distribution function as a means of correlating environmental pollution data. In this regard, the preceding analyses of oil pollution data attest both to the method's versatility and to its correlational ability. The interpretations that have been offered for these Weibull analyses are by no means conclusive; rather, they are merely intended to illustrate the interpretative potential of the Weibull threshold parameter.

I believe that the Weibull distribution may well be applicable to such diverse environmental problems as automotive emissions, pesticide and trace metal contaminant concentrations, marine fauna mortality, and a host of others. It may also provide new insight in evaluating the effectiveness of pollution control measures. Hopefully, investigators from other disciplines and fields of interest will explore these possibilities.

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References and Notes

1. Excellent reviews and bibliographies of the Weibull distribution function are presented by F. H. Steiger [*Chem. Technol.* **1**, 255 (1971)] and C. C. Harris [*ibid.*, p. 446].
2. Details of the natural oil seeps in the Santa Barbara Channel have been presented by A. A. Allen, R. S. Schlueter, and P. G. Mikolaj [*Science* **170**, 974 (1970)]; seeps in the Santa Monica Bay have been discussed by A. A. Allen, L. E. Fausak, V. N. Sutherland, P. G. Mikolaj, and R. S. Schlueter ("Santa Monica Bay natural oil seep investigation," final report to the El Segundo Refinery, Standard Oil Company of California, by Marconsult, Inc., Santa Monica, December 1971).
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4. The data points shown in Fig. 1, B–D, were obtained by arbitrarily grouping the actual field measurements. In all cases, the highest level of pollution shown coincides with the maximum field observation. Although grouping the data tends to introduce some variability into the resultant parameters, it simplifies considerably the task of obtaining a visual straight line. Other more sophisticated methods are also available (*1*).
5. "Oil Pollution Research Newsletter" (Environmental Protection Agency, Edison Water Quality Laboratory, Edison, N.J., July 1971), vol. 4.
6. G. A. Gilmore, D. B. Smith, A. H. Rice, E. H. Shenton, W. H. Moser, "Systems study of oil spill cleanup procedures; vol. 1, Analysis of oil spills and control measures," report prepared for the American Petroleum Institute by the Dillingham Corporation, La Jolla, Calif., February 1970).
7. This analysis does not include the *Torrey Canyon* spill which contaminated a reported 390 km of coastline. The remaining data included 12 incidents with a shoreline contamination ranging from 3 to 45 miles (4.8 to 72 km); in addition to these, two incidents were reported as minor and seven were reported as moderate. These latter incidents were assumed to involve less than 5 miles, and 5 to 10 miles of shoreline, respectively.
8. This work is a result of research sponsored by the National Science Foundation under grants GH 43 and GH 95, and by the National Oceanic and Atmospheric Administration Office of Sea Grant, Department of Commerce, under grant USDC 2-35208-5.

28 December 1971; revised 29 March 1972 ■

Statistical Thermodynamic Model for the Distribution of Crustal Heat Sources

Abstract. *The observed linear correlation of surface heat flux in continental regions with the radioactive heat release in the surface rocks is consistent with an exponential depth dependence of the concentration of the heat sources. We suggest that this depth dependence can be explained as an equilibrium distribution of the heat-producing elements in a gravitational field. The effective mean mass range of the elemental assemblies being differentiated is predicted, and it appears that potassium, uranium, and thorium diffuse upward in ionic complexes with other elements. Similar considerations should apply to the distribution of other elements in the crust and mantle.*

Quantitative evidence for the upward concentration of heat-producing elements within the continental crust in general and large plutonic bodies in particular has come from geochemical studies (*1*, *2*), from measurements of surface heat flow coupled with measurements of heat production in near-surface igneous bodies at the sites of the heat-flow measurements (*3–5*), and from direct measurements (*6*). It has been shown (*3–5*) that surface heat flow, q_s, in continental regions has a linear correlation with the radioactive heat release, H_s, in the near-surface rocks. This correlation is shown in Fig. 1 for the Sierra Nevada, Basin and Range, and tectonically inactive regions of the United States. Lachenbruch (*7*) has shown that these results are consistent with an exponential dependence of the radioactive heat release on depth.

So far no adequate explanation has been advanced for this exponential depth dependence, which is of fundamental significance in understanding the evolution of the continental crust and which is not readily understood in terms of conventional igneous differentiation processes. We have approached this problem by considering the equilibrium distribution of large ionic complexes in a high-temperature medium in a gravitational field. This technique has been used successfully to treat such problems as the distribution of gas molecules or particulates in the atmosphere. The applicability of this differentiation process to the crust has been suggested by Ramberg (*8*).

For the differentiation process to have resulted in the observed upward concentration of heat production it must have, to a greater or lesser extent, affected all three of the principal heat-producing elements, uranium, thorium, and potassium. This has been confirmed by the concentrations measured in borehole samples by Lachenbruch and Bunker (*6*).

Lachenbruch (*7*) shows that observations are consistent with an exponential fall in radioactivity with depth of the form

$$H = H_s \exp(-y/h) \qquad (1)$$

where y is the depth and h is the scale depth for the radioactive heat sources. Integration of Eq. 1 gives

$$q_s = H_s h + q_\infty \qquad (2)$$

where q_∞ is the heat flow from the lower crust and upper mantle. The linear correlation from Eq. 2 is given in Fig. 1 with $h = 10$ km. Within the experimental scatter the scale depth is the same for these very different geological provinces.

The exponential depth dependence and the nearly constant scale depth can be explained by equilibrium statistical thermodynamics, that is, the Boltzmann factor. In thermodynamic equilibrium the distribution of any group of particles is proportional to $\exp(-\varepsilon_i/kT)$, where ε_i is the energy of the particle i, k is Boltzmann's constant, and T is the absolute temperature. Taking the energy to be the gravitational potential energy we may write

$$H = H_s \exp\left(-\frac{\Delta\rho M g y}{\rho R T}\right) \qquad (3)$$

where M is the molecular mass of the assembly that is being differentiated, $\Delta\rho$ is the density difference between the molecular assembly and the surrounding medium, ρ is the density of that medium, R is the universal gas con-

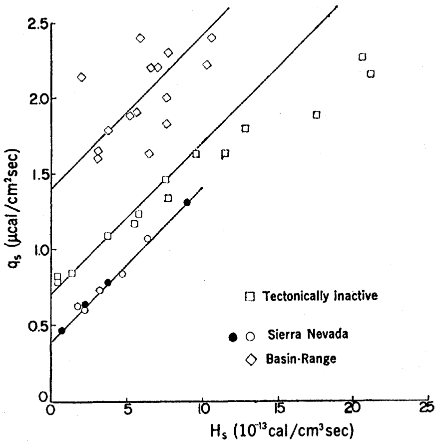

Fig. 1. Dependence of surface heat flux (q_S) on radioactive heat production (H_S) in near-surface rocks for three geological provinces in the United States. The open points are from Roy *et al.* (*4*) and the solid points are from Lachenbruch (*5*). The linear correlations are from Eq. 2, with $h = 10$ km.

stant, and g is the acceleration of gravity. The power of the statistical thermodynamic approach to this problem is that an understanding of the dynamic processes that lead to the differentiation is not required.

Comparing Eqs. 1 and 3 we find that the scale depth for radioactive heat release is given by

$$h = \frac{\rho RT}{\Delta\rho M g} \qquad (4)$$

Taking the observed value 10 km for h and using 10^3 cm/sec^2 for g we plot the required molecular mass M against $\Delta\rho/\rho$ for several temperatures in Fig. 2. Since the compositional equilibration must have occurred when the crustal rocks were hot the values 700°, 900°, and 1100°K should be an adequate range for T. We have assumed that the temperature in the region of differentiation is uniform. This represents a good approximation for large intrusive igneous bodies while intrusion and compositional equilibration is occurring. Lambert and Heier (*2*) have discussed the mechanism by which the upward transport of heat-producing elements in metamorphic terranes may take place. They tentatively favor a process involving the transport of large ionic complexes by means of an interstitial vapor-fluid phase.

Figure 2 shows that for complexes with densities 5 to 10 percent less than that of the surrounding medium there should be effective fractionation for molecular masses as low as 600 to 2000. Relatively small numbers of atoms of uranium or thorium are needed to give masses in this range. However, they must be associated with significantly greater numbers of light ions such as oxygen and hydrogen if the mean density of the complex is to fall in the required limits. Precise calculations cannot be undertaken without a knowledge of the form of the complexes as internal structural gaps will have a significant effect on the density. Since it is unlikely that the potassium is associated with the same molecular complexes as the uranium and thorium and since the scale heights for the three elements are nearly equal, it is likely that $\Delta\rho$ is larger for the potassium complex but M is smaller so that the product $M\Delta\rho$ is nearly the same.

As pointed out by Lachenbruch (*7*) erosion will change neither the exponential depth dependence of the radioactive heat release nor the scale depth. Only the magnitude of the heat release will be reduced by the ratio $\exp(-d/h)$ where d is the depth of the material that has been eroded away. The suggestion that low values of radioactive heat release are consistent with erosion to a deep level has been made by Clark and Ringwood (*1*) and by Lambert and Heier (*2*).

It is of importance to consider to what extent the exponential distribution proposed may be typical of all parts of the crust which have reached sufficiently high temperatures for an approach to equilibrium to have occurred. Equilibration is expected in near-surface regions only where magmatic or hydrothermal activity has substantially elevated the thermal gradient or where erosion has been deep enough to expose zones of former high temperature. It is also probable that the physical processes involved in the equilibration may differ in metamorphic and igneous terranes. The best evidence for the exponential distribution comes from the heat-flow measurements made on large, relatively homogeneous igneous or metaigneous bodies. In such bodies the temperature distribution shortly after intrusion and the presence of a fluid phase should favor equilibration over a depth which is a large fraction of the crustal thickness (*9*). Direct geological evidence from such bodies has been provided by a variety of studies (*6, 10*). Difficulties arise, however, from the relatively small fraction of the vertical scale depth that is accessible within a single pluton and the consequently small variations in concentration that must be recognized.

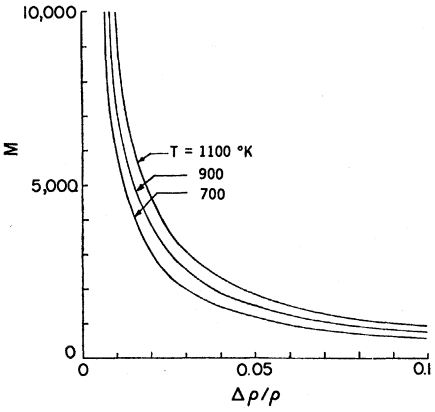

Fig. 2. Dependence of the molecular mass (M) of the assembly that is being differentiated on the ratio of the density difference to the density, from Eq. 4 for several temperatures and $h = 10$ km.

Lambert and Heier (*2*) in their studies of metamorphic terranes found little evidence for the movement of uranium, thorium, and potassium in rocks above the granulite facies of regional metamorphism. In such regions an uppermost homogeneous zone in which elemental distributions are governed by sedimentation, tectonic movement, and small-scale igneous intrusion is expected to overlie the zone of equilibration.

Although this model has been applied only to the heat-producing elements in the crust it should apply equally well to the distribution of other elements. In addition it should be applicable to chemical variations in those parts of the mantle which have not been homogenized by convective motions.

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1 October 1971; revised 19 April 1972