he himself wrote in his *System of Logic* (1843).

Whewell was a polymath who was as much at home delivering sermons at Great St. Mary's in Cambridge (he was a D.D.) as he was giving a Report to the British Association on Mathematical Theories of Electricity, Magnetism and Heat (1835). In his view a liberal education required classics as well as mathematics, by which he also meant science, because "the object of a liberal education is, not to make men eminently learned or profound in some one department, but to educe all the faculties by which man shares in the highest thoughts and feelings of his species. It is to make men truly men, rather than to make them men of genius, which no education can make them."

In Whewell's opinion education does not have a direct role in developing Discoverer's Induction or originality, but it is required for learning the method of sound reasoning that one needs in order to understand science. To be a scientist one needs to continue studying beyond a liberal education to learn the ways of what Whewell called the induction of proof.

HAROLD I. SHARLIN
Department of History, Iowa State University, Ames

A Mathematical Idea

Approximations with Special Emphasis on Spline Functions. Proceedings of a symposium, Madison, Wis., May 1969. I. J. SCHOENBERG, Ed. Academic Press, New York, 1969. xii + 492 pp., illus. $10. Publication No. 23 of the U.S. Army Mathematics Research Center, University of Wisconsin.

When a future historian of mathematics examines the sources of mathematical ideas in the last 25 years he will note that at the Aberdeen Proving Ground in the 1940's there began the development of the modern computer under the influence of von Neumann and the development of the modern theory of splines under the influence of the editor of this volume, I. J. Schoenberg. The U.S. Army continued the support of these ideas and, in particular, splines have been cultivated at the Mathematics Research Center, which also held an earlier Advanced Seminar on the subject. (The proceedings of that seminar were edited by T. N. E. Greville, who has long been connected with splines, originally from the actuarial point of view.) All concerned with the symposium of which the present book is a record deserve praise for the speed of publication.

It is tempting to conjecture that Peter the Great was concerned with (physical) splines in connection with naval architecture, but we have no evidence for this. A spline was, originally, the flexible ruler threaded between lead wedges with vertices placed at points on a drawing surface which had to be joined by a "smooth" curve. The theory of thin beams, due to Euler, shows that the spline takes a shape which minimizes $\int (y'')^2 dx$, the integral of (an approximation to) the square of the curvature, and this is a reasonable definition of smoothness.

The classical numerical analyst uses polynomials (algebraic or trigonometric, as appropriate) as the basis of his approximations, whether for interpolation, quadrature, or the solution of differential equations. It has been found that considerable advantages accrue if *piecewise* polynomials are used—these are the (mathematical) splines. In the simplest case, given a function $f(x)$, defined on the interval [0,1], and a set of nodes $0 \leq x_0 < x_1 < \cdots < x_n \leq 1$, a *spline* will be a function which is a polynomial of an assigned degree (in practice, cubics are usual) in each subinterval $x_i \leq x < x_{i+1}$ and which agrees with $f(x)$ (and perhaps certain of the derivatives agree too) at each node. The success of the approximation will depend on the distribution of the nodes and the degree of the polynomials.

One place where the advantage of splines is apparent is the following: if they are used as a basis instead of polynomials in the discretization of differential equations, the resulting system of linear equations may have a matrix which is sparse, and therefore convenient to handle on a computer.

The basic idea has been widely generalized. It is natural to consider many-dimensional splines: these have been proved to be of practical value in the shaping of automobile bodies by the General Motors Corporation and in engine design at the United Aircraft Corporation. In another direction, the polynomials may be replaced by other functions, for example trigonometric polynomials.

The editor has expressed the hope that the papers in this volume are beautiful, useful, or both. While this hope is realized, there is no doubt that they are highly technical. There is not so far available a genuine introduction to the theory, although there are several excellent expository articles. Such an introduction could be very successful, for there is available a considerable body of elegant yet quite elementary material accessible to those with a modest background in calculus and linear algebra; in addition, the exploitation of splines in practical computation is still in an early stage and full of promise.

JOHN TODD
Mathematics Department, California Institute of Technology, Pasadena

Physical Formulations

Elementary Particle Theory. Relativistic Groups and Analyticity. Proceedings of the eighth Nobel Symposium, Aspenäsgården, Lerum, Sweden, May 1968. NILS SVARTHOLM, Ed. Interscience (Wiley), New York, and Almqvist and Wiksell, Stockholm, 1969. 400 pp., illus. $31.75.

Groups and analyticity in elementary particle physics, the subject of this Nobel Symposium, are topics which have been studied in an attempt to circumvent the mathematical intractability of older formulations: the quantum field theory of Dirac and Heisenberg or the *S*-matrix bootstrap of Chew. Investigations in this area have the added attraction of being to a very large extent independent of the specific dynamics which govern fundamental particle processes. Thus powerful, general results can be obtained which are not too severely limited by our ignorance of the detailed nature of all the fundamental interactions except electromagnetism.

Several of the contributed papers summarize work which has elucidated the rather subtle constraints that a crossing symmetric scattering amplitude must satisfy when requirements of analyticity are conjoined with those imposed by the Poincaré group. Especially satisfying is the review by Domokos, who shows how at zero momentum transfer the Regge pole exchange terms must be arranged so that they "conspire" to satisfy these requirements. Furthermore, away from the forward direction, the O_4 group, which was a symmetry at that point, can be assumed to be broken in a well-defined and simple fashion. Such assumptions yield formulas for Regge trajectories which can be used to predict the positions of many resonances. These theoretical predictions have encouraging experimental support, although further investigation is still necessary, particularly in connection with the parity doublets which seem to be required by the theory. It is worth noting that subsequent research has established these results in a much more general context than the Bethe-Salpeter model used by Domokos.

In an application of analyticity and unitarity, Martin summarizes the large body of work which derives bounds on two-particle scattering amplitudes. These bounds have direct experimental content; but more importantly they place a check on model building, since many approximations to the scattering amplitude violate them. Rigorous results in field-theoretic model building are presented by Hepp. By a development of new mathematical tools which can handle the very singular operations that one is invariably led to in quantum field theory, it has been possible to prove the existence of nontrivial relativistic models. Such investigations provide a convenient theoretical laboratory; however, the relevance of the results to physics is not clear at the present time. Only very unrealistic interactions in one or two space dimensions have been analyzed so far, and after many difficult intermediate steps Hepp concludes that the results concerning the nature of divergences coincide with those of perturbation theory. As is pointed out by Thirring in subsequent discussions, the likely reason for this uninformative coincidence is that none of the models considered need infinite charge renormalization.

The second announced topic of the conference, groups, is somewhat underrepresented in these proceedings. Some studies concerning the origin of strong SU(3) breaking are reported. Cabibbo and Pais attempt to arrive at strong breaking in a "spontaneous" fashion; the only breaking effects that are explicitly present are the weak and electromagnetic forces, which one hopes become dynamically enhanced. On the other hand, Ne'eman proposes an explicit symmetry-breaking interaction. These investigations have not as yet produced satisfying results, and it is unfortunate that the very persuasive scheme for SU(3) $\times$ SU(3) breaking due to Glashow and Weinberg, as well as that of Gell-Mann, Oakes, and Renner, is not discussed.

The conference was held at an unfortunate moment, in that much of the contemporary activity in high-energy theory derives from observations which were made, it seems, only a few weeks after the close of the proceedings. Thus the published report is somewhat dated: it is not recognized in the discussion of current algebra that local commutators are considerably more model-dependent than had been assumed, nor is there any mention of the extremely important concept of duality in hadron physics which attempts to connect high-energy and low-energy phenomena. Indeed, the many articles in the book concerning infinite-component field theory achieve significance only now, in connection with duality.

The organizers of the conference might have avoided this common problem in a rapidly changing field if they had included more discussions which were rooted in physical fact. Michel's elegant review of neutral kaon physics and Gell-Mann's summary of the symposium stand out in their frequent reference to experiment. The absence of many other contributions in this style, the datedness of the theoretical emphasis, and the high price of the book make it difficult for me to recommend its purchase for purposes other than those of record.

ROMAN JACKIW
Department of Physics,
Massachusetts Institute of Technology, Cambridge

A Mostly Empirical Science

Surface and Colloid Science. EGON MATIJEVIĆ and FREDERICK R. EIRICH, Eds. Interscience (Wiley), New York, 1969. Vol. 1, viii + 264 pp., illus.; vol. 2, viii + 304 pp., illus. $14.95 each.

It is obvious to the most casual observer that academic colloid and surface science has suffered a considerable decline in importance over the past 20 years. With a few exceptions, course offerings on the subject have been discontinued; related topics in general physical chemistry courses have been deemphasized or eliminated altogether; and hiring practices have been such as to cause a considerable decrease in the number of active academic researchers in this field. It is likely that this phenomenon is due primarily to the feeling that colloid and surface science is short on fundamental principles and long on messy problems that yield only empirical rules of doubtful validity. However, changing patterns in research emphasis and funding may well presage a resurgence of activity in this practically important area.

The articles contributed to volumes 1 and 2 of the treatise being assembled by Matijević and Eirich have a dual function: they help dispel the illusion that problems involving surfaces and small particles are not susceptible to rigorous analysis, and they will be of great utility to workers coming into this field as well as to those who are already in it.

The editors intend the treatise to comprise a text rather than an "Advances" series. As one might expect, this is achieved with varying degrees of success. After a truly excellent beginning in the form of an article on the thermodynamics of fluid interfaces by F. C. Goodrich, the remainder of volume 1 comprises an extended discussion of the theory and measurement of surface tension by Padday. Although Padday's treatment of the theory is naive and somewhat outdated, the section on experimental techniques is authoritative and complete and is accompanied by a number of valuable numerical tables relating the size and shape of sessile and pendant drops to surface tension. As Princen points out in volume 2, all existing methods of measuring surface or interfacial tension rely ultimately on the analysis of interfacial shapes. Princen's discussion of the analytic expressions for the shapes of cylindrical and axially symmetric interfaces combines nicely with the tabular results in volume 1 for drops to give comprehensive coverage of interfacial shapes for systems of practical importance. Volume 2 also contains articles on wettability and contact angles by Johnson and Dettre; adsorption of solutions of nonelectrolytes by Schay; and aerosols by Hidy. After a skimpy treatment of the fundamentals, Johnson and Dettre give an excellent summary of present knowledge concerning advancing and receding contact angles for liquid drops on solid surfaces and the interpretation of these data. This article provides a particularly good example of the colloid chemists' ability to deduce interesting and useful conclusions from what might appear to be nonreproducible and poorly defined experiments. Schay's article on the adsorption of solutions comprises a review of the relevant thermodynamics followed by a description of the various types of adsorption isotherms encountered in practice. In its approach to the subject, this article typifies the treatise: strong on thermodynamics and authoritative in its discussion of experiment, but weak on theory and interpretation