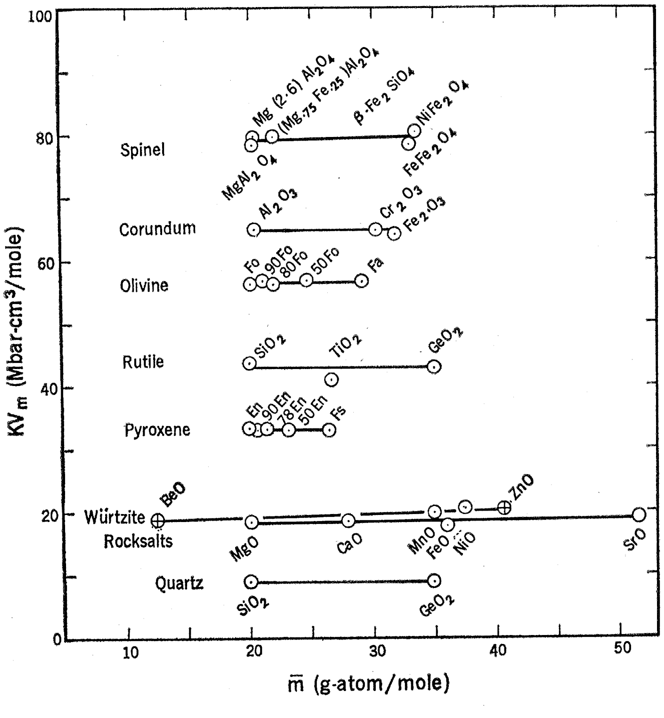
And, in the differential form of Eqs. 1 and 4, we have $d\ln V_1/d\rho = b/(a + b\rho)$ and $d\ln V_4/d\rho = \lambda/\rho$, where subscripts 1 and 4 refer to Eqs. 1 and 4, respectively; for $\rho = \rho_0$, we have

$$\frac{b}{a + b\rho_0} = \frac{\lambda}{\rho_0}\left(\equiv \frac{\gamma - 1/3}{\rho_0}\right) \qquad (6)$$

Thus, the parameters A and λ in Eq. 4 are given by Eqs. 5 and 6, respectively, in terms of Birch's constants and the density ρ_0.

Figure 1 is visual display of Eqs. 1 and 4 for substances for which $\bar{m}$ is 19 through 31, a range of $\bar{m}$ common to rocks and minerals. Let us consider one example (4). For substances with $\bar{m} = 21.0$ g-atom/mole, we find for the velocity of compressional waves V_p that $a = -2.0$ km/sec and $b = 3.0$ (g/cm³)/(km/sec) as given in Birch's original work (1).

Suppose we let $\rho_0 = 3.00$ g/cm³, a common value of density for minerals and rocks, for our illustration (at this density point, then, the two functions given by Eqs. 1 and 4 match exactly). From these Birch's constants, we find $A = 1.70$ and $\lambda = 1.286$ for the power law given by Eq. 4. As can be seen from Fig. 1, the linear law for the velocity-density relation due to Birch (1) provides virtually the same results as the power law derived from lattice dynamics within the density



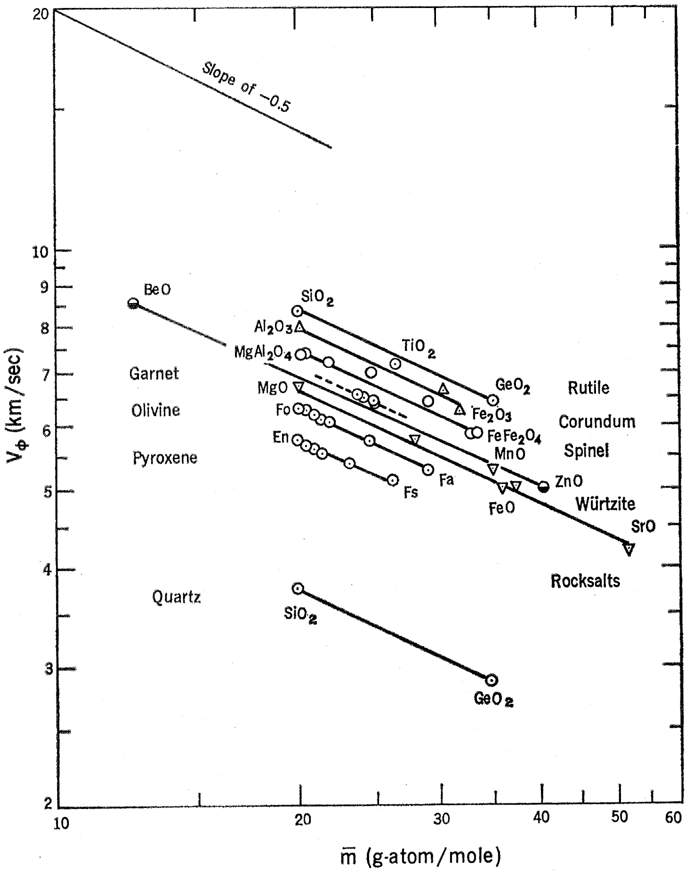


Fig. 2 (left). Illustration of the concept that the product NKV_a ($= KV_m$) is almost constant for materials with the same crystal structure. The observation that KV is constant is illustrated here with materials of eight different structures; K, bulk modulus; V_m, mean molar volume; $\bar{m}$, mean atomic weight (see text).

Fig. 3 (right). Illustration of the velocity–density–chemical composition relation in oxides and silicates; V_ϕ, bulk sound velocity; $\bar{m}$, mean atomic weight. The velocity-density relation for a change of chemical composition within the same crystal structure is, in the first-order approximation, normal to the velocity-density relation characterized by the power law. Similar diagrams for the velocities of compressional and shear waves can be constructed, but in these cases the slope is expected to deviate somewhat from −0.5 because of their sensitivity to the noncentral forces between constituent ions.

range of about 2.5 to 4.0 g/cm^3; beyond this the linear law deviates significantly from the power law. For this range of density, then, Birch's law is merely a linear approximation of the power law given by Eq. 4; as was shown earlier, both laws provide estimates of elastic parameters in a solid as it undergoes compression or phase changes as long as no alteration in chemical composition is involved (*5*). Rocks and minerals are polyatomic solids without group formation, that is, the average weight per unit volume is almost constant, and their densities and mean atomic weights (compared to those of metals and alloys) tend to cluster within a narrow range. Most rocks and minerals, in fact, have densities of approximately 2 to 4 g/cm^3. The apparent success of Eq. 1, which is a linearization of the power law over this limited range of density, arises from the empirical fact that most rocks and minerals do possess density values in that range.

A perusal of Birch's papers (*1*) suggests another important consequence of lattice dynamics. The velocity-density relation for a change of chemical composition within the same crystal structure is such that the variation of sound velocities with density is nearly perpendicular to the line provided by Eq. 1. This observation, made originally by Birch (*1*) and supplemented by many other authors (*2, 6, 7*) for various minerals and rocks, can be analyzed in terms of lattice dynamics.

From the nondispersive relation, Eq. 2, the velocity of elastic waves can take the form

$$V = \omega \left(\frac{4\pi}{3} V_a\right)^{1/3} = \omega \left(\frac{4\pi}{3} r_0^3\right)^{1/3} \quad (7)$$

where V_a is the mean atomic volume given by the mean interatomic separation r_0 at the equilibrium conditions. Since in lattice dynamics the frequency ω is specified by $\omega = (f/m)^{1/2}/2\pi$, where m is the mass of a body in harmonic oscillation and f is the restoring force given by the second derivative of an interatomic potential $U(r)$ with respect to r, Eq. 7 can be evaluated for a particular potential. For all practical purposes in geophysics, the frequency ω can be estimated with confidence from the Born-Mayer type of potential of the form: $U(r) = -\alpha z_1 z_2 e^2/r + B/r^n$, where z_1e and z_2e are the ionic charges, α the Madelung constant, B a constant, and n the Born repulsion parameter (*8*). Rewriting $U(r)$ in a form such that the potential is a minimum at the r_0 point, we find then

$$U(r) = -\frac{\alpha z_1 z_2 e^2}{r_0}\left[\frac{r_0}{r} - \frac{1}{n}\left(\frac{r_0{}^n}{r^n}\right)\right] \quad (8)$$

Thus, Eq. 7 takes the form

$$V = \frac{1}{2\pi}\left(\frac{4\pi}{3}\right)^{1/3}\left(\frac{\alpha z_1 z_2 e^2\,(n-1)}{r_0}\,\frac{1}{m}\right)^{1/2} \quad (9)$$

where the product $\alpha z_1 z_2 e^2 (n-1)/r_0$ corresponds to a macroscopic quantity NKV_a, where N is Avogadro's number and K is the bulk modulus; this quantity is almost constant for materials with the same crystal structure, as illustrated in Fig. 2. What is implied in Fig. 2 is that the quantity $\alpha z_1 z_2 e^2$ is approximately invariant within a given structure and that both n and r_0 increase with increasing atomic weight. The variation of sound velocities with varying $\overline{m}$ (thus density within the same structure) is then in essence the variation of m given by $\overline{m}/N$; taking the partial derivative of Eq. 9 with respect to $\overline{m}$, we obtain the desired relation

$$\left(\frac{\partial \ln V}{\partial \ln \overline{m}}\right)_{\mathrm{CS}} \simeq -1/2 \quad (10)$$

where CS stands for constant structure. Shankland (*9*) independently found the same expression as Eq. 10 above.

Experimental information on sound velocities and mean atomic weights for various isostructural oxides and silicates are found in the literature (*2, 6, 7*). As an illustration, a $\ln V_\phi - \ln \overline{m}$ diagram is constructed in Fig. 3 (V_ϕ is the bulk sound velocity). Figure 3 shows that the slope of this relation among oxides and silicates is indeed in the neighborhood of -0.5 as implied by Eq. 10.

I have described how Birch's law arises in the physics of solids. It is simply a linearization, over a certain range of density, of the power law (Eq. 4) resulting from lattice dynamics. Beyond that range of density, Birch's law should be used with caution. Within that range of density, either Birch's law or the power law should be broadly applicable in geophysics and materials science, and below and above it the power law should be used with wide generality. Finally, the velocity-density relation for a change of chemical composition within the same crystal structure arises essentially from the effects of mass variation; the velocity-density relation in that case is, in the first-order approximation, perpendicular to the velocity-density relation characterized by the power law.

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References and Notes

1. F. Birch, *Geophys. J. Roy. Astron. Soc.* **4**, 295 (1961); *J. Geophys. Res.* **66**, 2199 (1961).
2. Historically, Birch observed, on the basis of his systematic studies [*J. Geophys. Res.* **65**, 1083 (1960); *ibid.* **66**, 2199 (1961)] with pressures up to 10 kbar, that the compressional wave velocity V_p in minerals and rocks is proportional to the density ρ for substances with similar mean atomic weights $\overline{m}$. Birch then concluded that the velocity of compressional waves in isotropic aggregates of oxides and silicates is mainly a function of ρ and $\overline{m}$ and that the variation of ρ for substances with the same $\overline{m}$ reflects differences in structure and composition. Since then, McQueen *et al.* (*6*) observed that for similar materials the bulk sound velocity V_ϕ is proportional to density. C. Wang [*J. Geophys. Res.* **74**, 1451 (1969)] also observed a linear variation of V_ϕ with ρ in shock-compressed periclase. From their measurements of the velocity of shear waves V_s in naturally occurring minerals and rocks, G. Simmons [*J. Geophys. Res.* **69**, 1123 (1964)] and N. I. Christensen [*ibid.* **70**, 3549 (1966); *ibid.* **71**, 5921 (1966)] concluded that this velocity also is a linear function of ρ, although a few exceptions with calcium-rich minerals were noted by Simmons. O. L. Anderson, E. Schreiber, R. C. Liebermann, and N. Soga [*Rev. Geophys.* **6**, 491 (1968)] also demonstrated Birch's law for several oxides by measuring the ultrasonic velocities in these materials. Chung (*5*), on the basis of a systematic investigation of the substitution of iron for magnesium in olivine [*J. Geophys. Res.* **75**, 7352 (1970); *Geophys. J. Roy. Astron. Soc.* **25**, 511 (1971)] and in pyroxene [*Trans. Amer. Geophys. Union* **52**, 919 (1971)], concluded that Birch's law essentially holds for these mineral systems for a wide range of iron/magnesium ratios. He then extended Birch's law for use in the construction of equations of state of high-pressure solid phases.
3. Here, V can be a compressional velocity V_p or a shear velocity V_s, or both, depending on the mode of Grüneisen's parameter, which arises from $\delta \ln \omega / \delta \ln \rho$, where ω may be in the longitudinal or the transverse mode, or both. The bulk sound velocity $V_\phi \equiv (K_s/\rho)^{1/2}$, where K_s is the adiabatic bulk modulus, is then given by $V_p - 4/3\ V_s$.
4. For substances with other values of $\overline{m}$, it is of interest to note that (i) Grüneisen's parameter γ depends on $\overline{m}$ in such a way that the larger the value of $\overline{m}$ the greater the value of γ and (ii) an error introduced in the power law constant A by the choice of the reference density ρ_0 is negligible, whereas an error introduced in λ by the choice of ρ_0 is expected to be relatively large. The reason for the latter is that γ is rather sensitive to the choice of ρ_0, thus to $\overline{m}$ by implication. The velocity-density relations inferred from Eqs. 1 and 4 for substances for which $\overline{m}$ is 19 through 31 are plotted in Fig. 1; for each value of $\overline{m}$ the range of density in which both Birch's law and the power law result in the same velocity-density relation is seen.
5. D. H. Chung, *Nature*, in press.
6. R. G. McQueen, J. N. Fritz, S. P. Marsh, *J. Geophys. Res.* **69**, 2947 (1964).
7. N. I. Christensen, *Tectonophysics* **6**, 331 (1968); *J. Geophys. Res.* **77**, 369 (1972); R. C. Liebermann, *ibid.* **75**, 4029 (1970).
8. C. Kittel, *Introduction to Solid State Physics* (Wiley, New York, ed. 3, 1967).
9. T. J. Shankland, *J. Geophys. Res.*, in press.
10. The ideas in this report were presented by the author in a series of seminars at Boston College (physics); University of California, Berkeley (engineering geoscience); University of Rochester (space sciences); Texas A & M University (geophysics); and Massachusetts Institute of Technology (earth and planetary science). I wish to express a deep sense of gratitude to Francis Birch for his advice and to Frank Press for his encouragement during my research on the equations of state of mantle materials. I thank Thomas J. Shankland for discussing his work, and Pierre-Yves F. Robin for assistance in the computation and for his comments. Supported by NSF grant GA-11091.

24 March 1972