The Logic of Discovery

William Whewell's Theory of Scientific Method. ROBERT E. BUTTS, Ed. University of Pittsburgh Press, Pittsburgh, 1969. xii + 360 pp. $8.95.

This volume will contribute to the growing appreciation by historians and philosophers of science of William Whewell. Whewell serves well as an introduction to 19th-century thought, and in that way to thought today, because he was one of the few scientists of his time who explicitly examined the accepted scientific method. He can be taken as a spokesman for his scientific era. Not all of his contemporaries agreed with him, but he shared with the other scientists of the time a concern with the investigation, proof, and testing of hypotheses.

Whewell was one of the few scientists in Britain whose personal influence extended beyond scientific circles. There were other men, Tyndall and Huxley, for example, who were better known, but these men were primarily interpreters or popularizers of science. Whewell demonstrated what scientific theory meant in morals, religion, education, and philosophy. The debate he carried on with John Stuart Mill exemplifies problems that arise in attempts to extend scientific ideas to other spheres of activity. Whewell the scientist objects to the misapplication and misunderstanding of his ideas, whereas Mill the interpreter has no intention of direct application of those ideas but is trying to trace their social or philosophical implications.

The book organizes selections from Whewell's writings in such a way as to present his views as they developed over the years. Butts has chosen for inclusion essays that expound what he considers the central thesis of Whewell's philosophy of science, Whewell's theory of necessary truth and his theory of induction. Butts also provides an introduction to the selections. In this introduction Whewell the scientist-turned-philosopher is compared with Kant, Descartes, Hume, and other philosophers who have been interested in science. Regarded as an account of the strict and formal reasoning of philosophy Whewell's position is found "somewhat puzzling." If Whewell is to be understood, Butts argues, he must be seen as trying to explain how scientific investigation is conducted rather than as trying to construct a system of philosophy. Such ideas as Whewell's intuition of necessary truth (a necessary truth being one the contrary of which cannot be conceived "distinctly") "had a quite concrete historical meaning" stemming from Whewell's view of the progressive nature of science. Changes in the methods and goals of science are better understood in Whewell's terms, that is, by historical rather than by analytical dissection.

It is Whewell's view of science as a historical process that accounts for his ideas about induction that philosophers found so objectionable. Whewell invented the Inductive Table, according to which the evolution of theories is depicted as a progression from lower-order ideas to higher-order ones. The lower order of observations can be affirmed deductively from the higher order of generalizations. For example, in Whewell's Inductive Table of Astronomy the highest generalization is Newton's theory of universal gravitation and at the opposite end of the table is the observation that eclipses of the sun and moon often occur. Some objected that the table did not fit the logic of induction but Whewell, Butts says, thought of the logic of induction as being the logic of discovery as much as the logic of proof. In the formal, the Aristotelian, employment of logic those who agree to the rules of induction will always arrive at the same conclusion given the same premises. What Whewell was trying to show was the way a scientist arrives at a hitherto unknown idea by means of induction. This induction of discovery is not predictable but contains original thought sometimes labeled, for want of a better word, intuition. *Afterwards* the new theory is demonstrated to other scientists by means of the rigid rules of logic and is tested according to standard rules of deduction. In other words, a scientist uses one system of reasoning in arriving at a theory and another system of reasoning in convincing others of the rightness of the theory.

The selections in *William Whewell's Theory of Scientific Method* exhibit the wide range of Whewell's thought in one of his several areas of interest. Butts's organization is sensible, and the selections all tie together. The book requires concentration, and the editor's introduction offers insight but is no easy key. Although Whewell's theory deals with the scientific method generally, it offers such an intensive analysis of the ways of thought that more than once I found myself having to reread paragraphs. Of all the selections I gained the most from "Of the transformations of hypotheses in the history of science." Perhaps this article is of more interest to historians of science than to scientists, but it addresses a question I think all scientists need to consider—why men of the past have resisted evidence of a new theory with, as Whewell put it, "a degree of obstinancy and captiousness which now appears to us quite marvellous." "It cannot but seem strange . . .," Whewell wrote, "that in a matter which depends upon mathematical proofs, the whole body of the mathematical world should pass over . . . from an opinion confidently held, to its opposite." What happens, he contended, is that "rival theories pass into one another. . . . And thus, when different and rival explanations of the same phenomena are held, till one of them, though long defended by ingenious men, is at last driven out of the field by the pressure of facts, the defeated hypothesis is transformed before it is extinguished."

There has been no full-scale biography of Whewell since 1881, to my knowledge, and a new one is needed. In the present work Butts gives a very brief account of Whewell's life. Master of Trinity College for 25 years, Whewell was one of the most powerful men at Cambridge during the period of educational reform. His influence as a scientist was primarily through his writing. In 1819, at age 25, he published *An Elementary Treatise on Mechanics*, which was the first English textbook to make use of the calculus. The work went through seven editions. He received a Royal Society medal for his work on tides and furnished Faraday with the nomenclature he wanted for his work in electricity. The terms "anode," "cathode," "ion," "anion," and "cation" are due to Whewell. It was his writings on the history, philosophy, and implications of science that made Whewell widely known, and these books also helped make science of wide public interest. He was asked to write one of the Bridgewater Treatises, which he called *Astronomy and General Physics Considered with Reference to Natural Philosophy* (1833). His *Thoughts on the Study of Mathematics as a Part of a Liberal Education* was published in 1835. The very popular *History of the Inductive Sciences* came out in 1837, followed by *The Philosophy of the Inductive Sciences* in 1840. The *History* and the *Philosophy* were credited by Mill as being the basis for much of what he himself wrote in his *System of Logic* (1843).

Whewell was a polymath who was as much at home delivering sermons at Great St. Mary's in Cambridge (he was a D.D.) as he was giving a Report to the British Association on Mathematical Theories of Electricity, Magnetism and Heat (1835). In his view a liberal education required classics as well as mathematics, by which he also meant science, because "the object of a liberal education is, not to make men eminently learned or profound in some one department, but to educe all the faculties by which man shares in the highest thoughts and feelings of his species. It is to make men truly men, rather than to make them men of genius, which no education can make them."

In Whewell's opinion education does not have a direct role in developing Discoverer's Induction or originality, but it is required for learning the method of sound reasoning that one needs in order to understand science. To be a scientist one needs to continue studying beyond a liberal education to learn the ways of what Whewell called the induction of proof.

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A Mathematical Idea

Approximations with Special Emphasis on Spline Functions. Proceedings of a symposium, Madison, Wis., May 1969. I. J. SCHOENBERG, Ed. Academic Press, New York, 1969. xii + 492 pp., illus. $10. Publication No. 23 of the U.S. Army Mathematics Research Center, University of Wisconsin.

When a future historian of mathematics examines the sources of mathematical ideas in the last 25 years he will note that at the Aberdeen Proving Ground in the 1940's there began the development of the modern computer under the influence of von Neumann and the development of the modern theory of splines under the influence of the editor of this volume, I. J. Schoenberg. The U.S. Army continued the support of these ideas and, in particular, splines have been cultivated at the Mathematics Research Center, which also held an earlier Advanced Seminar on the subject. (The proceedings of that seminar were edited by T. N. E. Greville, who has long been connected with splines, originally from the actuarial point of view.) All concerned with the symposium of which the present book is a record deserve praise for the speed of publication.

It is tempting to conjecture that Peter the Great was concerned with (physical) splines in connection with naval architecture, but we have no evidence for this. A spline was, originally, the flexible ruler threaded between lead wedges with vertices placed at points on a drawing surface which had to be joined by a "smooth" curve. The theory of thin beams, due to Euler, shows that the spline takes a shape which minimizes $\int (y'')^2 dx$, the integral of (an approximation to) the square of the curvature, and this is a reasonable definition of smoothness.

The classical numerical analyst uses polynomials (algebraic or trigonometric, as appropriate) as the basis of his approximations, whether for interpolation, quadrature, or the solution of differential equations. It has been found that considerable advantages accrue if *piecewise* polynomials are used—these are the (mathematical) splines. In the simplest case, given a function $f(x)$, defined on the interval [0,1], and a set of nodes $0 \leqq x_0 < x_1 < \cdots < x_n \leqq 1$, a *spline* will be a function which is a polynomial of an assigned degree (in practice, cubics are usual) in each subinterval $x_i \leqq x < x_{i+1}$ and which agrees with $f(x)$ (and perhaps certain of the derivatives agree too) at each node. The success of the approximation will depend on the distribution of the nodes and the degree of the polynomials.

One place where the advantage of splines is apparent is the following: if they are used as a basis instead of polynomials in the discretization of differential equations, the resulting system of linear equations may have a matrix which is sparse, and therefore convenient to handle on a computer.

The basic idea has been widely generalized. It is natural to consider many-dimensional splines: these have been proved to be of practical value in the shaping of automobile bodies by the General Motors Corporation and in engine design at the United Aircraft Corporation. In another direction, the polynomials may be replaced by other functions, for example trigonometric polynomials.

The editor has expressed the hope that the papers in this volume are beautiful, useful, or both. While this hope is realized, there is no doubt that they are highly technical. There is not so far available a genuine introduction to the theory, although there are several excellent expository articles. Such an introduction could be very successful, for there is available a considerable body of elegant yet quite elementary material accessible to those with a modest background in calculus and linear algebra; in addition, the exploitation of splines in practical computation is still in an early stage and full of promise.

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Physical Formulations

Elementary Particle Theory. Relativistic Groups and Analyticity. Proceedings of the eighth Nobel Symposium, Aspenäsgården, Lerum, Sweden, May 1968. NILS SVARTHOLM, Ed. Interscience (Wiley), New York, and Almqvist and Wiksell, Stockholm, 1969. 400 pp., illus. $31.75.

Groups and analyticity in elementary particle physics, the subject of this Nobel Symposium, are topics which have been studied in an attempt to circumvent the mathematical intractability of older formulations: the quantum field theory of Dirac and Heisenberg or the *S*-matrix bootstrap of Chew. Investigations in this area have the added attraction of being to a very large extent independent of the specific dynamics which govern fundamental particle processes. Thus powerful, general results can be obtained which are not too severely limited by our ignorance of the detailed nature of all the fundamental interactions except electromagnetism.

Several of the contributed papers summarize work which has elucidated the rather subtle constraints that a crossing symmetric scattering amplitude must satisfy when requirements of analyticity are conjoined with those imposed by the Poincaré group. Especially satisfying is the review by Domokos, who shows how at zero momentum transfer the Regge pole exchange terms must be arranged so that they "conspire" to satisfy these requirements. Furthermore, away from the forward direction, the O_4 group, which was a symmetry at that point, can be assumed to be broken in a well-defined and simple fashion. Such assumptions yield formulas for Regge trajectories which can be used to predict the positions of many resonances. These theoretical pre-