and the women who make history. This has been eloquently stated by the Cuban poet Padilla, who has recently been released from prison in his own country. The final lines of his poem "Important Occasions" (*25*) read:

History's going to save us—we were thinking.
Going to save us—were we dreaming?
It wasn't all just uprisings, barricades, bonfires:
in our heads it was a dress of bubbling foam, a
Rhine maiden with clear eyes, smiling, standing
at the door, hand outstretched
toward a hungry and waiting people.
But there was no one in the doorway. Nor in the house.
Instead we stumbled. They shoved us inside. We broke our teeth going in, got our jaw smashed.
We found tools and weapons and we
fought, we struggled, we worked and continued
fighting. But it's true, old Marx,
that History is not enough.
Important occasions,
man makes them.

It's a real, live man who does it,
who masters it, who will fight.
History by itself does
nothing, dear friends.
It does absolutely nothing.

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Circumscribed Halos

Circumscribed halos which appear around the sun are simulated by a computer treatment of a simple model.

Robert G. Greenler and A. James Mallmann

Ice crystals in the atmosphere are responsible for a variety of optical phenomena in the sky. One of the most familiar of these phenomena is a circle of light around the sun, with an angular radius of 22°, as shown in Fig. 1. This halo results from sunlight deviated by about 22° by refraction through randomly oriented hexagonal ice crystals. Occasionally this 22° halo is circumscribed by another halo which is tangential to the 22° halo at the top and bottom. Unlike the circular 22° halo, the shape of the circumscribed halo varies markedly with the altitude of the sun (as can be seen in Fig. 7). In this article we analyze the source and form of this circumscribed halo.

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The Model

Ice crystals in the air commonly exist in the form of hexagonal prisms. If the length of the crystal along the axis is short as compared with its width, the crystal will have the form of a flat hexagonal plate. If the axial dimension is long, the crystal will have the shape of a hexagonal column which we refer to as a pencil crystal, after its resemblance to the shape of the common wooden pencil. These crystals, falling in still air, tend to assume the orientation which provides maximum air resistance. This means that flat plate crystals will tend to fall with their axes vertical and the pencil crystals with their axes horizontal. The details of the orientations of a group of falling ice crystals will determine the pattern of light in the sky which results from sunlight refracted by these crystals. Our approach has been to calculate the path of light rays through a distribution of ice crystals in order to find that distribution which produces the observed forms of the circumscribed halo.

The Calculation

Instead of dealing with a multitude of crystals at different locations in the sky, we have used one crystal, at one location, and have assigned to it all the desired orientations. With the invaluable aid of the computer, we have, for each crystal orientation, calculated the direction of the light ray leaving the crystal. Figure 2 shows that light passing through alternate faces of a pencil crystal is refracted as if it had passed through a 60° prism. If a light ray from the sun reaches an observer after being deviated through an angle δ by an ice crystal, the observer will have to look an angle δ away from the sun to see that light ray (Fig. 3). The angle of minimum deviation for a 60° ice prism is 22°; rays with either greater or smaller angles of incidence on the crystal face will be deviated by more than 22° in passing through the prism. The nature of this minimum in the deviation angle determines two important features of a halo: (i) the angular radius of the inside of a halo is equal to the minimum deviation angle, and (ii) the halo is brightest around its inside edge. The case of refraction by a prism treated in most optics textbooks is for a light ray that is perpendicular to the axis of the prism, and for this case the angle of minimum deviation is easily calculated. For rays that are not perpendicular to the prism axis (skew rays) the angle of minimum deviation depends on the skewness of the rays. Earlier investigators (*1*) have calculated the pattern of light in the sky that would result only from the rays with minimum deviation. Such an approach necessarily omits intensity information and the contributions of all other refracted rays.

Our calculation includes intensity information resulting from the dependence of deviation of a light ray on the orientation of the prism. In addition, we have accounted for three other factors that influence the intensity distribution. First, the intensity of the exit ray relative to the incident ray is diminished because of reflection losses at each surface. The intensity loss due to reflection depends on the angle of incidence and on the index of refraction of the crystal. Using Fresnel reflection coefficients (*2*), we have accounted for this reflection factor. The other two intensity factors that we have included are geometrical factors. For the circumscribed halo we are concerned with light that is refracted through two surfaces that make an angle of 60° with each other. The cross-sectional area of the light beam incident on the first crystal face depends on the cosine of the angle of incidence; when the projected area of the crystal face in the direction of the incident light is small, the relative intensity of the exit beam is low. These first two intensity factors have been treated exactly in our formulation. A third factor allows for the fact that only for the case in which the light passes through the crystal at minimum deviation, and normal to the crystal axis, will all the light that enters the first face pass through the second face. For all other paths some of the light transmitted by the first face will strike other faces of the crystal. We have accounted for this last geometrical factor by applying the correction appropriate to light rays that are perpendicular to the crystal axis. By using this approximate correction we have also neglected any effects due to the ends of the pencil crystals. Using these intensity-influencing factors, we calculated relative intensities for the exit rays which ranged between zero and unity. Before plotting the data we compared the relative intensity of the exit ray with a random number between zero and unity. Only if the relative intensity of the exit ray was greater than the random number did that ray contribute to the plot. According to this procedure, the effect would be, for example, that only one-tenth of the spots associated with rays having a relative intensity of 0.1 would be plotted.

The orientation of a crystal is determined by the specification of three angles. We assign sets of random values to the three angles with appropriate weighting factors to produce the desired distribution. The following approach is used to make a plot of the

Fig 1. Photograph of a 22° halo.

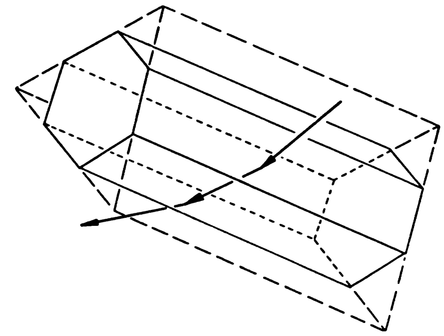

Fig. 2. Light ray passing through a hexagonal ice crystal.

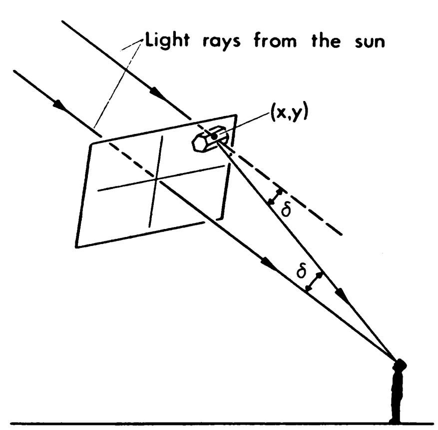

Fig. 3. An observer viewing a light ray refracted by an ice crystal.

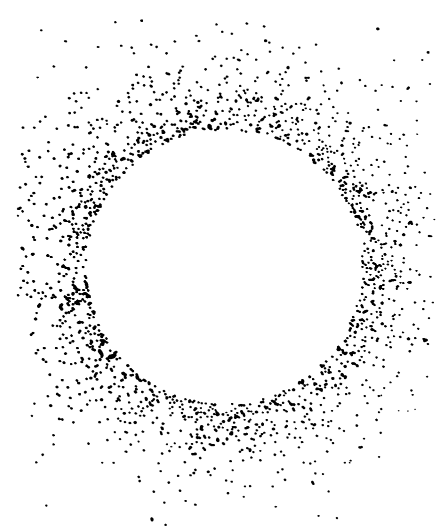

Fig. 4. Simulation of a 22° halo.

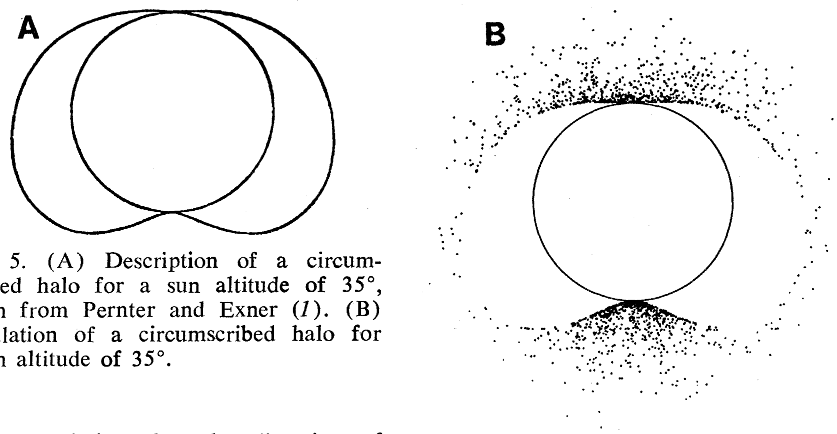

Fig. 5. (A) Description of a circumscribed halo for a sun altitude of 35°, taken from Pernter and Exner (*1*). (B) Simulation of a circumscribed halo for a sun altitude of 35°.

simulated halo: after the direction of the exit ray is determined, we calculate the coordinates of the point in a plane (normal to a line between the observer and the sun) at which the crystal can be located to give an exit ray which points toward the observer. The situation is depicted in Fig. 3. We calculate the coordinates of several thousand points for each simulation and plot them automatically. The number of points per unit area is proportional to the intensity of the light.

Results and Discussion

As a special case of our formulation, a random distribution of orientations of the ice crystals should produce a simulation of the 22° halo. The result is shown in Fig. 4, which may be compared with the photograph of an actual halo shown in Fig. 1. The simulated halo shows the intensity distribution for red light. In all of our simulations we have ignored the diameter of the sun; our results are those predicted for a point sun. To take the diameter of the sun (0.5°) into account, we could smear the pattern over a 0.5° circle, but this operation would not change any of the essential features we describe.

The simplest distribution of crystals that should produce a circumscribed halo is one in which all the pencil crystals have horizontal axes, including all orientations resulting from rotation about that axis and from rotation of that axis in the horizontal plane. Figure 5B shows the pattern resulting from such a distribution for the sun at an altitude of 35°. About 20,000 different crystal orientations were used to produce this figure. In this simulation we have drawn a circle representing the inner edge of the 22° halo. Figure 5A, taken from Pernter and Exner (*1*), shows their locus of rays with minimum deviation for horizontal crystals for this same sun altitude. The locus of minimum deviation matches the inner boundary of our intensity distribution. It is clear, however, that the intensity information contained in the dot diagrams adds a great deal of predictive value to the simulation.

Figure 6 shows the resulting intensity distribution for pencil crystals with horizontal axes for several different values of the sun altitude. A horizon has been added to each of the figures along with a circle of 22° radius. For sun altitudes less than about 30°, there is no light at the sides of the halo and the halo separates into two segments. These are generally referred to as the upper and lower tangential arcs. For sun altitudes between 30° and about 40°, the circumscribed halo extends completely around the sun, in principle, but our plots predict such a low intensity at the sides of the halo as to be unobservable. For higher sun altitudes, the

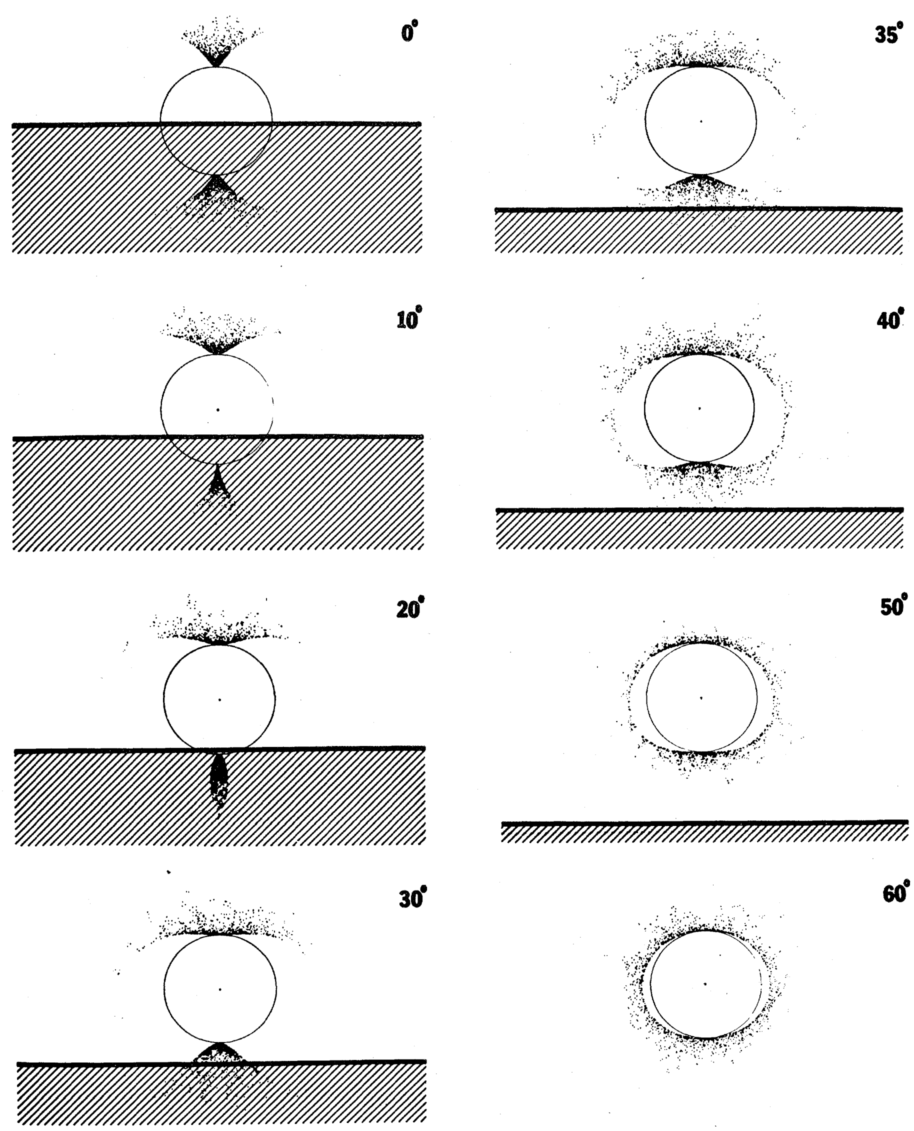

Fig. 6. Simulation of circumscribed halos for various altitudes of the sun, based on the assumption that the ice crystal axes are horizontal. The heavy line represents the horizon, and the circle marks the inner edge of the 22° halo.

circumscribed halo becomes more uniformly intense around its perimeter and shrinks to match the circular shape of the 22° halo.

We have taken photographs with which to compare these predictions, some of which are shown in Fig. 7. Figure 7, C and F (sun altitudes of 25° and 60°), show the 22° halo not as a circle but somewhat horizontally elongated. These two photographs were taken with the camera facing a reflecting glass sphere (*3*), and the resulting distortion is the price one pays for obtaining a photograph which originally covered the entire sky. For altitudes of 0° and 25° (Fig. 7, A and B) the upper tangential arc is essentially complete. It is not expected, even in the most intense display, to continue around to the sides of the 22° halo. For sun altitudes less than 22° the lower tangential arc will be below the horizon, but an observer in an airplane may see it.

The 22° halo and the circumscribed halo can occur independently. Figure 1 illustrates the more common situation in which only the 22° halo is seen. Five of the photographs in Fig. 7 (A, B, C, E, and F) show both halos, but Fig. 7D (for a sun altitude of 40°) shows the upper arc of the circumscribed halo with no trace of the 22° halo. The simultaneous occurrence of both halos could result from groups of ice crystals with different orientation distributions located at different altitudes in the sky; or they might result from two different distributions in the same region of the sky resulting from crystals of different sizes or different length-to-width ratios. Flat plate crystals oriented with their axes nearly vertical will give rise to bright spots located at about 22° away from the sun on either side. These parahelions or "mock suns" are commonly observed. The techniques used in this investigation appear to produce satisfactory simulations of these effects also (*4*).

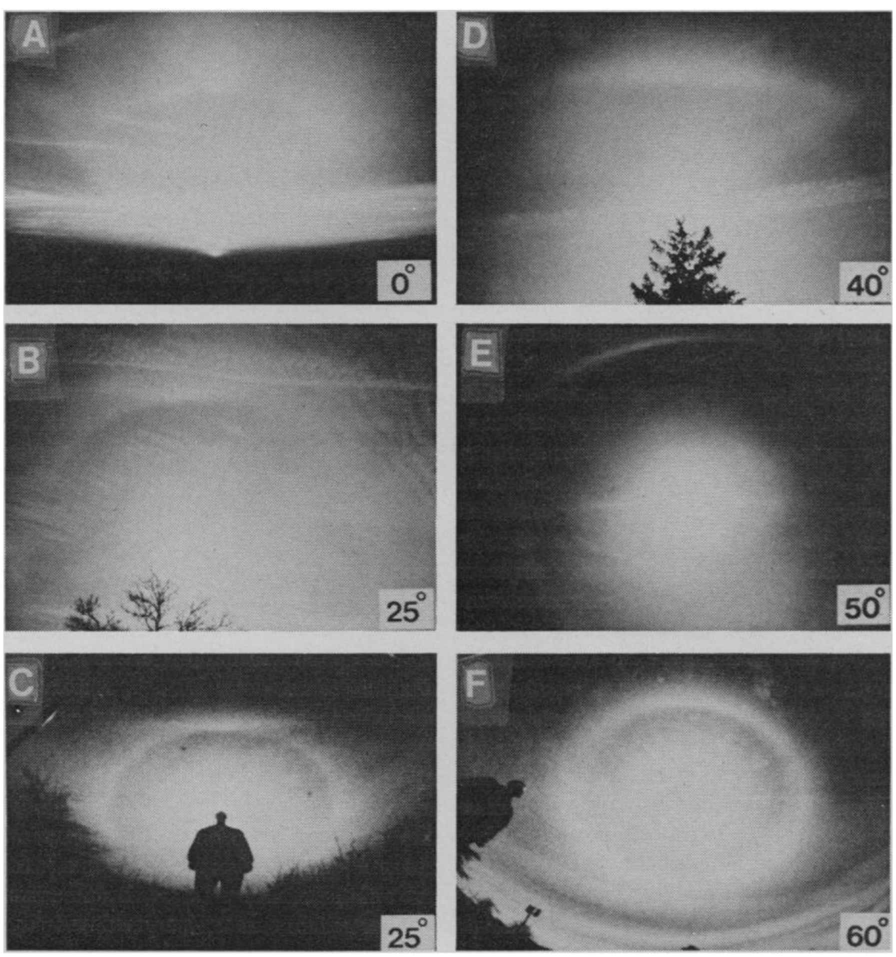

Fig. 7. Photographs of circumscribed halos for various altitudes of the sun.

Conclusion

The detailed agreement between the form of the circumscribed halo and our simulations for a range of sun altitudes would seem to provide convincing evidence that hexagonal ice crystals with nearly horizontal axes are responsible for this phenomenon. In general, slight deviation from the horizontal will tend to smear the upper and lower arcs out along the 22° halo, the extreme case of deviation from the horizontal resulting in a turning of the circumscribed halo into the 22° halo. Our simulation shows that the features of the circumscribed halo are reproduced by crystals with exactly horizontal axes, no tilt from the horizontal being necessary.

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2. See, for example, M. Born and E. Wolf, *Principles of Optics* (Pergamon, Oxford, ed. 4, 1970), pp. 38–42.
3. The reflecting sphere used to obtain Fig. 7, C and F, is actually a Christmas tree ornament. The composition of Fig. 7C is not intended to imply anything about the character of the senior author who appears in it, but results from an attempt to reduce a lens flare by blocking out the sun.
4. See R. G. Greenler, M. Drinkwine, A. J. Mallmann, G. Blumenthal, *Amer. Sci.*, in press.