Cross-Correlation and Cross-Spectral Methods for Drift Velocity Measurements

Abstract. *Cross-correlation and cross-spectral methods are used in a laboratory modeling experiment to compute the drift velocity of a structure which undergoes internal rearrangement as it drifts in a uniform direction. Emphasis is placed on the problems of making such measurements in remote probing experiments, particularly in the study of irregularities in the solar wind plasma by radio star scintillation techniques, where the number of observing sites is limited to two or three.*

Scintillation techniques are becoming increasingly important as tools in the study of irregular structure in the ionospheric and interplanetary plasmas (*1*), in the neutral atmosphere, and in the oceans. Scintillation can also be used to deduce information about the angular structure of the radiation source (*2*).

The general availability of modern high-speed digital computers has contributed greatly to the usefulness of such methods, which by their very nature involve massive statistical processing of the data. Because many of the scintillation geometries of practical interest have not been fully treated theoretically, and because the various signal-processing techniques which are available for this work need direct evaluation, a laboratory modeling experiment for the scintillation process has been undertaken. This experiment was prompted by the work of Bartusek and Felgate (*3*). Figure 1 shows a general schematic drawing of the experiment. An ultrasonic sound source illuminates a diffracting screen of warm turbulent air. The resulting scintillation signal is probed with ultrasonic microphones in the region behind the diffracting screen. In addition, direct measurements of the screen structure are made with resistance wire thermometers (*4*). All the experimental outputs are digitized and sent on-line into a small laboratory computer to allow for the fullest application of digital data processing techniques.

As a preliminary exercise in this modeling experiment, two of the available techniques for velocity measurement, cross-correlation and cross-spectral analysis, were used to compute the drift velocity of the turbulent screen. The measurements were made directly in the screen turbulence with the resistance wire thermometers.

The velocity of a structure which translates, without undergoing change, is easily measured. But when the structure changes as it moves, the measurement of a drift velocity becomes more difficult and it is then important to investigate the strength and limitations of the various signal-processing techniques which are available. In some cases the interpretation of experimental results may be complicated by the fact that internal rearrangement in the structure may result from both turbulence and dispersive wave effects. Only turbulence effects are important in the experimental results reported here. The added complications introduced by dispersive wave effects will be discussed in the concluding paragraphs.

We shall describe first the cross-correlation measurements and then two forms of cross-spectral measurements, the first of which computes the mean phase difference as a function of frequency between two fixed probes, and the second which sorts the cross-spectral amplitude on the basis of the phase of individual realizations. The results will be compared and then discussed as they apply to solar wind studies.

Many of the ideas discussed in this report have been developed independently in the radiophysics and turbulence literatures. We have found no cross references between these two developments in any of the papers we have read. In the early radiophysics literature the cross-correlation ideas were developed by Briggs, Phillips, and Shinn (*5*). Several authors have expanded on these ideas. One recent refinement has been published by Fedor (*6*). Cross-spectral methods have been used extensively by Gossard (*7*). In turbulence studies, cross-correlation methods have been used by Favre, Gaviglio, and Dumas for many years (*8*). Willmarth and Wooldridge (*9*), Fisher and Davies (*10*), and Wills (*11*) have done similar work, as have many others. Several workers, including Wills, have worked with the two-dimensional Fourier transform of the cross-correlation.

There are two ways in which the velocity of an irregular strcuture can be deduced by correlation methods. Suppose first that one has two probes separated by a fixed spacing ξ_0 along the flow direction. The outputs of the two probes are cross-correlated to form $\rho(\tau, \xi_0)$ and the delay τ' which maximizes the cross-correlation is found. An apparent velocity, $V' = \xi_0/\tau'$, can then be computed. Briggs, Phillips, and Shinn (*5*) have shown that in the case where the structure rearranges itself as

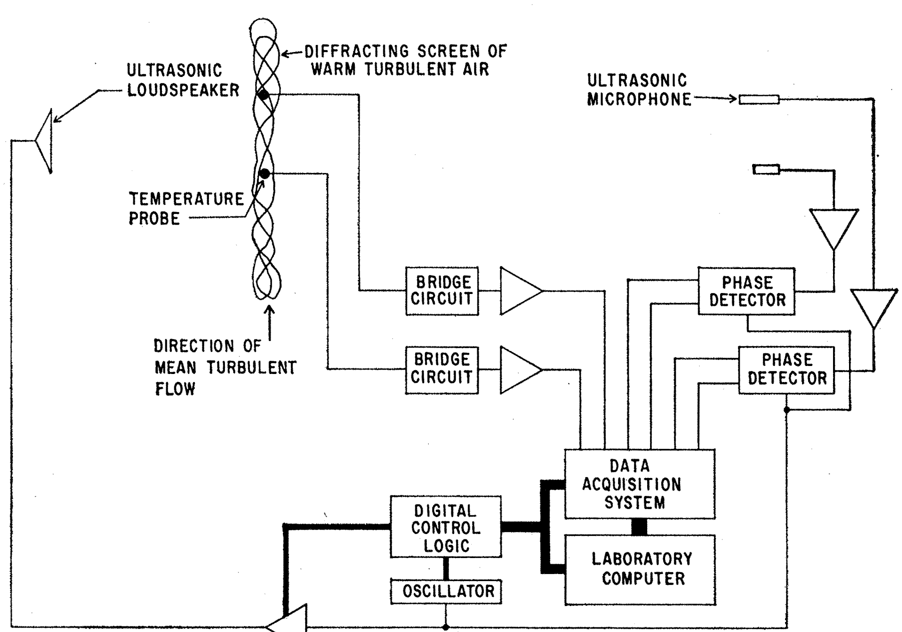

Fig. 1. A schematic drawing of the scintillation modeling experiment. An electrostatic loudspeaker illuminates a diffracting screen of warm turbulent air with ultrasonic sound in the frequency range of 40 to 100 khertz. The resulting scintillating signal is received with ultrasonic microphones, phase detected, and sent to a digital computer. Resistance wire thermometers are used to measure the structure in the turbulent screen directly.

it drifts, this apparent velocity overestimates the true drift velocity. Alternatively, the moving structure could be sampled separately with two probes at time t and $t + \tau_0$, where τ_0 is a fixed time delay. The separation of the probes, ξ, could be varied to form a second cross-correlation function $\rho(\tau_0, \xi)$. The separation ξ' which maximizes this correlation function could be found, and a drift velocity, $V = \xi'/\tau_0$, computed. This second velocity is the one at which an observer must move to minimize the rate at which the screen appears to be rearranging itself. This velocity is usually defined in radiophysics as the true drift velocity, or in turbulence as the convection velocity.

More generally we can consider a two-dimensional correlation surface $\rho(\tau, \xi)$. Most authors in radiophysics assume that a two-dimensional Gaussian correlation function adequately describes the drift process, with the result that the contours of constant correlation are ellipses. In reality this need not be so; indeed, it is clearly not the case in many boundary-layer flows (*8*).

We have measured the correlation surface $\rho(\tau, \xi)$ for the temperature fluctuations in the diffracting screen of the model experiment. Cross-correlation functions were computed between several temperature probes spaced at a collection of separations along the flow direction. From these data the correlation surface $\rho(\tau, \xi)$ can be directly constructed (Fig. 2). The apparent and true drift velocities can be scaled from $\rho(\tau, \xi)$ as the slopes of two lines, the first through the maxima of correlation for fixed probe separations and variable time delay, the second through the maxima of correlation for fixed time delay and variable probe separations (*12*). The measurement illustrated in Fig. 2 yields $V = 46 \pm 4$ cm/sec and $V' = 60 \pm 4$ cm/sec.

Table 1. A summary of the velocities obtained by the two methods of cross-spectral analysis. $V_{\langle\Delta\Phi\rangle-S}$ is the velocity computed from the mean cross spectrum. V_1 and V_2 are the two velocities scaled from the plot of cross-spectral amplitude sorted as a function of phase. ξ is the separation between the probes.

ξ	$V_{\langle\Delta\Phi\rangle-S}$	V_1	V_2
1 cm	60 ± 2	64 ± 4	52 ± 4
2 cm	55 ± 2	57 ± 5	46 ± 5
3 cm	54 ± 3	61 ± 6	46 ± 6
Average	56	60	48

While it is easy to use several probes in the laboratory, it is usually not possible to make measurements for many-probe separations in a geophysical experiment. If a signal-processing method could be found which allowed the computation of V directly from the output of two fixed sensors, it would be extremely useful in many remote probing problems. To this end, Gossard (*7*) has recently employed cross-spectral analysis to obtain the velocity of rather discrete wave structures traveling through the lower ionosphere, with striking success. We believed that the same technique might provide similar information for a turbulent structure. The measurement has proved to be somewhat more complicated than was expected, but it does appear that the velocity V can be obtained with data from just two fixed probes.

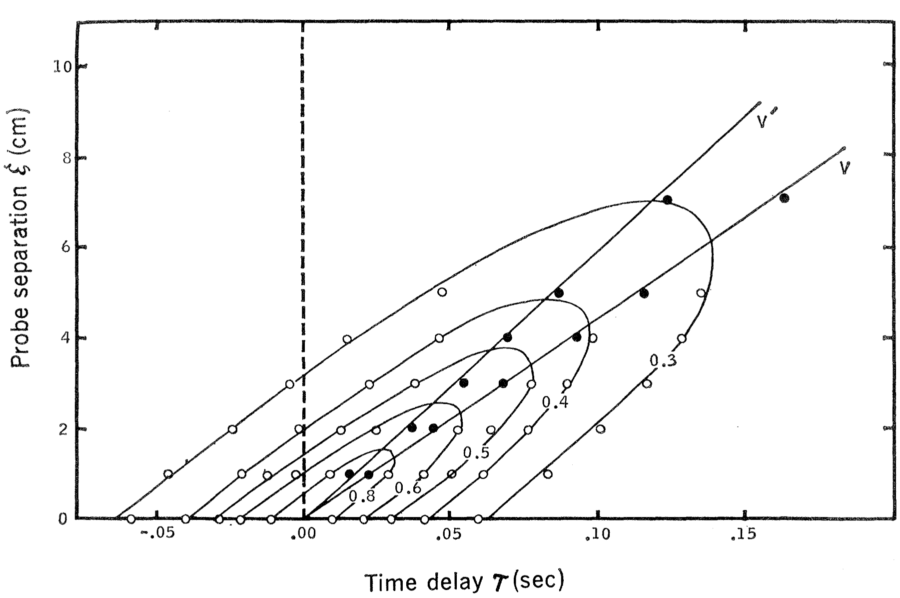

Fig. 2. The two-dimensional cross-correlation surface $\rho(\tau, \xi)$. $V = 46 \pm 4$ cm/sec, $V' = 60 \pm 4$ cm/sec. The curve was constructed from six simultaneously measured cross-correlation functions for separations of 1 to 5 cm and 7 cm along the flow direction. The curves are computed via the spectral domain by use of the fast algorithm for the Fourier transform. Each curve is composed of 1000 data blocks with 32 real data points per block. Sampling rate was 64 hertz. The solid points along the line V' mark the maxima of the cross-correlations for fixed spacings. The solid points along the line V were computed as described in reference *12*.

A drifting irregular structure may be thought of as consisting of a collection of spatial Fourier components, which in the case of neutral gas turbulence have little physical identity, but which in some instances, particularly in plasmas, have individual identities as propagating waves. One such component has been sketched in Fig. 3A along with two probes separated by a distance ξ_0 in the direction of the motion. The present discussion will be restricted to turbulent structures, without wave effects, because this is the case which corresponds to the model experiment. The frequency of the signal observed at one of the probes is proportional to the drift velocity v, such that spatial wave number k and frequency s are related by $s = kv/2\pi$. We denote the phase difference between the signals observed by the two probes as $\Delta\phi(s)$. Now consider the full collection of spatial Fourier components which constitute the irregular structure. We plot $\Delta\phi$ versus s for a fixed probe separation ξ_0. The very low frequency components will have small values of $\Delta\phi$, and $\Delta\phi$ will increase with increasing s. The resulting $\Delta\phi$ versus s plot, for the case when all spatial Fourier components move with the same velocity, will be a straight line. We would expect the velocity to be given by:

$$v = \frac{2\pi\xi_0}{\Delta\phi(s)/s} \qquad (1)$$

Measurements of $\Delta\phi$ as a function of s have been made with the same data that were used in the correlation analysis. The time series $f_1(t)$ and $f_2(t)$ from the two probes were subdivided into convenient segments each of which was then processed by use of the finite discrete Fourier transform, yielding $F_1(s)e^{i\phi_1(s)}$ and $F_2(s)e^{i\phi_2(s)}$. For each segment, or realization, a cross spectrum $C(s) = F_1(s)F_2(s)e^{i[\phi_1(s) - \phi_2(s)]}$ was then computed. The mean of many realizations, $\langle C(s) \rangle$, was computed and its argument $\{\Delta\phi(s)\}$ was measured. Substituting $\{\Delta\phi(s)\}$ for $\Delta\phi(s)$ in Eq. 1 we obtained an apparent velocity,

$V_{\{\Delta\phi\}-s}$, which was very close to V', the erroneous velocity, rather than to V, the true velocity. A typical $\{\Delta\phi(s)\}$ versus s plot is reproduced in Fig. 3B.

This result prompted a simple mathematical demonstration which shows that when the cross-correlation function $\rho(\tau,\ \xi_0)$ is symmetric, with its maximum displaced by τ' from the origin, this measurement will yield exactly the erroneous velocity V'. We write:

$$\rho(\tau,\ \xi_0) = \int f_1(t) f_2^*(t-\tau)dt \rightleftarrows F_1(s)F_2(s)e^{i[\phi_1(s)-\phi_2(s)]} \qquad (2)$$

Now $\rho(\tau,\ \xi_0)$ has been taken as symmetric, so by the shift rule of Fourier transforms,

$$\rho(\tau,\ \xi_0) \rightleftarrows F_1(s)\,F_2(s)e^{i2\pi s\tau'} \qquad (3)$$

Thus

$$\{\Delta\phi(s)\} = 2\pi s\tau' \qquad (4)$$

On substitution into Eq. 1 we obtain $V' = \xi_0/\tau'$ which is just the apparent drift velocity. In the simple case of symmetric $\rho(\tau,\ \xi_0)$, then, the $\{\Delta\phi\}$ versus s analysis yields the same result as the uncorrected cross-correlation between two fixed probes.

A physical argument can be given which explains in a more general way why this measurement does not yield the true drift velocity for such turbulent structures. It is a common feature of turbulent flows that the smaller turbulent structure is convected along by the very large structure (*13*). In addition, the various structure sizes may exhibit random velocity fluctuations of their own. A particular spatial wave number in the turbulence will be detected by the temperature probe as a higher frequency when its "instantaneous drift velocity" is greater than the mean drift velocity, and as a lower frequency when its "instantaneous drift velocity" is less than the mean drift velocity. Thus, over time, a particular frequency observed by the temperature probe corresponds to a range of spatial wave numbers as the "instantaneous drift velocity" in the neighborhood of the probe fluctuates. If the spectrum of irregularities in the turbulence were flat the influence of this effect on our measured velocity would tend to disappear following averaging of a large number of independent samples. However, the spectrum is a typical turbulence spectrum, which falls rapidly with increasing frequency. The result is that the measurement of $\{\Delta\phi(s)\}$ by use of the mean cross-spectrum at any particular frequency is weighted more strongly during those periods when the drift velocity of the large irregularities is more rapid than usual. In other words, the larger scale, and hence stronger, spatial components are Doppler shifted up to the frequency being observed.

In making the $\{\Delta\phi(s)\}$ versus s measurement from the mean cross-spectrum we have essentially used that value of $\Delta\phi$ which yields the maximum amplitude of the cross-spectrum for a fixed frequency. In order to measure V we would like to be able to find the $\Delta\phi$ which yields the maximum amplitude for a fixed k. This suggests that rather than averaging on the cross-spectrum we should sort the individual realizations according to $\Delta\phi$, and in this way compute the distribution of the amplitude of the cross-spectrum as a function of $\Delta\phi$ and s. This measurement has been made for probe separations of 1, 2, and 3 cm. The 1-cm measurement is reproduced in Fig. 3C. Since $\Delta\phi = k\xi$, and ξ is fixed, lines of constant $\Delta\phi$ are lines of constant k. We can draw lines through the points of maximum amplitude for fixed frequency and for fixed k, and using Eq. 1 we can compute corresponding velocities V_1 and V_2. The results for three probe separations are summarized in Table 1.

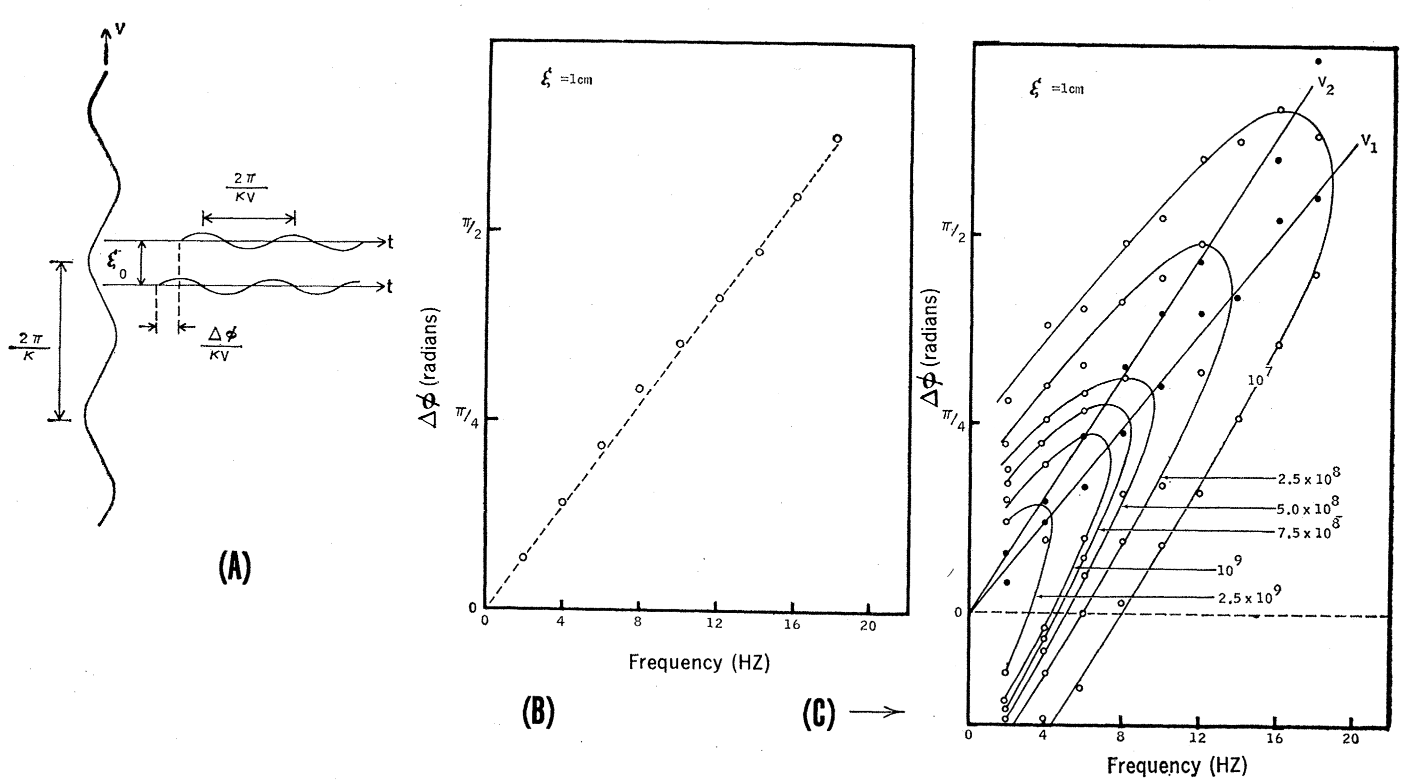

Fig. 3. (A) The geometry for cross-spectral measurements. A single Fourier component is shown. (B) A $\{\Delta\phi(s)\}$ versus frequency cross-spectral plot for a probe separation of 1 cm. The line between the probes was parallel to the flow direction. (C) A plot of the unnormalized cross-spectral amplitude sorted as a function of $\Delta\phi$. The probe separation $\xi_0 = 1$ cm. This plot contains 1000 realizations for each frequency. The frequency resolution is 2 hertz. The phase was sorted into 20 slots of equal width between 0 and 2 π. The velocity obtained by taking cuts at fixed frequencies is 64 ± 4 cm/sec. The velocity for cuts at constant k ($\Delta\phi = k$ since $\xi_0 = 1$ cm) is 52 ± 4 cm/sec.

We previously measured from $\rho(\tau, \xi)$, $V' = 60 \pm 4$ cm/sec and $V = 46 \pm 4$ cm/sec. The agreement between V' and V_1 and between V and V_2 is clearly within the accuracy of the experiment. The error estimates are intended to indicate only the size of the random errors resulting from a finite number of samples. We would expect $V_{\{\Delta\phi\}-s}$, the velocity measured from the $\{\Delta\phi\}$ versus s plot, to be equal to V_1 if the distribution of cross-spectral amplitude were symmetric in $\Delta\phi$. The distribution is not symmetric and the difference between $V_{\{\Delta\phi\}-s}$ and V_1 is the difference between the mean and the point of maximum of the distribution.

Additional work is clearly required to properly evaluate this technique. At the moment, however, it appears to be a very promising method for measuring the true drift velocity V with data from only two fixed probes separated along the direction of motion.

The observations described in this report are for simple turbulence without the presence of dispersive wave phenomena. The Doppler shift effect, which distorts the simple $\{\Delta\phi\}$ versus s velocity measurement for the turbulence case will not apply to linearly superimposed waves. The $\{\Delta\phi\}$ versus s plot for dispersive waves will usually not be a straight line but will display some curvature from which the dispersion relation for the waves might be scaled. Such a measurement would be very valuable in an ionosphere or solar wind scintillation experiment. However, even if the observer knows that he is measuring a cross-spectrum resulting only from wave effects, cross-spectral analysis gives unambiguous measurements of velocity and dispersion only for restricted geometries.

An attempt to make such measurements with ionospheric scintillations has recently been reported by Briggs and Golley (*14*). They conclude that the apparent dispersive effect which they measure was probably due to structure moving with different velocities at different altitudes rather than to true dispersive wave modes.

Similarly, if a wave structure which is generating scintillations is not all propagating in the same direction, but exists as an angular spectrum of waves, the cross-spectral results will be confused. The scintillation measurement will respond only to the projection of the wave lengths and velocities of the diffracting wave structure on a plane normal to the direction of the probing ray. The resulting cross-spectrum may assume very rich structure which need not be uniquely related to the diffracting screen geometry.

When turbulence and dispersive wave phenomena are combined, as they probably are in the solar wind, the situation becomes even more complicated. It may prove possible to scale a dispersion relation from the $\Delta\phi$ versus s cross-spectral distribution surface in some cases. We find that when used in scintillation studies cross-correlation and cross-spectral measurements will yield unambiguous information about the diffracting screen only when the observer can safely make additional assumptions about the screen structure.

While these results demonstrate certain limitations of the cross-correlation and cross-spectrum analysis methods, they do not express a fundamental limit to the amount of information available about the irregular structure from diffraction measurements. It may be possible to obtain additional information with techniques of higher-order spectral analysis (*15*). Further, it is often true that special characteristics of the medium under study allow logical resolution of the ambiguities limiting the diffraction techniques. For example, plasma resonances may be expected in the ionospheric or interplanetary plasmas, and these resonances should often be useful in the experimental interpretations.

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References and Notes

1. See, for example, A. Hewish, P. A. Dennison, J. D. H. Pilkington, *Nature* **209**, 1188 (1966); P. A. Dennison and A. Hewish, *ibid.* **213**, 343 (1967).
2. See, for example, M. H. Cohen, E. J. Gundermann, H. E. Hardebeck, L. E. Sharp, *Astrophys. J.* **147**, 449 (1967); M. H. Cohen, E. J. Gundermann, D. E. Harris, *ibid.* **150**, 767 (1967).
3. K. Bartusek and D. G. Felgate, *Australian J. Phys.* **19**, 545 (1966).
4. G. R. Ochs, paper presented to the 1967 Fall URSI meeting at Palo Alto, California (U.S. National Meeting of the International Scientific Radio Union).
5. B. H. Briggs, G. J. Phillips, D. H. Shinn, *Proc. Roy. Soc. London Ser. B* **63**, 106 (1950).
6. L. C. Fedor, *J. Geophys. Res.* **72**, 5401 (1967).
7. E. E. Gossard, *ibid.*, p. 1563.
8. A. Favre, J. Gaviglio, R. Dumas, *Phys. Fluids Suppl.* S138 (1967).
9. W. W. Willmarth and C. E. Wooldridge, *J. Fluid Mech.* **14**, 187 (1962).
10. M. J. Fisher and P. O. A. L. Davies, *ibid.* **18**, 97 (1963).
11. J. A. B. Wills, *Proc. Intern. Congr. Appl. Mech. 11th Munich*, p. 889 (1964). [See also *J. Fluid Mech.* **20**, 417 (1964).]
12. The line for fixed time delay and variable probe separation may be hard to draw with accuracy. An equivalent way of deducing V is to construct the envelope of the cross-correlation curves $\rho(\tau, \xi_0)$. This envelope will be the moving frame auto-correlation. The points where the moving frame auto-correlation is tangent to the cross-correlation curves can be scaled as τ_n. If ξ_n are the corresponding probe separations then $V = \xi_n/\tau_n$.
13. J. L. Lumley, *Phys. Fluids* **8**, 1056 (1965).
14. B. H. Briggs and M. G. Golley, paper presented to the IAGA/URSI symposium Upper Atmosphere Winds, Waves, and Ionospheric Drifts, St. Gallen, Switzerland, October 1967.
15. See, for example, K. Hasselmann, W. Munk, G. MacDonald, "Bispectra of Ocean Waves," in *Time Series Analysis*, M. Rosenblatt, Ed. (Wiley, New York, 1963); T. Madden, "Spectral, cross-spectral, and bispectral analysis of low frequency electromagnetic data," in *Natural Electromagnetic Phenomena Below 30 kc/s*, D. F. Bleil, Ed. (Plenum, New York, 1964); M. P. Aubry, *Ann. Astrophys.* **29**, 389 (1966) and **30**, 101 (1967).
16. We thank R. D. Abney, C. W. Van Atta, M. H. Cohen, W. A. Coles, and C. H. Gibson for their helpful suggestions and discussions. The work was supported partially by NASA grant NsG-538, partially by Project DEFENDER 67/68-181, and partially by NSF grant GA-744.

20 June 1968

Murray Fracture Zone: Westward Extension

Abstract. *The Murray Fracture Zone is one of the principal east-west rifts in the crust of the northeast Pacific basin. As judged by bathymetric and magnetic surveys, the Murray approaches the Hawaiian Archipelago as a well-defined zone of ridges and troughs accompanied by strong, linear magnetic anomalies. It loses its topographic expression on encountering the Hawaiian Arch but can be traced magnetically to its intersection with the Hawaiian Ridge in the vicinity of Laysan Island (near 172°W). All evidence tends to discount a previously suggested genetic relation between the Murray Fracture Zone and the Necker Ridge.*

The long, linear fracture zones (*1*), which trend in an east-west direction across the northeast Pacific basin and offset well-defined patterns of north-south trending magnetic anomalies, are of primary significance and must be considered in any geological model of the earth. One of these, the Murray Fracture Zone, lies between and parallels two others, the Mendocino and the Molokai fracture zones. The Murray Fracture Zone has been traced from a point off the coast of southern California westward for more than 4000 km. It has been suggested (*2, 3*) that before reaching the Hawaiian Archipelago, the Murray bends abruptly southwestward and continues into the Necker Ridge which trends southwestward from Necker Island (Fig. 1). De-