east and not heretofore apparent in regional maps of Alaska. This orthogonal fracture set may reflect a conjugate set of fractures within the crust, which has exerted significant control over the geologic history of the state. Increased resolution in other images from space platforms, such as the resolution of 200 to 650 feet (60 to 200 m) planned for the satellite television cameras of the ERTS program (*20*), will permit the discernment of finer detail and a greater accuracy in identifying and locating geologic features.

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21. I thank my colleagues W. P. Brosgé, A. Grantz, R. A. Loney, and W. W. Patton, Jr., for helpful criticism of the manuscript and for stimulating discussion; J. M. DeNoyer of the National Aeronautics and Space Administration for providing the original image for interpretation; Goddard Space Flight Center of the National Aeronautics and Space Administration for prompt provision of high-quality prints and negatives; and the clouds over Alaska for cooperating so beautifully with Nimbus IV. Publication of this article is authorized by the director of the U.S. Geological Survey.

Measurement Structures and Psychological Laws

Measurement of psychological variables is closely linked to the testing of qualitative psychological laws.

David H. Krantz

Measurement is employed in two quite different ways in scientific research. Usually, the investigator searches for quantitative laws, relying on previously established physical measurement. This is what chemists do, for example, in weighing various products of reactions. Such weight measurements led ultimately to the periodic table and to the chemical theories suggested by it. Similarly, physiologists and psychologists measure variables such as pupil diameter in order to discover laws of functioning of the visual system and the brain.

The second use for measurement is to represent an empirical structure by an analogous or homomorphic numerical structure. It is this second use that leads to construction of measurement scales de novo. For example, the weight measure was originally introduced because the properties of numerical weights gave a convenient representation to qualitative empirical observations. Objects can be compared qualitatively in a pan balance and the resulting empirical ordering is represented by the numerical ordering of the scale values (weights) assigned to them. Similarly, putting two objects together is represented by adding their weights. A set of empirical relations that leads to construction of measurement scales in this fashion is called a measurement structure.

In physics, measurement structures are very common. In most other sciences, previous physical measurement is used to generate new empirical relations. The laws satisfied by these latter relations lead to formulation of theories, but they do not lead to introduction of new measurement, that is, the empirical relations do not constitute measurement structures, even though they were generated by use of previously established physical measures. In psychology, however, the program of introducing new numerical functions, to measure such variables as intelligence, utility, or sensation magnitude, has long been attractive. In some cases, appropriate qualitative psychological laws do lead to such new, nonphysical measurement.

In this article I review some kinds of lawful structures that lead to measurement, briefly first in physics, then more thoroughly in psychology. I conclude by emphasizing that many areas of psychology should and do operate in the same way as biology or chemistry, so that previously established physical measures are used to discover new psychological theory, but not to establish new measurement scales. In still other areas, psychological measurement structures seem quite promising.

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Measurement Structures in Physics

In some cases, physical measurement is based on quantitative laws and their importance for measurement is generally recognized. For example, interval-scale measurement of temperature is based on the law of linear expansion of a thermometric substance; while measurement of temperature ratios (Kelvin scale) depends on the third law of thermodynamics. Another sort of example is provided by measurement of density, which depends on the fact that the ratio of the mass to the volume of a homogeneous substance is independent of the volume.

The measurement of physical quantities such as length, mass, and duration is logically prior to the formulation of the quantitative laws of physics. Nevertheless, philosophers and physicists investigating the foundations of physics have long understood that even these so-called fundamental measurements are based on certain qualitative physical laws. For example, suppose that two straight rods, a and b, are laid side by side (Fig. 1) and that b extends beyond a at both ends (that is, b is longer than a, denoted $b > a$). Similarly, suppose that $b' > a'$. If now b and b' are concatenated by laying them end to end, and if, alongside them, a and a' are concatenated similarly, then the combination of b and b' is longer than the combination of a and a'. Denoting the operation of end-to-end concatenation by *, we have the following simple law:

$$\text{If } b > a \text{ and } b' > a', \quad \text{then } b * b' > a * a' \qquad (1)$$

In the fundamental measurement of length, two qualitative (that is, nonnumerical) empirical relations are used, the comparison relation $>$ and the concatenation operation *. These empirical relations satisfy various qualitative laws, such as that shown in Eq. 1. It can be proved (*1*) that the qualitative laws satisfied by $>$ and * imply that a real-valued function, or length measure, can be constructed, such that larger function values correspond to longer rods and such that the length of $a * b$ is the sum of the lengths of a and b.

Fundamental measurement of mass and duration also requires qualitative comparison relations and concatenation operations, and these empirical relations satisfy the same abstract laws, like the one in Eq. 1, that hold in the case of length. From a formal standpoint, the procedures for assigning numbers as measures of objects are identical for length, mass, and duration: one concatenates many identical objects (for example, centimeter-lengths, gram-masses, or pendulum periods) and counts how many such identical objects are needed to match approximately the object to be measured. Such measurement, based on concatenation, is called extensive measurement (*1*). (Some of the most interesting lengths, masses, and times are those occurring on cosmic or molecular scales, and these, of course, are measured indirectly by using the much more elaborate quantitative laws of physics to relate them to quantities on a laboratory scale, such as the length of a photographic track or the arc length along a meter scale.)

Measurement Structures in Social Science

Extensive measurement has had few applications to fundamental measurement of psychological variables, because no concatenation operations are known for variables such as loudness, utility, intelligence, thirst, or anxiety, which satisfy appropriate qualitative laws. In particular, Eq. 1 can be shown to fail for many natural concatenation operations. Comparison relations which resemble $>$ abound, but in the absence of additional structure, such as that provided by *, they lead only to ordinal measurement. However, concatenation is not the only possible source of additional structure; various other kinds of structure can combine with a comparison relation and, if appropriate qualitative laws are satisfied, interval- or ratio-scale fundamental measurement results.

The possibility that nonextensive structures may lead to satisfactory measurement was first widely recognized after the publication by von Neumann and Morgenstern (*2*) of a set of qualitative axioms leading to interval-scale measurement of utility. They assumed a comparison relation $>$, where $a > b$ is interpreted as "a is preferred to b," and a family of combining operations for options a, b: for any probability p, $0 < p < 1$, and any a, b, they assumed that a combined option $[pa, (1-p)b]$ exists, where this is interpreted as a lottery in which option a (which may itself be a lottery) is obtained with probability p and option b with probability $1 - p$. Their axioms for this structure constitute a set of qualitative laws for "rational" decisions among risky options. These qualitative laws imply the existence of a real-valued (utility) function u over the set of all options, such that $a > b$ if and only if $u(a) > u(b)$ and

$$u[pa, (1-p)b] = pu(a) + (1-p)u(b)$$

In recent years, a great many nonextensive structures, with corresponding systems of qualitative laws leading to interval- or ratio-scale measurement, have been studied mathematically (*3–5*). Most of these structures (including von Neumann and Morgenstern's) reduce to, or are closely related to, one particular class of structures, in which there is a comparison relation, $>$, and the objects compared can be regarded as elements of a Cartesian product. That is, the ordinal position of any object, with respect to $>$, depends on the levels of two or more independently controllable (or at least identifiable) factors. Measurement based on such structures requires the simultaneous development of scales for the ordered variable and for the contributions of each factor, and so it has been called simultaneous conjoint measurement (*6*); the empirical relational structures may be termed conjoint structures.

Qualitative Laws in Conjoint Structures: Independence

To illustrate a conjoint structure, let us consider the psychological variable "response strength" and its dependence on three independently controllable factors, amount of food deprivation (a), quality of food reward (b), and degree of previous experience (c). To be even more concrete, consider rats traversing a long straight alley. Assume that the response strengths of the rats are ordered (inversely) by observing the total times required to traverse the alley (*7*). Deprivation (a) may be indexed by percentage of normal body weight, reward (b) by concentration of sucrose solution offered at the end of the alley, and previous experience (c) by number of previous runs through the alley (say, with no deprivation but with sucrose available). The word "indexed" is used because, for purposes of conjoint measurement, only a nominal scale on each factor is required a priori; in the pres-

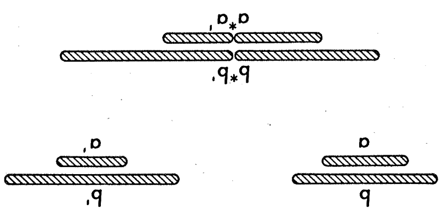

Fig. 1. Illustration of qualitative law underlying additive measurement of length. If rod b is longer than rod a and b' is longer than a', then the end-to-end concatenation of b and b' is longer than the concatenation of a and a'.

ent example, the effect of reduction in body weight on running speed might well be nonmonotonic.

The simplest types of qualitative laws that can be tested in a conjoint structure are independence laws. To illustrate, suppose that four groups of rats are tested in conditions abc, $a'b'c$, abc', and $a'b'c'$, shown in Fig. 2. Assume that a represents more deprivation than a' (in a range where increasing deprivation increases running speed), but b represents less reward than b'. Therefore there will be doubt, a priori, as to whether treatment combination ab or $a'b'$ produces more response strength. Let c, c' be any two different levels of the experience factor. Let the four experimental groups be matched by taking sets of four littermates and randomly assigning them to distinct groups. Now suppose the following results are obtained: most of the rats in group abc traverse the alley in less time than do their littermates in group $a'b'c$; and most in group abc' are faster than their littermates in group $a'b'c'$. We conclude that the combination ab produces more response strength than $a'b'$, independent of the choice of a fixed level of experience factor, c or c'. If the result were replicated for enough different levels of a, b, c, a', b', and c', one would conclude that the ordering of combined deprivation-reward effects, at constant experience, is independent of the level of experience. This qualitative law is formally expressed as follows:

$$abc > a'b'c \text{ if and only if } abc' > a'b'c' \quad (2)$$

The experimental situation of comparing concatenations of rods (Fig. 1) yields the first qualitative law (see Eq. 1), which is the key to construction of an additive length measure. Likewise, the second qualitative law (Eq. 2), tested in conjoint structures by the experimental design of Fig. 2, is the key to additive (or multiplicative) conjoint measurement. In fact, if the second law is valid for a three-factor conjoint structure, and if the two symmetric independence laws hold, by which the ordering of ac, $a'c'$ combinations is independent of the fixed level of b, and the ordering of bc, $b'c'$ combinations is independent of the fixed level of a, then, with the addition of one or two technical assumptions (for example, continuity), one can prove (*8*) that there exists a monotonic transformation of the ordinal dependent variable that decomposes into the sum of the three scales, one for the contribution of each factor. This monotone transformation is unique up to linear transformations (changes of origin and units); so we obtain interval-scale measurement, over the objects compared, and over each factor as well. Sometimes the exponentials of these various scales are used, resulting in a multiplicative combination of the factor effects.

There are other kinds of qualitative laws for conjoint structures, corresponding to more complicated rules of combination of the factor effects. But independence laws are so important that it pays to dwell on them a bit more, and to present a few somewhat disguised examples.

First, note that there are really two sorts of independence laws: single-factor independence, in which the ordering of the effects of one factor is the same, no matter what fixed levels are chosen in all other factors; and joint-factor independence, in which the ordering of the joint effects of two or more factors is independent of the fixed levels of the remaining factors. The second law illustrated joint-factor independence: the ordering of the factor combinations ab, $a'b'$ was considered, for different levels of the remaining factor. Single-factor independence is quite a weak condition: it means only that each factor can be ordered in such a way as to contribute monotonically to the overall effect. In other words, scales can be constructed such that the rule of combination of the factors is monotone in each variable, but no other constraint on the rule is imposed. Often the orderings on single factors are given in advance, by the very operations that define the factors. For example, suppose we compare the momenta of objects that move in a fixed direction. The two relevant factors are mass and speed. But the very operation

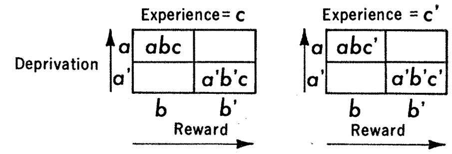

Fig. 2. Illustration of experimental design to test whether the ordering of the joint effect of deprivation and reward is independent of experience. Independence holds when the ordering of ab relative to $a'b'$ is the same at c as at c'.

that defines a nominal scale of mass also defines an ordering, consistent with the ordering obtained by observing momenta, holding speed fixed.

One striking class of violations of single-factor independence is suggestive of multiplicative combination of factors, with positive, zero, and negative multipliers. A reversal in the sign of a multiplier reverses totally the ordering of the products. A zero multiplier produces a degenerate ordering. For an example, consider an experiment in which subjects rate the moral connotation of adverb-adjective combinations, such as "slightly evil." Not surprisingly, "slightly evil" is rated higher than "very evil" but "slightly pleasant" is rated lower than "very pleasant" (*9*). This suggests that scale values of adjectives are both positive and negative and multiply the adverb scale values. When the only violations of independence involve total reversals of ordering or degenerate ordering, we speak of sign dependence; this generalizes the notion of independence (*10*, *11*).

In contrast to single-factor independence, joint-factor independence is a very strong condition. As remarked above, when all possible joint-factor independence laws hold, we obtain additive measurement. Note that in the two-factor case, the only possible independence laws are single-factor ones; so other kinds of laws, of a kind mentioned below, must be studied to determine whether additive measurement is possible.

Example of Independence Laws

The first example is a fairly transparent one: the theory of consumer choice, in economics. The objects compared are commodity vectors $(x_1, \cdots, x_n)$, where x_i is the amount of the ith commodity. The comparison relation is defined by the consumer's choice or preference between competing com-

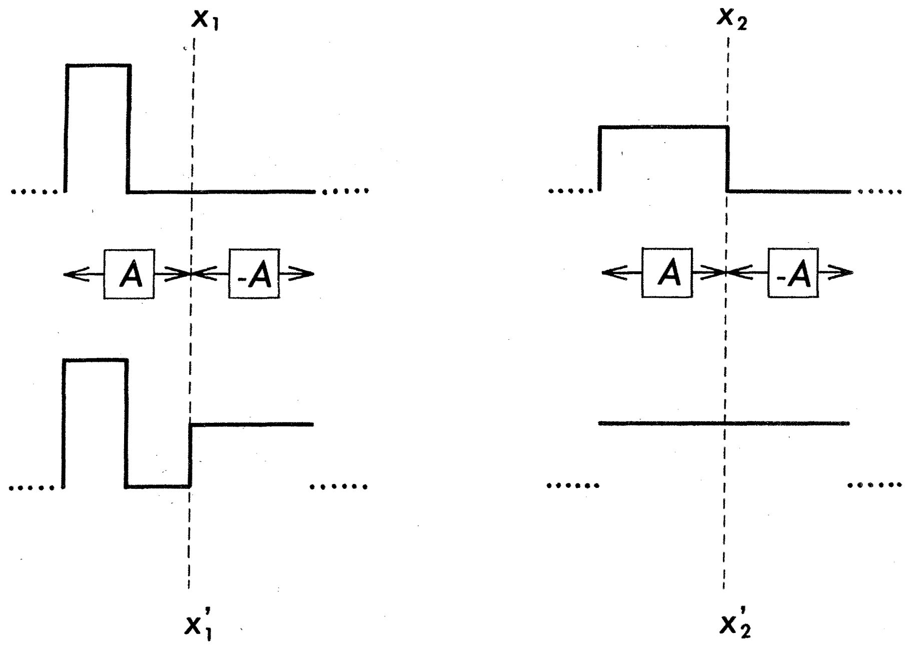

Fig. 3. Inputs x_1 and x_2 coincide in the second half of each period ($-A$). Inputs x_1', x_2' are formed from x_1, x_2 by adding a common input that is zero except in $-A$. According to the generalized superposition law, the output ordering of x_1 and x_2 is the same as that of x_1' and x_2'.

modity vectors available to him. Single-factor independence merely means that amounts of each commodity can be ordered so that it is a good: values higher in the ordering are always preferred to lower ones, the amounts of other commodities being held constant. Joint-factor independence means lack of complementarity between commodities (*12*). Suppose, for example, that the second law (Eq. 2) is violated, with $abc > a'b'c$ and $a'b'c' > abc'$. If a is ordered above a', b' above b, and c above c' (by single-factor independence) then the first and third commodities are complementary, since with more of the third, the consumer also wants more of the first, and is willing to sacrifice some of the second commodity for that end.

The second example concerns a reformulation of the von Neumann and Morgenstern utility theory in terms of conjoint structures (*13, 14*). A risky operation can be regarded as a vector $(x_1, \cdots, x_n)$ where the outcome x_i is obtained if event E_i occurs, and $E_1, \cdots, E_n$ is a set of events, exactly one of which will occur. Joint-factor independence now states that if the result x_i is the same for two risky options, then the value of x_i—good or bad—is disregarded in choosing between the options; the decision-maker focuses on the events E_j, $j \neq i$ and chooses as though one of those events were bound to occur.

This principle is regarded as a normative law of rational decision-making, and is often known as the extended sure-thing principle. It was emphasized by Savage (*13*). As a descriptive law of human choice, it often fails (*15*), but there is little doubt about its value as a normative principle, and it is therefore valuable to teach people to act in accordance with it. For example, contract-bridge teachers tell students that in choosing between two plays, they should treat as irrelevant distributions of opponents' cards for which the two plays will have the same outcome, and attend only to the consequences for card distributions where the two plays may have different outcomes.

A third example arises in testing for linearity of an input-output transformation, where the input may be measurable only on a nominal scale and the output only on an ordinal scale (*16*). For an example, the input $x(\tau)$ could be light intensity, as a function of time τ, and the output $y(t)$ could be neural-firing rate of some visual-system unit, as a function of time t. Usually, linearity is defined by the superposition law: if $x'(\tau) = x(\tau) + \bar{x}(\tau)$, then the corresponding output functions satisfy $y'(t) = y(t) + \bar{y}(t)$. If, however, the input is subject to an arbitrary distortion and the output to a monotone distortion, we can define linearity by a generalized superposition law. Let x_1 and x_2 be two input functions that coincide for some values of τ (see Fig. 3, where x_1, x_2 are periodic inputs that coincide in the second half, designated $-A$, of each period). Let x_1' and x_2' be modifications of x_1 and x_2, where only the common part is modified (see lower half of Fig. 3). The generalized superposition law affirms that for any t, the outputs y_1 and y_2 are ordered in the same way as the modified outputs y_1' and y_2'; that is,

$$y_1'(t) \geqslant y_2'(t) \text{ if and only if } y_1(t) \geqslant y_2(t) \quad (3)$$

This generalizes the usual superposition principle, because modifying x_1 and x_2 to obtain x_1' and x_2' is like adding a "difference input" $\bar{x}$ to both x_1 and x_2, where $\bar{x}$ is restricted to being zero except for values of τ where the original inputs coincide.

To understand why the generalized superposition law is an instance of joint-factor independence, note that each τ defines a factor. Input function x is a vector, with factor-level $x(\tau)$ on the τ factor. The ordering of outputs $y_1(t)$ and $y_2(t)$ depends on the joint effect of all factor-levels $x_1(\tau)$ and $x_2(\tau)$, where these do not coincide. The ordering of $y_1'(t)$ and $y_2'(t)$ depends on exactly the same joint effects. Hence Eq. 3 follows from joint independence.

This formulation leads, for example, to methods for testing linearity of neural processing, where neural-firing rate is taken as only an ordinal measure of the output of a hypothesized linear process.

For a final example, I turn to the psychological literature on impression formation (*17*) and multidimensional psychophysics (*18*). In general, several different factors may contribute to a percept or an impression. The example of adjective and adverb factors contributing to the impression produced by phrases such as "slightly pleasant" was cited above; other examples include impressions of job candidates produced by test scores for intelligence and motivation, and perception of size as a function of visual angle and perceived distance. In each of these cases, one has a fairly obvious conjoint structure, and independence laws may be examined.

Slightly more disguised is the application of qualitative independence laws to multidimensional geometric models

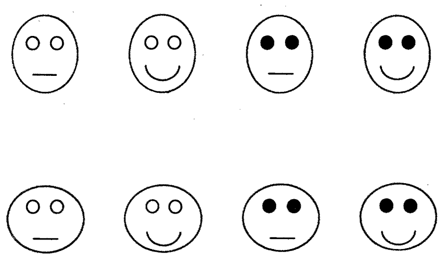

Fig. 4. Eight faces varying on three binary attributes, used to test whether the contributions of the differences on the three attributes contribute additively to overall dissimilarity.

of proximity or dissimilarity. In these models (*18*), one tries to represent stimuli by vectors $x = (x_1, \cdots, x_n)$, $y = (y_1, \cdots, y_n)$, for example, in such a way that stimulus dissimilarities vary monotonically with (Euclidean) distance, that is, with

$$d(x,y) = [\sum_{i=1}^{n} (x_i - y_i)^2]^{1/2}$$

This model postulates, among other things, that an impression of dissimilarity varies monotonically with an additive combination of contributions from the differences along each of several dimensions. This postulate can be studied in isolation, that is, in the context of the much more general model

$$d(x,y) = F[\sum_{i=1}^{n} \varphi_i (x_i, y_i)] \quad (4)$$

where F is an arbitrary monotone function and each φ_i is a function of two (nominal-scale) variables x_i, y_i. This, and other generalizations of the Euclidean model have been studied by conjoint-measurement methods (*19*). Equation 4 leads immediately to joint-factor independence laws: if the dissimilarity produced by some combination of differences on $n-1$ dimensions exceeds that produced by some other combination on those same dimensions (the difference on the nth dimension remaining constant), then that same ordering holds, as the constant difference on the nth dimension is varied.

In a study of dissimilarities of schematic faces, by Tversky and Krantz (*20*), the independence laws were well supported. The stimuli, shown in Fig. 4, were made up with two levels of each of three attributes: long versus wide face, empty versus filled eyes, straight versus curved mouth. Each pair of faces could have either a large difference or zero difference, on each of the three attributes. Figure 5 exhibits a typical test of joint-factor independence. The top two pairs of faces compare the joint effect of large mouth and zero eye difference to the joint effect of zero mouth and large eye difference, at a fixed (zero) level of shape difference; the bottom two pairs compare the same two joint effects, at a different fixed level of shape difference.

Many other examples of conjoint structures and independence laws could be cited; but these four, together with the example for strength of response in the preceding section, may give something of a feeling for the variety of different situations subsumed under the same formal structure.

Other Qualitative Laws in Conjoint Structures

Qualitative laws more complicated than independence laws have been used in the analysis of two-factor conjoint structures (*6*), where independence does not suffice for additive measurement, and in the analysis of nonadditive combination rules for conjoint structures with three or more factors (*11*). Here I present a single example with a very common nonadditive rule, in which the effects of two variables are added and then the sum is multiplied by the effect of the third variable. That is, we have scales φ_1, φ_2, φ_3, such that the scale

$$\varphi(a,b,c) = [\varphi_1(a) + \varphi_2(b)]\varphi_3(c)$$

preserves the ordering of comparisons $>$. In other words,

$$abc > a'b'c' \text{ if and only if } [\varphi_1(a) + \varphi_2(b)]\varphi_3(c) > [\varphi_1(a') + \varphi_2(b')]\varphi_3(c') \quad (5)$$

Note that this hypothesis implies one of the three possible joint-factor independence laws: if we assume that the φ_3 values are all positive, then the ordering of joint effects of the a and b factors is independent of the fixed level in the c factor (Eq. 2). However, it is easy to show that the other two joint-factor independence laws fail.

Consider the experimental design shown in Fig. 6, where there are eight conditions, a 2×2 factorial design at level c of the third factor and another 2×2 factorial at level c'. Suppose that cells 1, 2, 3, 4 in the first 2×2 design are, respectively, compared with cells 1′, 2′, 3′, 4′ in the second one, with the results $1 > 1'$, $2 > 2'$, but $3' > 3$ and $4' > 4$. If we use Eq. 5 these comparisons translate into the following inequalities in φ_i values:

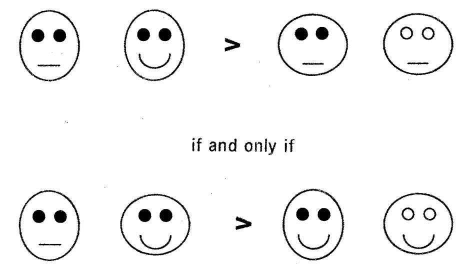

Fig. 5. A test of joint-factor independence for the stimuli of Fig. 3. The comparison of the joint effect of (large mouth difference, zero eye difference) with the joint effect of (zero mouth difference, large eye difference) should yield the same results, regardless of the kind of shape difference (zero or large) that is present.

$$[\varphi_1(a) + \varphi_2(b)]\varphi_3(c) > [\varphi_1(a'') + \varphi_2(b'')]\varphi_3(c')$$
$$[\varphi_1(a') + \varphi_2(b')]\varphi_3(c) > [\varphi_1(a''') + \varphi_2(b''')]\varphi_3(c')$$
$$[\varphi_1(a''') + \varphi_2(b'')]\varphi_3(c') > [\varphi_1(a') + \varphi_2(b)]\varphi_3(c)$$
$$[\varphi_1(a'') + \varphi_2(b''')]\varphi_3(c') > [\varphi_1(a) + \varphi_2(b')]\varphi_3(c)$$

By adding the first two inequalities we obtain

$$[\varphi_1(a) + \varphi_2(b) + \varphi_1(a') + \varphi_2(b')]\varphi_3(c) > [\varphi_1(a'') + \varphi_2(b'') + \varphi_1(a''') + \varphi_2(b''')]\varphi_3(c')]$$

But by adding the second pair of inequalities we produce the contradictory inequality. This shows that Eq. 5 implies a new qualitative law: if $1 > 1'$ and $2 > 2'$, then $3' > 3$ and $4' > 4$ cannot both hold. A similar argument would apply if we started with $1 > 2'$ and $2 > 1'$, or even with $1 > 3'$, $2 > 4'$. Thus we have the following generalization:

> If one diagonal of a 2×2 factorial design, at a fixed level of the third factor, dominates one diagonal of another 2×2 factorial at a different fixed level of the third factor, then the other diagonals cannot have reversed dominance. (6)

The laws shown in Eqs. 2 and 6 are both necessary if scale values satisfying Eq. 5 are to be constructed; however, they are not sufficient to guarantee that such scale values exist. A more complete discussion of these kinds of laws can be found in (*11*) and (*5*). For an empirical example in which Eq. 5 seems to hold, and where Eqs. 2 and 6 were tested to infer this, see Coombs and Huang (*21*). They studied perceived riskiness of games consisting of multiple plays of even-chance two-outcome gambles; the additive factors were the regret or disparity between the two outcomes (*a*) and the expected value per single play (*b*); the multiplicative factor (*c*) was the number of plays per game.

Direct Estimates of Sensation

One of the most impressive bodies of experimental results in psychology has been amassed by S. S. Stevens' technique, which requires observers to assign numbers "in proportion to" their sensations. Particularly important is the consistent prediction of cross-modality matches (*22*), cross-context matches (*23*), and heterochromatic matches (*23, 24*) from such numerical assignments.

What has not been done is to set forth explicitly the empirical relational structure generated by these experiments and the qualitative laws that lead to measurement. Stevens' claim (*25*) that the sone scale of loudness is a prime example of ratio-scale measurement in psychology seems to be based on a mere pun: the instruction to assign numbers "in proportion" to sensation.

To justify Stevens' claim that his "proportion" instructions lead to a ratio scale, note that the requirement to assign numbers "in proportion" demands that the observer judge the presented stimulus relative to previous stimuli. One may hypothesize that the observer orders pairs (a,b) of stimuli with respect to some perceived quality of such pairs. The perceived quality may be called the sensation "ratio" of the pair. The quotation marks indicate that this quality is not actually a ratio, rather, it is a perceptual attribute of stimulus pairs within a single modality. Thus, the assumed relational structure is simply a comparison relation $\succ$ over pairs of stimuli:

> $(a,b) \succ (c,d)$ if the sensation "ratio" produced by a relative to b is at least as great as the sensation "ratio" produced by c relative to d.

This hypothesis leads, therefore, to a two-factor conjoint structure, in which the first factor is the stimulus that is judged and the second one is the standard relative to which it is judged. This formulation is not too different from others that have been given in connection with direct estimation of sensation (*26*), and it was, in essence, suggested by Stevens (*27*), who rejected it on the grounds that it leads only to a logarithmic interval scale, rather than a ratio scale, for sensation measurement.

The conjoint structure for a single sensory modality can be extended, however, to include cross-modality comparisons of sensation ratios, for example, $(a,b) \succ (a',b')$ where a,b come from one modality and a',b' from another. Elsewhere (*28*), I have shown that this extended structure is capable of subsuming Stevens' major results, including the results of numerical estimation and cross-modality experiments, provided that appropriate qualitative laws are assumed. The key qualitative law is the following:

$$\text{If } (a,b) \succ (a',b') \text{ and } (b,c) \succ (b',c'), \text{ then } (a,c) \succ (a',c') \quad (7)$$

[The law shown in Eq. 7 was discussed by Hölder in his 1901 analysis of interval measurement on a line (*1*).] From this law, and some technical assumptions, it can be shown that a measurement scale exists for each modality, such that the order of ratios of scale values predicts within- and cross-modality comparisons of sensation "ratios." With an additional law, one can prove that the measurement scales are correctly given by numerical estimation. Moreover, the scales are indeed ratio scales (unique except for choice of units), except for the possibility of transforming all of them by a fixed power transformation (in effect, changing the numerical response scale by raising it to a power).

The theory just sketched provides an example of the way in which a sufficiently rich body of empirical results can sometimes be recast as a measurement structure, satisfying appropriate qualitative laws. Once this has been done, some of the focus of attention changes. The empirical question, "What is the form of the numerical estimation function as stimulus energy varies?" loses some importance; instead, we ask what is the nature of the underlying quality of pairs that is judged, and what is the relation of such a perceived quality of pairs to sensory processing.

Measurement Structures in Color Perception

Color perception provides the best examples of measurement structures in psychology. In classical trichromatic color measurement, as well as in the empirical observations underlying both the Young-Helmholtz theory of color blindness and the Hurvich-Jameson theory of opponent colors, the structures that arise are not conjoint structures at all, but a quite different sort, which I term Grassmann structures. Other aspects of color perception give rise to a variety of conjoint structures: the attempt to represent color discrimination or similarity in uniform color spaces requires multidimensional scaling structures; the scaling of hue and brightness requires sensation "ratio" structures; the relation of saturation to hue and whiteness exemplifies the $(\varphi_1 + \varphi_2)\varphi_3$ structure discussed above; and there are yet other examples. Furthermore, color perception provides a kind of microcosm of psychology. The empirical relational structure of color mixture and color matching, with its qualitative laws (Grassmann's laws) leading to trichromatic color measurement, is extremely well established; modern measurement analysis just formalizes what Thomas Young knew. The Grassmann structures involved in color blindness and opponent-color perception are very natural and probably correct, but more detailed investigation of their qualitative laws is still needed. In the area of color discrimination and color similarity, there is a great mass of empirical relations, but the appropriateness of measurement analysis is quite uncertain. The idea of representing saturation comparisons by a three-factor composition rule, [red-or-green + yellow-or-blue] × [white-or-black]$^{-1}$, is a bit speculative; we do not really know what the best way of making saturation comparisons is, nor what qualitative laws to expect. Finally, areas such as color memory or esthetics are still preparadigm: there are some good studies, but we have no clear idea what sorts of empirical structures may eventually appear, nor whether they will be measurement structures.

I will now give a brief sketch of the (Grassmann) structures that arise in color matching and in opponent-colors theory.

Trichromatic color measurement is based on metameric matching of lights and on two operations on lights, additive color mixture and scalar multiplication. A light is identified by a curve giving its energy density as a function of wavelength, for wavelengths in the visible electromagnetic spectrum. Two lights a,b are metameric (denoted $a \equiv b$) if they look exactly alike in color (hue, saturation, and brightness). For example, a light with a broad spectrum, such as the output of a tungsten-filament lamp, can be matched exactly by a light with energy concentrated at just two wavelengths (for example, 470 and 580 mm). Two lights a,b can be mixed to form $a \oplus b$, the light that has at each wavelength the sum of the energies of a and b; and a light a can have its energy at each wavelength multiplied by a fixed nonnegative constant t, to form $t * a$. The empirical relational structure consists of the set of lights and the relations $\equiv$, $\oplus$, and $*$; $\equiv$ depends on a human observer, while $\oplus$ and $*$ depend only on physics.

The qualitative laws that involve only $\oplus$ and $*$ are physical laws, for example, $t * (a \oplus b) = (t * a) \oplus (t * b)$. The qualitative laws that also involve $\equiv$ were first stated by Grassmann (*29*). They can be reformulated as follows:

$$\equiv \text{ is transitive, reflexive, and symmetric} \quad (8)$$

$$a \equiv b \text{ if and only if } a \oplus c \equiv b \oplus c; \text{ and if } a \equiv b, \text{ then } t * a \equiv t * b \quad (9)$$

For any four lights, a positive linear combination of two of them or three of them matches a positive linear combination of the remaining two or one, respectively; and there are three lights such that no positive linear combination of any two of them matches the remaining one. (10)

In Eq. 10 we define a positive linear combination of lights, $a_1, \cdots, a_m$ to be an additive mixture of scalar multiples, $(t_1 * a_1) \oplus \cdots \oplus (t_m * a_m)$, where $t_i \geqq 0$ and at least one t_i is strictly positive.

A structure that satisfies both Eqs. 8 and 9 may be termed a Grassmann structure; if Eq. 10 also holds, it is a trichromatic Grassmann structure.

The law shown by Eq. 8 is usually considered to be a test of the adequacy of the empirical operations defining $\equiv$; the law shown by Eq. 9 is interpreted as implying that $\equiv$ is determined at the level of retinal cones, before any nonlinearities in the sensory transduction process; and the law shown by Eq. 10, trichromacy, restricts the number of independent channels, at the level of the cones and above, to three. For a discussion of those empirical laws, see Brindley (*30*).

The measurement consequences of these laws are embodied in the standard systems of color measurement (*31*). There exists a vector-valued measure, $\varphi = (\varphi_1, \varphi_2, \varphi_3)$, defined over lights, such that metameric lights correspond to equal vectors [$a \equiv b$ if and only if $\varphi(a) = \varphi(b)$] and additive mixture and scalar multiplication of lights correspond to ordinary addition of vectors and multiplication by scalars. The measure φ is determined only up to arbitrary changes of linear coordinates, that is, it can be transformed by an arbitrary 3×3 matrix with a nonzero determinant.

Much of color theory can be regarded as attempting to consider additional empirical relations, besides $\equiv$, $\oplus$, $*$, and on the basis of the qualitative laws satisfied by such additional empirical structure, to find a more tightly defined coordinate system that gives a simple representation of all the empirical relations. For example, the empirical structures defined by color-discriminability data have been used for such purposes since Helmholtz (*32*). Such use of color discriminability has only been partially successful, mainly because of our lack of knowledge of the qualitative laws by which small differences on different dimensions or channels combine to determine overall discriminability.

Differences among color theories lie mainly in different choices of which additional empirical relations should be given first consideration. The Hurvich-Jameson opponent-colors theory emphasizes observations that certain lights, viewed under achromatic adaptation and surround conditions, appear neutral with respect to one of the main hue attributes. For example, some lights appear reddish, others greenish, but some are neither reddish nor greenish. This last neutral set includes the "unique" yellows, the "unique" blues, and the achromatic lights. Denote it by C_1. Likewise, there is a set C_2 of unique reds-greens, which are neutral on the yellow-blue attribute. Lights that are in both C_1 and C_2 are achromatic.

Jameson and Hurvich measured the "redness" of a light by the intensity of a standard green needed to just cancel out the red, that is, such that the mixture is in C_1. That is, two lights are equally red (or green) if admixture of the same third light brings them to neutrality. This defines an equivalence relation $\equiv_1$ on lights:

$$a \equiv_1 b \text{ if and only if for some } c, \text{ both } a \oplus c \text{ and } b \oplus c \text{ are in } C_1$$

A yellowness-blueness equivalence relation, $\equiv_2$, may be defined similarly. Their first main empirical result (*33*) is that the measures of redness-greenness and yellowness-blueness obtained by cancellation to C_1 and C_2 neutrality are linear combinations of the trichromatic color-matching functions. It can be shown that this is the same as the assertion that the empirical relations $\equiv_1$, $\oplus$, $*$ and $\equiv_2$, $\oplus$, $*$ yield 1-chromatic Grassmann structures (Eqs. 8, 9, and 10, with 1-chromacy replacing trichromacy) compatible with the trichromatic structure $\equiv$, $\oplus$, $*$. In particular, this requires that the following two qualitative laws hold:

$$\text{If } a \equiv b \text{ and } a \text{ is in } C_1, \text{ then } b \text{ is in } C_1 \text{ (and likewise for } C_2) \quad (11)$$

$$\text{If } a \text{ is in } C_1, t * a \text{ is in } C_1, \text{ and } b \text{ is in } C_1 \text{ if and only if } a \oplus b \text{ is in } C_1 \text{ (and likewise for } C_2) \quad (12)$$

The law shown in Eq. 11 is surely correct. Their results give indirect support to Eq. 12 and there is some direct evidence for it, but a more careful test is needed. It should be remarked in this connection that C_1 and C_2 (like the set of lights that appear chromatic) vary as a function of adaptation and surround color, and that these sets may not satisfy Eq. 12 except with achromatic or near achromatic conditions, such as those used by Jameson and Hurvich.

When these laws hold, it is possible to get a tightly constrained measurement representation $\varphi = (\varphi_1, \varphi_2, \varphi_3)$, where φ gives trichromatic color measurement, φ_1 measures redness-greenness, and φ_2 measures yellowness-blueness. Here, φ_1 and φ_2 are ratio scales. Under conditions where whiteness-matching also yields a 1-chromatic Grassmann structure, φ_3 can be required to measure whiteness on a ratio scale. These measures are based on cancellation and matching, or both, and will not coincide numerically with the results of direct-estimation experiments; the qualitative laws linking direct estimation of hue to cancellation measures remain to be elucidated.

Discussion

The above examples all indicate that measurement of psychological variables is a consequence, rather than a forerunner, of lawfulness. In many experiments, one collects quantitative data, relying on counting, on physical measurement, or on some well established method of psychological measurement.

In such cases, one does not have new measurement of a new psychological variable; one simply has a count, or a measure of a well-known physical or psychological quantity, which is often hypothesized to be correlated with the new psychological variable under investigation. New measurement—that is, a new numerical function—is introduced only to provide a simpler representation of the results of such experiments. When such new measurement has been possible, the data (quantitative or qualitative) have yielded a measurement structure.

For example, consider the Hurvich-Jameson method of measuring redness by canceling via admixture of a standard green light. Their procedure was bound to lead to a number—the mean intensity of the green light at which their observers were satisfied with the adjustment to neutrality. This number is a physical measure of the intensity of the green cancellation light. One can, by fiat, identify this physical measure, as an operational definition of "redness," but to do so would be foolish. Suppose some other green cancellation light led to results that were not proportional to those obtained by the first light; which, then, would measure "redness?" Once one determines that the results are unaffected by the choice of the green cancellation light, one already has lawfulness; indeed, this invariance result is essentially equivalent to the law shown in Eq. 12. Jameson and Hurvich derived their confidence in their measurements from a different manifestation of Eq. 12: the fact that the results were lawfully, indeed linearly, related to color-matching functions. If one were to have no such lawfulness, then blindly stating that one particular set of measurements constitutes the operational definition of "redness" would merely conceal the fact that one had really only measured the physical intensities of some particular green light.

Moreover, once new functions have been introduced, giving a simple representation of lawful empirical relations, intuitive names like "redness" may give way to new names that describe better the nature of the functions introduced. These sharper names correspond to new empirical distinctions, which might never have been discovered if one were content merely to identify a count or a physical measure as an operational measure of a psychological variable.

How much lawfulness is sufficient for psychological measurement? This question has no general answer, of course, but recent theoretical studies in measurement foundations go some way toward answering it. If the empirical observations can be cast in the form of a conjoint structure with three or more factors, then the independence laws are essentially sufficient for additive measurement; or other sufficient sets of laws are known for some kinds of nonadditive measurement. Similarly, Stevens' instructions for "proportional judgment" suggested a generalization of two-factor conjoint structures with the law shown in Eq. 7 as the sufficient condition of ratio-scale measurement. Likewise, knowledge of Grassmann structures permits one to recognize the laws that must hold for the Hurvich-Jameson cancellation procedure to yield a linear transformation of color-matching functions, which measures redness-greenness, and so on. In short, research on measurement structures (*3–5*) has amassed a collection of abstract measurement structures; if data can be made to fit some variant or some extension of these known structures, then various measurement representations with convenient properties can be constructed.

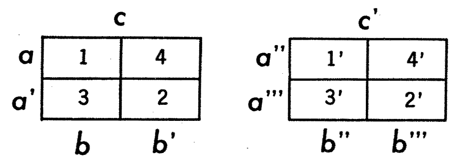

Fig. 6. Experimental design to test a qualitative law that is necessary in a measurement structure where scale values of a and b factors are added, and their sum is multiplied by the c scale value. The law states that if one diagonal of the unprimed (c) matrix dominates one diagonal of the primed (c') matrix, then the other pair of diagonals cannot have reversed dominance (c' dominating c).

The known abstract measurement structures provide a tool that can be powerful, but also perhaps dangerous. The power comes from the discovery of experiments that really should have been done before but were not done. For example, there have been a number of papers in which investigators have attempted to test whether, in the response-strength example given earlier, drive, incentive, and habit combine by $\varphi_1 + \varphi_2 + \varphi_3$ or $(\varphi_1 + \varphi_2)\varphi_3$; one now sees how to do much more decisive experiments (*11*) by testing joint-factor independence and other laws, such as Eq. 6. In general, tests of joint-factor independence are likely to be scientifically fruitful: when they fail, they may fail systematically, in highly interesting ways (the examples of complementary or substitutable commodities were cited above). Likewise, Eq. 7 for relative judgments of stimulus pairs and Eq. 12 for color appearance deserve direct investigation.

The danger of this tool, however, lies in the fact that for many substantive areas of psychology, there is absolutely no reason to expect qualitative laws leading to measurement structures. The goal of scientific psychology is not measurement, but theory. And theory comes in various and unexpected forms.

To take an example from another science: one of the seminal qualitative laws of chemistry was the law of stoichiometric proportions. This law might at first seem to suggest a sort of chemical measure, that is, valence. But the fact that valence is a multivalued and partially periodic function of atomic number leads in a quite different theoretical direction: to the periodic table, and ultimately, to the electron-shell theory.

In fact, biology and chemistry have not produced measurement structures. In these sciences, physical measurement is employed extensively, and qualitative and quantitative laws abound; these laws have, in turn, led to various sorts of theories. One might well expect that many areas of psychology should proceed similarly; and indeed, many of the best established and most lawful areas do so. The idea of measuring psychological variables such as loudness, utility, and intelligence is still very attractive, and in some cases, successful; but a focus that is devoted exclusively to measurement may be sterile. It would seem much more fruitful to look for any kinds of regularities and laws, using known abstract measurement structures merely as a sometimes useful heuristic tool. Occasionally, newly discovered laws may generate novel measurement structures, but more often, other kinds of theory may result.

I have not yet discussed what is actually the largest and best known body of literature on psychological measurement: psychometrics. In all of the various areas of psychometrics, measurement still depends critically on lawfulness. The laws that are involved, however, are mostly much more complicated than any of the ones described above. One postulates a geometric and a statistical model, or both; the law that holds is simply that the model fits the data (with a sufficiently low number of dimensions in the case of geometric models); and the measures are coordinates in the geometric model, or parameters of the hypothetical probability distributions. In other words, the laws in question usually do not have simple statements directly in terms of observable empirical relations; they cannot be summarized in any better way than by saying that such-and-such a model fits the data, in so many dimensions.

When a psychometric model does give a spectacular fit in a low dimensionality, with measurement parameters that make sense and vary appropriately under psychological manipulations of various sorts, the scientific value of the results can scarcely be doubted. The main disadvantages of such an approach are:

1) Because the laws are so complex, failures to fit are both harder to detect (violations may be hidden in random error) and less informative when detected, as compared with tests of the sort of qualitative laws described here.

2) The approach presupposes measurement as the goal, and is therefore even more subject to the dangers of inappropriate theorizing that I have just described above.

The first point can be met, in some cases, by reducing the complex model to a series of simpler qualitative or quantitative laws. This has been partially accomplished for Thurstonian scaling (*34*) and for multidimensional scaling (*19*).

Conclusions

Empirical laws in psychology may be based on physical measurements (for example, voltages, times), counting, ordering, or just classifying. It is a pointless, though widespread practice to use a physical measure or a count as a "definition" of a psychological variable; this practice obscures the fact that all one has done is measured a physical variable, or counted. What is important are the empirical laws that are established by use of such quantitative or qualitative observations. Some kinds of empirical relations and laws yield measurement structures, akin to the qualitative structures underlying fundamental measurement in physics. Measurement structures are empirical structures that can be described most simply by introduction of a new numerical function; such a function is a new measure, and is typically interpreted as measuring some particular psychological variable.

Measurement structures, formulated abstractly, sometimes provide valuable tools for formulating new empirical hypotheses to be tested; but in many instances, other kinds of theory may be more appropriate. The main focus of research ought always to be the discovery of simple laws; these may or may not lead to new measures.

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