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# Interstellar Scattering of Pulsar Radiation and Its Effect on the Spectrum of NP0532

Abstract. *Angular scattering in the interstellar medium results in multipath dispersion which can amount to more than one pulse period for pulsars of short period and high dispersion measure. The dispersion, if operative, imposes on the pulsation flux a cutoff inversely proportional to the fourth power of the observing wavelength. The low-frequency pulse shape of pulsar NP0532 suggests that this pulsar is subject to such scattering and that the observed low-frequency cutoff in the apparent spectrum is not an intrinsic property of the pulsar. In fact, there is evidence that NP0532 may be identified as the compact, low-frequency source in the Crab Nebula and that the pulsar may radiate in accordance with its high-frequency spectrum down to frequencies as low as 10 megahertz, although the periodic time variations are suppressed by the scattering below 100 megahertz.*

The time and frequency structure of the intensity fluctuations of pulsars is evidence for diffraction arising from irregularities in the density of free electrons in the interstellar medium (*1–3*). Important confirmation of the scintillation mechanism has been provided by recent observations of highly correlated intensity fluctuations over a long base line with time delays appropriate for a diffraction pattern traveling at a few tens of kilometers per second (*4*).

If one analyzes the intensity scintillations, it is possible to deduce the scale of the angular scattering associated with the diffraction (*2, 3*). The angular scattering will, in turn, give rise to single frequency temporal dispersion as a result of multipath propagation. If the temporal dispersion amounts to more than the pulsar period, the pulsation flux will appear to be reduced; the scattering mechanism transfers pulsation flux into continuous flux (of course, the *mean* pulsar flux is not affected by the scattering, which is an energy-conserving phenomenon).

In this report the parameters of the temporal dispersion function and the scattering cutoff function are derived and applied to the pulsar NP0532. The analysis indicates that the spectrum of the pulsar should be severely distorted by the scattering. The possibility that the low-frequency flux of the pulsar may not be cut off at all (contrary to the apparent pulsation flux) is considered. The angular size, flux, and position of the "low-frequency compact source" (*5–7*) in the Crab Nebula, and the excess flux in the overall low-frequency emission of the Crab (*8*), are good indications that the pulsar may radiate in accordance with the spectrum of its high-frequency pulsation flux down to 10 Mhz.

The geometrical relationship between angular scattering and temporal dispersion is depicted in Fig. 1. Radiation from a point source at a distance $L$ from the observer and scattered through an angle $\theta$ (as seen by the observer) at a distance $R$ is delayed relative to unscattered radiation by a time $\tau$:

$$\tau = \alpha R\theta^2/2c \qquad (1)$$

where $\alpha = L/(L-R)$ and $c$ is the free-space speed of light; with $\theta$ in seconds of arc and $L$ and $R$ in kiloparsecs, $\tau = 1.25\ \alpha R\theta^2$ seconds.

From the intensity scintillation of pulsars it is known (*2*) that the scattering is strong, that is, the root-mean-square phase deviation through the medium is much larger than unity. Thus it is a good approximation to take the distribution function for the two-dimensional angular scattering to be Gaussian regardless of the spatial spectrum of the irregularities of the electron density (*9*). If the scattering is circularly symmetric, the two-dimensional distribution of scattered flux, defined as $f'(\theta,\phi)$, may then be written as:

$$f'(\theta,\phi) = \exp(-\theta^2/2\theta_0^2)/2\pi\theta_0^2 \qquad (2a)$$

where $\phi$ is the azimuthal angular coordinate and $\theta_0$ is the root-mean-square scattering angle. Thus the fractional flux between $\theta$ and $\theta + d\theta$ is

$$f(\theta)d\theta = \int_0^{2\pi} f'(\theta,\phi)\theta d\theta\, d\phi = (\theta/\theta_0^2)\exp(-\theta^2/2\theta_0^2)\,d\theta \qquad (2b)$$

where $f(\theta)$ is the radial flux distribution function. From the change of variables given by Eq. 1 the fractional flux arriving, between time $\tau$ and $\tau + d\tau$ after the leading edge of a single pulse has arrived, is:

$$g(\tau)d\tau = (1/\tau_0)\exp(-\tau/\tau_0)d\tau \qquad (3)$$

where $g(\tau)$ is the temporal flux distribution function and $\tau_0 = \alpha R\theta_0^2/c$.

If the emitted radiation is a periodic pulse train, then at some time $\tau$ after the leading edge of the most recent pulse has arrived, an observer will see not only the contribution $g(\tau)$ of the most recent pulse but also the contributions $g(\tau + T)$ of the previous pulse, $g(\tau + 2T)$ of the pulse which preceded it, and so on. Thus the overall temporal distribution is given by $G(\tau)$:

$$G(\tau) = \sum_{n=0}^{\infty} g(\tau + nT), 0 \leqq \tau < T \qquad (4)$$

where $g(\tau + nT)$ is the flux contributed by the $n$th preceding pulse and $T$ is the pulse period. From Eq. 3:

$$\mathrm{G}(\tau) = \exp(-\tau/\tau_0)/\{\tau_0[1-\exp(-T/\tau_0)]\}, \quad 0 \leqq \tau < T \qquad (5)$$

Thus the observed pulse shape should appear as a sharp rise followed by an exponential decay with time constant $\tau_0$; this is precisely what is observed for the pulses of NP0532 at frequencies of $\lesssim$ 150 Mhz (*10, 11*). The scattering parameters may be deduced from the time constant of the pulse decay as:

$$LR\theta_0^2/(L-R) = 0.4\ \tau_0 \qquad (6)$$

Since $\theta_0$ is proportional to $\lambda^2$ [see Ratcliffe (*9*)], $\tau_0$ is proportional to $\lambda^4$.

Pulse energy that has not decayed by the end of the pulse period will appear only as continuous flux. Thus the fraction of mean pulsar flux which appears as continuous flux is $TG(T)$, and the fraction which appears as pulsation flux is $S_p = 1 - TG(T)$:

$$S_p = 1 - \{y\exp(-y)/[1-\exp(-y)]\} \qquad (7)$$

where $y = T/\tau_0$.

According to Eqs. 5 and 7 the reduction of flux may be estimated by measuring the time constant from the average pulse shape $G(\tau)$. The true pulsar flux may then be found as $S$:

$$S = S_{\mathrm{app}}/S_{\mathrm{p}} \qquad (8)$$

where $S_{\mathrm{app}}$ is the apparent pulsar flux. At short wavelengths, $y \gg 1$ and $S_{\mathrm{p}} = 1$; for $y = 1.26$, $S_{\mathrm{p}} = \frac{1}{2}$ and at long wavelengths, $y \ll 1$ and $S_{\mathrm{p}} = y \propto \lambda^{-4}$. Thus the pulsation flux of any pulsar will be sharply cut off at long wavelengths.

For NP0532 the period $T$ is 33 msec (*12*) and the time constant of the exponential pulse decay is 13 msec at 115 Mhz (*10*). Plotted in Fig. 2 is the predicted apparent pulsation flux for NP-0532 computed as $S_{\mathrm{p}} \times S$, where $S$ is the flux extrapolated from the high-frequency ($\gtrsim$ 150 Mhz) spectrum of the pulsar (*13*). The limits on the observed pulsar spectrum are drawn in on the basis of the stated measurement errors for the pulsar flux. The turnover point on the spectrum is consistent with the turnover predicted from the pulse decay time constant at 115 Mhz and the theoretical cutoff function (Eq. 8). However, the predicted apparent flux at 73.8 Mhz is somewhat higher than the measured pulsation flux. The discrepancy can be accounted for by two factors.

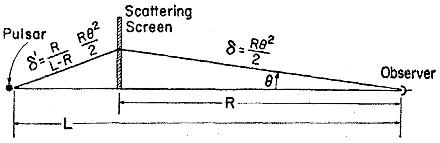

Fig. 1. Geometrical relationship between angular scattering and spatial dispersion (for temporal dispersion, divide by $c = 10^{-11}$ kiloparsec sec$^{-1}$). The excess path length is given for each segment of the scattering trajectory; the total excess is $\alpha\theta^2 R/2$, where $\alpha = L/(L - R)$.

1) The time constant of 13 msec at 115 Mhz was based on measurements of individual strong pulses, whereas the pulsation flux estimates are based on the coherent addition of several thousand pulses. Refractive effects, which manifest themselves as random variations in the time of arrival of pulses, would increase the time constant of the average pulse shape and further reduce the pulsation flux.

Table 1. Flux of compact source, NP0532 pulsation flux, and excess flux. Column 1 gives the observational frequency in megahertz, column 2 the estimated flux (in flux units, that is, $10^{-26}$ watt m$^{-2}$ hz$^{-1}$) of the pulsar or compact source, column 3 the published error in the estimated flux, column 4 the technique, and column 5 the reference for the measured flux. For the estimates of excess flux the error is the root-mean-square sum of the published error in the measured flux and the error in the high-frequency spectrum (*21*) with both the estimated error in the index and that in the reference flux taken into account. The techniques (column 4) used to estimate the flux attributed to either the compact source or the pulsar are: E, excess flux, that is, the difference between the observed overall flux of the Crab Nebula, as given in the reference, and flux as given by an extrapolation of the high-frequency spectrum of the overall flux as given by Long *et al.* (*21*); IPS, interplanetary scintillation analysis; LBI, long ($\geqq$ 7000 $\lambda$) base-line interferometer; SBI, short ($\leqq$ 2000 $\lambda$) base-line interferometer; O, lunar occultation; P, apparent pulsation flux; and T, method based on the time constant of the pulse decay (as discussed in the text).

| $F$ (Mhz) | $S$ (f.u.) | $\Delta S$ (f.u.) | Technique | References |
|---|---|---|---|---|
| 10.0 | 3220 | 950 | E | (*8*) |
| 10.0 | 1150 | 1220 | E | (*22*) |
| 10.0 | 0 | 410 | E | (*23*) |
| 12.6 | 1920 | 810 | E | (*8*) |
| 13.1 | −890 | 770 | E | (*24*) |
| 14.7 | 2160 | 810 | E | (*8*) |
| 16.7 | 800 | 570 | E | (*8*) |
| 20.0 | 290 | 480 | E | (*8*) |
| 25.0 | 710 | 500 | E | (*8*) |
| 26.3 | 800 | 300 | IPS | (*14, 25*) |
| 26.5 | 1070 | 270 | O | (*5, 26*) |
| 38.0 | 380 | 60 | LBI | (*27, 28*) |
| 38.0 | 600 | — | SBI | (*6, 29*) |
| 38.0 | 440 | +160, −90 | IPS | (*15, 17*) |
| 38.0 | 340 | 20 | IPS | (*30*) |
| 40.0 | 280 | 50 | O | (*19*) |
| 60.0 | 300 | 60 | O | (*19*) |
| 73.8 | 4.8 | 3 | P | (*13*) |
| 81.5 | 195 | 30 | IPS | (*15, 31*) |
| 86.0 | 140 | 35 | IPS | (*31*) |
| 111.5 | 19 | 5 | P | (*13*) |
| 115.0 | 1.3 ($S_{\mathrm{p}}$) | — | T | (*10, 32*) |
| 196.5 | 6.4 | 1 | P | (*13*) |
| 210.0 | 70 | 20 | O | (*19*) |
| 404 | < 25 | — | O | (*18*) |
| 408 | 25 | — | SBI | (*5, 26, 33*) |
| 430 | 0.85 | 0.3 | P | (*13*) |
| 535 | 22 | 8 | O | (*34*) |
| 611 | 0.3 | 0.1 | P | (*13*) |
| 1407 | < 0.88 | — | LBI | (*35*) |
| 1422 | 7.5 | 1.6 | SBI | (*26, 29, 36, 37*) |
| 1422 | < 1.5 | — | SBI | (*36*) |

2) The pulse structure of NP0532 is not a simple train of ideal spikes; 66 percent of the pulse energy is in the main pulse, and the remainder is in an interpulse which follows the main pulse by 14.5 msec (*12*). If the details of the structure are considered, the pulsation flux is reduced by another 20 percent at frequencies less than 100 Mhz (*14*). Of course, there is also the possibility that the pulsar is undergoing an intrinsic cutoff.

Now let us consider the apparent angular diameter of the pulsar. From Eqs. 6 and 7 the angular diameter, $\theta_{\mathrm{d}}$ (total half-intensity width), of the pulsar at 115 Mhz should appear to be:

$$\theta_{\mathrm{d}}(115\ \mathrm{Mhz}) = 0.17/(R\alpha)^{1/2}\ \mathrm{arc\ sec} \qquad (9)$$

If we make the usual assumption (*2, 4*) that the irregularity structure of the interstellar medium is uniform between earth and the pulsar, and that therefore the effective location of the scattering screen is halfway between earth and the Crab Nebula ($R = 1$ kiloparsec, $\alpha = 2$), $\theta_{\mathrm{d}}$ (115 Mhz) is equal to 0.12 arc sec. With $\theta_{\mathrm{d}}$ proportional to $\lambda^2$, the angular diameter at 80 Mhz should be 0.24 arc sec—in excellent agreement with the estimate of 0.2 ± 0.1 arc sec for the angular size of the compact low-frequency source in the Crab (*15*). If the identification of the pulsar with the compact source is accepted, then, from the limits on the angular size at 80 Mhz, the analysis may be inverted to show that the *scattering* is more than 0.3 kiloparsec from the pulsar—far removed from the structure of the nebula.

If the angular diameter of the compact source is an artifact of interstellar scattering, it must be proportional to $\lambda^2$ at frequencies below 80 Mhz. From the analysis of interplanetary scintillations (IPS) the angular diameter at 26.3 Mhz has been estimated to be 2 ± 1 arc sec, which is consistent with a $\lambda^2$ variation (*14*). A comparison of IPS observations of the compact source at 38 and 80 Mhz taken by Antonova and Vitkevich (*16*) indicates that the angular diameter at 38 Mhz is about 1 arc sec. (*14*). Bell and Hewish (*15*) estimated the angular diameter to be 0.2

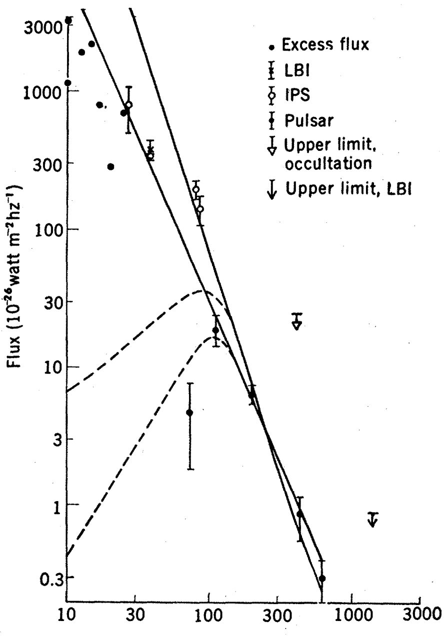

Fig. 2. Measured and predicted fluxes relevant to NP0532. (Solid lines) Limits on the spectrum of the mean flux of NP-0532 determined from measurements at frequencies higher than 200 Mhz and extrapolated to frequencies lower than 200 Mhz. Except for pulsar flux, measurements are explicitly only of continuous flux. (Dashed lines) Limits on predicted apparent pulsation flux, made on the assumption that the limits of mean flux are as given by solid lines, the time constant of pulse decay at 115 Mhz is 13 msec, and the intrinsic pulse structure is a single spike.

arc sec at 38 Mhz but instrumental limitations may have seriously affected their estimate (*17*).

If the compact source is to be identified as the pulsar, its flux must also be consistent with that of an extrapolation of the high-frequency spectrum of the pulsar. The presence of the compact source in the middle of the high flux density of the Crab Nebula demands unusually high instrumental resolution for accurate flux measurements. Thus the techniques of interferometry, IPS analysis, and lunar occultation have all been used with varying degrees of success. The presence in the Crab Nebula of other small-scale (~ 20 arc sec) emission regions (*18, 19*) has resulted in the contamination of flux estimates of the compact source derived from interferometers of insufficient (< 5000 λ base line) resolution. The lunar occultation technique theoretically enables one to scan a source with extremely high spatial resolution (< 5 arc sec), but for the occultations of the Crab Nebula the signal-to-noise ratio has not been high enough to permit such resolution. None of the estimates of the flux of the compact source derived from lunar occultations have been based on the observation of diffraction fringes. Thus the lunar occultation estimates could also be contaminated by small-scale structure other than the compact source. The IPS technique severely discriminates against structure with scale size more than a few seconds of arc, although the analysis of IPS is not as well defined at the frequencies for which the compact source is prominent (< 80 Mhz) as it is at higher frequencies.

There is strong evidence that upward curvature in the low-frequency spectrum (*8*) of the overall flux of the Crab Nebula can be accounted for by the flux contributed by the pulsar in accordance with its high-frequency spectrum. Of course, even if the upward curvature is not as pronounced as that measured by Braude *et al.* (*8*), the high-frequency spectrum of the pulsar can still be accommodated within the overall low-frequency spectrum if there is H II absorption in the interstellar medium (*20*).

Table 1 presents a reasonably complete summary of all measurements of flux attributed to either the compact source or the pulsar. In Fig. 2 only those flux estimates are plotted which are based on long base-line interferometry, pulsation flux, excess flux, and relevant upper limits. The excess flux is merely suggestive; flux estimates based on measurements made with short base-line interferometers and on lunar occultations are not plotted because in all probability these techniques gave an overestimation of the compact source flux.

Interferometric observations at frequencies less than 150 Mhz over base lines up to $10^4/\lambda$ km (where λ is in meters) are of crucial importance in establishing the identity of the compact source as NP0532. In any case there can be no doubt that interstellar scattering is a contributing factor to the spectrum of NP0532.

WILLARD M. CRONYN

*Space Disturbances Laboratory,*
*Environmental Science Services*
*Administration Research Laboratories,*
*Boulder, Colorado 80302*

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