he himself wrote in his *System of Logic* (1843).

Whewell was a polymath who was as much at home delivering sermons at Great St. Mary's in Cambridge (he was a D.D.) as he was giving a Report to the British Association on Mathematical Theories of Electricity, Magnetism and Heat (1835). In his view a liberal education required classics as well as mathematics, by which he also meant science, because "the object of a liberal education is, not to make men eminently learned or profound in some one department, but to educe all the faculties by which man shares in the highest thoughts and feelings of his species. It is to make men truly men, rather than to make them men of genius, which no education can make them."

In Whewell's opinion education does not have a direct role in developing Discoverer's Induction or originality, but it is required for learning the method of sound reasoning that one needs in order to understand science. To be a scientist one needs to continue studying beyond a liberal education to learn the ways of what Whewell called the induction of proof.

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A Mathematical Idea

Approximations with Special Emphasis on Spline Functions. Proceedings of a symposium, Madison, Wis., May 1969. I. J. SCHOENBERG, Ed. Academic Press, New York, 1969. xii + 492 pp., illus. $10. Publication No. 23 of the U.S. Army Mathematics Research Center, University of Wisconsin.

When a future historian of mathematics examines the sources of mathematical ideas in the last 25 years he will note that at the Aberdeen Proving Ground in the 1940's there began the development of the modern computer under the influence of von Neumann and the development of the modern theory of splines under the influence of the editor of this volume, I. J. Schoenberg. The U.S. Army continued the support of these ideas and, in particular, splines have been cultivated at the Mathematics Research Center, which also held an earlier Advanced Seminar on the subject. (The proceedings of that seminar were edited by T. N. E. Greville, who has long been connected with splines, originally from the actuarial point of view.) All concerned with the symposium of which the present book is a record deserve praise for the speed of publication.

It is tempting to conjecture that Peter the Great was concerned with (physical) splines in connection with naval architecture, but we have no evidence for this. A spline was, originally, the flexible ruler threaded between lead wedges with vertices placed at points on a drawing surface which had to be joined by a "smooth" curve. The theory of thin beams, due to Euler, shows that the spline takes a shape which minimizes $\int (y'')^2 dx$, the integral of (an approximation to) the square of the curvature, and this is a reasonable definition of smoothness.

The classical numerical analyst uses polynomials (algebraic or trigonometric, as appropriate) as the basis of his approximations, whether for interpolation, quadrature, or the solution of differential equations. It has been found that considerable advantages accrue if *piecewise* polynomials are used—these are the (mathematical) splines. In the simplest case, given a function $f(x)$, defined on the interval [0,1], and a set of nodes $0 \leqq x_0 < x_1 < \cdots < x_n \leqq 1$, a *spline* will be a function which is a polynomial of an assigned degree (in practice, cubics are usual) in each subinterval $x_i \leqq x < x_{i+1}$ and which agrees with $f(x)$ (and perhaps certain of the derivatives agree too) at each node. The success of the approximation will depend on the distribution of the nodes and the degree of the polynomials.

One place where the advantage of splines is apparent is the following: if they are used as a basis instead of polynomials in the discretization of differential equations, the resulting system of linear equations may have a matrix which is sparse, and therefore convenient to handle on a computer.

The basic idea has been widely generalized. It is natural to consider many-dimensional splines: these have been proved to be of practical value in the shaping of automobile bodies by the General Motors Corporation and in engine design at the United Aircraft Corporation. In another direction, the polynomials may be replaced by other functions, for example trigonometric polynomials.

The editor has expressed the hope that the papers in this volume are beautiful, useful, or both. While this hope is realized, there is no doubt that they are highly technical. There is not so far available a genuine introduction to the theory, although there are several excellent expository articles. Such an introduction could be very successful, for there is available a considerable body of elegant yet quite elementary material accessible to those with a modest background in calculus and linear algebra; in addition, the exploitation of splines in practical computation is still in an early stage and full of promise.

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Physical Formulations

Elementary Particle Theory. Relativistic Groups and Analyticity. Proceedings of the eighth Nobel Symposium, Aspenäsgården, Lerum, Sweden, May 1968. NILS SVARTHOLM, Ed. Interscience (Wiley), New York, and Almqvist and Wiksell, Stockholm, 1969. 400 pp., illus. $31.75.

Groups and analyticity in elementary particle physics, the subject of this Nobel Symposium, are topics which have been studied in an attempt to circumvent the mathematical intractability of older formulations: the quantum field theory of Dirac and Heisenberg or the *S*-matrix bootstrap of Chew. Investigations in this area have the added attraction of being to a very large extent independent of the specific dynamics which govern fundamental particle processes. Thus powerful, general results can be obtained which are not too severely limited by our ignorance of the detailed nature of all the fundamental interactions except electromagnetism.

Several of the contributed papers summarize work which has elucidated the rather subtle constraints that a crossing symmetric scattering amplitude must satisfy when requirements of analyticity are conjoined with those imposed by the Poincaré group. Especially satisfying is the review by Domokos, who shows how at zero momentum transfer the Regge pole exchange terms must be arranged so that they "conspire" to satisfy these requirements. Furthermore, away from the forward direction, the O_4 group, which was a symmetry at that point, can be assumed to be broken in a well-defined and simple fashion. Such assumptions yield formulas for Regge trajectories which can be used to predict the positions of many resonances. These theoretical pre-