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24. A volume of distilled water approximating the gram weight of a mature rat is placed in a thick-walled vessel of foamed polystyrene at the ambient temperature of the exposure cavity. The water (which is continuously agitated or stirred) is then irradiated and the average available power is obtained by the formula $w = T_{\Delta}V/kt$, where w is the average available (thermalized) power in watts, T_{Δ} is the temperature increment in degrees Celsius, V is the volume of water in milliliters (weight in grams), k is Joule's conversion factor (0.239), and t is the duration of irradiation in seconds. Temperatures are measured either by expanded-scale spirit thermometers or by electronic thermometers with thermistor sensors.
25. Since the unit-mass (watts per gram) dose is an estimate of absorbed energy, and the conventional unit-surface (watts per square centimeter) dose is a planar index of field density, the two doses cannot be precisely equated. Maximum limiting values of the unit-surface dose can be approximated (*5*) and for the unit-mass doses as given are approximately 20, 15, 7.5, 3.75, and < 2 mw/cm^2.
26. Individual binomial probabilities were derived as follows: (i) The number of responses generated by an animal during an *S* interval was compared to the number generated during the succeeding *W* interval; (ii) if the former number was higher, an instance of cueing was noted; if equal or lower, an instance of no cueing was noted; and (iii) frequencies of positive and negative instances were cumulated across a total set of sessions and evaluated for reliability by use of the binomial theorem.
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28. Frey (*2*) has discussed the practice by some investigators of assuming that irradiation which does not lead to measurable thermalization is ipso facto "nonthermalizing." Such an operational fiat may lead one to overlook the possibility of nonlinear heating of tissues of a biological preparation or of relative insensitivity of thermal measurements.
29. See the discussion by L. D. Sher (*5*, p. 192).
30. The only creature suspected of possessing a specialized sensory-motor apparatus capable of receiving and transmitting microwaves is the corn earworm; see the *J. Microwave Power* **5**, 149 (1970).
31. Supported by 8200 Research Funds from the U.S. Veterans Administration and by contract funds (DADA17-68-C-8021) from the Surgeon General, U.S. Army, to D.R.J. We thank E. L. Wike, C. L. Sheridan, H. F. Fisher, and D. G. Cross for technical advice and criticism.

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Bispectrum Analysis of Electroencephalogram Signals during Waking and Sleeping

Abstract. *The degree of interaction of component waves making up a single electroencephalogram trace was strongly correlated with alpha activity, lead placement, and state of consciousness. Significant quadratic coupling of the waves was found only for awake subjects with high alpha activity. For these subjects about 50 percent of beta activity can be attributed to harmonic coupling with the alpha peak. During sleep, the degree of interaction was of borderline significance and did not follow a consistent pattern with respect to subject, frequency, state, or lead.*

The statistical description of the electrical characteristics of the brain has been greatly advanced by applying the method of time series analysis to electroencephalogram (EEG) records (*1, 2*). With one exception (*3*) the analysis methods have dealt with only second moment statistical quantities—that is, the spectrum and the cross spectrum. As might be expected, the spectrum approach has raised some important questions: for instance, "Is there any coupling (interaction) between the various wave components that make up the EEG trace?" This report supplies a partial answer to this question.

Ordinary spectrum analysis gives no information on the degree of coupling between the (idealized) component waves that make up an EEG record. However, by computing and analyzing the bispectrum (third movement) of the EEG activity, it is possible to discern interacting components. If the bispectrum of the record is nil, we can infer that none of the component waves are related or coupled to each other. Hence the EEG can be considered as a linear superposition of statistically independent component waves. A significant bispectrum, on the other hand, indicates a quadratic coupling (or phase locking) among various of the component waves. In such a situation the bispectrum provides a quantitative measure of the cross coupling. For simplicity, we shall identify this measure by the term "bicoherence," which can be considered as merely a suitably normalized version of the bispectrum. The bicoherence has an expected value of zero for a signal that represents a purely random process. Significant deviations from zero, although small, occur for different spectrum components whose phase and amplitude are quadratically related. A good description of the bispectrum and its properties can be found elsewhere (*4*).

Table 1. Bicoherence values at the frequency coordinates (10 hz, 10 hz). Values above 0.02 are significant at the 95 percent confidence level.

Subject	Lead area		
	F_3	O_1	C_3
1	0.00	0.01	0.01
2*	.03	.02	.04
3*	.09	.22	.08
4	.00	.00	.00
5	.02	.06	.01
6	.01	.01	.00
7*	.09	.13	.09
8*	.14	.12	.22

* Subject with high alpha activity.

Values of bicoherence that were significant at the 95 percent level were found in the awake alpha region for our subjects with high alpha activity (see Table 1). In the awake state, then, the nonzero bispectra appear to be highly correlated with high alpha intensity in the power spectrum. Though significant bicoherence values were found in all leads (electrode sites), their magnitude was dependent on lead placement. The largest bicoherence (R^2) values occurred in the left central (C_3) and left occipital (O_1) leads. Only one of the subjects with low alpha activity had a significant alpha R^2 value, and this was in the O_1 lead.

The waking bispectra, where significant, were characterized by sharp peaks that indicated interactions within narrow frequency bands. Of the observed significant bicoherences, 50 percent occurred in the (10 hz, 10 hz) frequency space. This essentially means that waves with a frequency of 10 hz have associated with them a phase-locked first harmonic component of frequency 20 hz. Of the other significant bicoherences, 20 percent fell in the (10 hz, 20 hz) region, 10 percent in the (10 hz, 30 hz) area, and 20 percent at approximately the (7 hz, 2 hz) region. These results indicate that for subjects with high alpha activity, their alpha activity is composed not only of a pure sine wave of approximately 10 hz but also of its second harmonic (20 hz) and, in some instances, its third harmonic (30 hz). To a much smaller degree, energy in the alpha band appears to be due to interactions between frequencies of approximately 7 hz and 2 hz.

Estimates of the bispectrum can also be manipulated to provide information on the relative phase relation or "biphase" between the interacting waves. In the above four frequency regions the coupled waves were found to be either in phase or totally out of phase. The biphases were both lead and subject dependent with no clear pattern.

Interpretation of our data leads to the conclusion that, during the awake state, part of the spectrum energy in the beta region (20 hz) is harmonically related to the alpha rhythm (10 hz), a conclusion that supports earlier suggestions of such a relationship (*5, 6*). Dumermuth *et al.* (*5*), noting that no beta peaks occurred other than those harmonically related to alpha activity, interpreted this finding as indicating that beta activity was biologically dependent on the alpha activity. Bispectrum analysis allows us to actually estimate the amount of beta energy due to harmonic effects and the amount due to "uncoupled" (statistically independent) generators plus noise. When the background activity is taken into account, approximately one-half of the remaining beta intensity can be described in terms of harmonic coupling with the dominant alpha frequency. It is thus clear that brain activity in the beta region cannot be discussed solely in terms of a single independent beta generator, but neither does it appear to be entirely dependent on the alpha activity.

During sleep, no clear bicoherence pattern emerged, though one or more significant bicoherence values were found in all but one of the subjects. These values, although significant, were all below 0.07, with a modal value of 0.04. They were not related to stage of sleep, but they were dependent upon lead placement, with most significant values occurring in the (left frontal) F_3 and O_1 leads. In contrast to waking, during sleep the bicoherence values for subjects with high alpha activity did not differ from those with low alpha activity.

The character of the wave couplings was also different during sleep. The bispectrum peaks were low and diffuse, indicating that the interactions were between broad, weakly coupled regions in frequency space. The (10 hz, 10 hz) region where the most intense waking activity was found showed little bispectrum activity during sleep. It was only during stages 3 and 4 that any significant pattern appeared to emerge. This pattern indicated broad interactions among frequencies below 7 hz.

In all stages of sleep the biphase estimates, although scattered, were in the neighborhood of 0° or 180°. The biphases were again dependent on both lead and subject.

The bispectrum values were obtained from EEG activity recorded during waking and during all night spontaneous sleep from eight male subjects (age range, 17 to 20 years). While awake with eyes closed, four of the subjects had high alpha activity and four had low alpha activity. The EEG activity was recorded from the eight subjects on both polygraph and frequency-modulated tape recorder at a tape speed of 1.875 inches per second (3.762 cm/sec). The EEG signals were passed through a 0.2- to 50-hz bandpass filter before analog tape recording. The analog tapes were digitized at a rate of 125 samples per second. For the bispectrum analysis, we selected sections of the analog tape corresponding to 66-second, artifact-free periods of the polygraph tracing for waking and for each of the five stages of sleep (*7*) (rapid eye movement and stages 1, 2, 3, and 4). [For details of the data collection system, see (*2*). The linearity of the data collection system was carefully investigated.]

The bispectra $B(f_1,f_2)$ were computed for time series of 8192 points by the fast Fourier transform method. The definition of B in this context is

$$B(f_1f_2) = \langle A(f_1)A(f_2)A^*(f_1+f_2)\rangle$$

where the brackets indicate ensemble averages, the A's are the complex Fourier coefficients of the original time series, the asterisk denotes complex conjugation, and f_1 and f_2 are true frequencies (in hertz). For our computations the frequency resolution was 1.25 hz (*8*). We also require definitions of the bicoherence (R^2) and the biphase (ϕ):

$$R^2(f_1,f_2) = \frac{[B(f_1,f_2)]^2}{\langle [A(f_1)]^2[A(f_2)]^2[A(f_1+f_2)]^2\rangle}$$

and

$$\phi(f_1,f_2) = \tan^{-1}\left[\frac{\mathrm{Im}\ (B)}{\mathrm{Re}\ (B)}\right]$$

With these definitions, $0 \leqslant R^2 \leqslant 1$ and $-\pi \leqslant \phi \leqslant \pi$. In addition to the bispectrum computation on real data, we also made bispectrum estimates on computer-generated, random Gaussian data for control purposes. These controls were complemented and supplemented by theoretical determinations of bispectrum significance (*3, 9*).

The results of our bispectrum analysis of EEG activity have provided a statistical means for quantitatively investigating the quadratic interaction of brain waves. In the interpretation of these results, the problem of possible nonstationarity of the EEG must be recognized (*5, 10*). We have conducted extensive tests to ensure that our basic data are at least quasi-stationary over the lengths of records that have been analyzed.

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