Book Reviews

On the Derivation of Space Groups

Symmetry of Crystals. E. S. Federov. Translated from the Russian edition (Moscow, 1949) by David and Katherine Harker. American Crystallographic Association, New York, 1971 (distributor, Polycrystal Book Service, P.O. Box 11567, Pittsburgh, Pa.). x, 316 pp., illus. $25. ACA Monograph No. 7.

Modern crystallographers as well as historians of science can derive considerable profit from a study of this book. It is a translation of a collection of five of the most significant articles of the Russian scientist E. S. Federov (1853–1919), published by the Akademiya Nauk S.S.S.R. in 1949. The appearance of these important portions of Federov's work in English is most welcome.

Though their original publication spanned the decade 1885–1895, the articles form a coherent whole. Together, they clearly display the progress of Federov's thought and work in modifying standard analytic geometry so that crystallographic problems might be more easily handled, in developing the symmetry properties of finite figures by the use of the corrected geometry, in deriving the 230 possible space groups, and in applying his methods to the actual structure of crystals.

The crucial correction in analytic geometry that Federov made was to define the coordinates of a point by the perpendicular projections of that point on the axes of the coordinates, thus making the coordinates of the point independent of the inclination of the other axes. This allowed him to give a convenient notation of the positions of points related by axes of rotational symmetry and forms the basis for his following treatment of point groups and space groups.

Federov is critical of the definition, advanced by Bravais, of the concept of symmetry as a combination of a center, axes, and planes of symmetry, pointing out that Gadolin, in 1869, found a form with sphenoidal symmetry which had neither planes nor a center of symmetry, thus partially vitiating Bravais's definition. Thus, in the study of the symmetry of finite figures, Federov prefers rotations about axes, reflections through planes, and combined rotation-reflections as symmetry operations, and uses a twofold reflection-rotation instead of the operation of inversion through a point. This tactic, David Harker believes, explains "the persistence of rotation-reflection axes in Russian crystallographic literature to this day."

In deriving the 230 possible space groups, Federov, in addition to simple translation, includes the screw axis and the glide plane. Each possible combination of symmetry operations is notated by a set of three equations, which defines the coordinates of an infinite set of points interchanged in the course of the operations. Harker points out that Federov's notations introduce significant economies that might well be an improvement on modern tables, although he notes that Federov's schematic diagrams illustrating the symmetries of the space groups (included in the book) are not as useful as those available today.

The fourth article, "Comparisons of the crystallographic results of Mr. Schoenflies with mine," is the most interesting for the historian. In this Federov traces the different paths by which he and Schoenflies arrived almost simultaneously at the same result—the derivation of the 230 possible space groups. It is clear that Federov felt his definitions and analytic procedures were more elegant than those of Schoenflies, and he believed also that the designation of the space groups by algebraic equations, though more difficult, was superior to the symbolization employed by Schoenflies. In addition, Federov comments favorably on the early work of J. F. C. Hessel, criticizes that of Sohncke (who did not take glide planes into consideration), and presents a table showing the correspondence between the space groups derived by Schoenflies and his own, as well as noting those that were identified by Sohncke.

The final and longest article is also the most difficult, and Harker believes that it can be eminently useful to those scientists who are unacquainted with the Russian crystallographic tradition. Federov assumes that crystals are composed of polyhedra with pairs of mirror equal opposite faces which fill space by being juxtaposed in parallel orientation, an assumption that immediately calls to mind the early crystallographic theory of René Just Haüy. Federov, however, proceeds to derive the possible shapes of the polyhedra, and then divides them into "stereohedra," which are not symmetrical but which can be interchanged through the symmetry operations of the space group in question. He arrives at a total of 1182 stereohedra, which, he states, is the total number of possible crystal structures. In this article, Federov identifies five space groups that he believes cannot correspond to actual structures. Harker notes, however, that each is now known to be represented by several actual crystal structures, and, in the preface, he calls for a solution to the problem as to where Federov erred. W. Nowacki, of the Department of Crystallography at the University of Bern, has clarified the situation in a recent report in *Science* (**174**, 52–53 [1 Oct. 1971]), noting that a solution was given by Federov in an article published in 1900, and also by himself in 1935, in his doctoral dissertation.

A biography of Federov and a bibliography of his scientific work were appended to the Russian volume but were not translated for inclusion in the present book. It is unfortunate that not even a brief biographical sketch is included, inasmuch as Federov, in the introduction to two articles, makes enigmatic references to circumstances that prevented him from devoting his full efforts to his scientific work. However, Cecil Schneer, a member of the editorial board of the *Dictionary of Scientific Biography*, advises that in volume 5 of that publication a biography of Federov (under the spelling "Fyodorov") will appear, which will, it is hoped, provide a good account in English of the career of this important and influential scientist.

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