Celestial Mechanics Experiment

Abstract. *Equipment on Mariner V has yielded values for the masses of Moon and Venus more accurate than any previously reported. Range and Doppler radio tracking data necessary for precise space navigation of the spacecraft from Earth to Venus can also be used to obtain data on the orbits of Earth and Venus.*

The range and Doppler radio tracking data from Mariner V have been fitted by the method of weighted least squares to obtain the masses and other parameters of Venus and the Moon.

The direct determination of the mass of Venus from the encounter orbit is fundamentally expressed in terms of the universal gravitational constant G times the mass $M♀$ and is

$$GM_♀ = 324{,}859.61 \pm 0.49 \text{ km}^3/\text{sec}^2$$

If the radar value of the astronomical unit (A.U. = 149,597,890 km) is assumed, the corresponding Sun-Venus mass ratio from Mariner V is 408,522.6 ± 0.6.

The monthly periodic motion of Earth about the center of mass (barycenter) of the Earth-Moon system is appreciable in both range and Doppler data. Therefore, the amplitude of this motion can be determined from the four months of data from the cruise, between the end of the mid-course maneuver and planetary encounter, and, from this determination of amplitude, a value of the Earth-Moon mass ratio μ^{-1} can be deduced. Because of the range data, which have not been available from other spacecraft in interplanetary space, the determination of μ^{-1} from Mariner V is potentially the most accurate value reported to date. Indeed, the expected uncertainty in μ^{-1} is about ±0.0002. However, various fits to the range and Doppler data have produced values from 81.2999 to 81.3009. Evidently, systematic errors are entering current solutions, and until these error sources can be removed the best that can be done is to set μ^{-1} equal to 81.3004 and to increase the uncertainty figure to ±0.0007.

To obtain some measure of the goodness of fit of the solutions of encounter and cruise the root mean square (RMS) and mean residuals are listed respectively in Tables 1 and 2 for each tracking station and for the total of data from all stations. If there are N residuals (δz_i; $i = 1,2\ldots,N$) for which mean residuals are to be computed, then the RMS residual is defined by

$$\left[\sum_{i=1}^{N} (\delta z_i)^2/N\right]^{1/2}$$

and the mean residual is simply

$$\sum_{i=1}^{N} \delta z_i/N$$

The RMS residual does not account for the distribution of residuals with respect to sign; the mean residual indicates whether the residuals are distributed about a zero mean.

In all solutions, the RMS Doppler residual is less than 1 mm/sec, an indication of the accuracy of the data. There are two encounter solutions represented in Table 1, the first a fit to Doppler data only from 5 days before encounter to 5 days after, and the second a fit to Doppler and range data during the same period. Moreover, the Doppler solution without range data includes only corrections to the six orbital elements of the spacecraft and the mass of Venus, while the Doppler and range solution includes, in addition, corrections to the position vector of Venus at encounter, a solar radiation pressure constant, and the locations of the tracking stations. Table 1 shows that the fit is good in each solution. The mass of Venus that best fits the data is the same in both; the only difference amounts to about one-third the standard deviation of ± 0.49 km³/sec² on $GM_♀$. An error of ± 1.67 mm/sec has been assumed for each encounter Doppler measurement in computing standard deviations (Table 1). When the total RMS residual is divided by this assumed error, a measure of about 0.425 for both solutions for dimensionless overall goodness of fit is obtained.

The cruise solution that gives a value for the Earth-Moon mass ratio (Table 2) shows mean residuals similar to those in Table 1 in the Doppler data, and the range residuals reveal an RMS residual of about 50 m. This value results from a numerical "round-off error" arising when the range data are computed to only eight significant figures (single precision on the IBM 7094 computer), and the value does not represent the measurement error of about 10 to 20 m. The range residuals from Station Goldstone, 14 (the 63-m station) are much greater than from the other 25.5-m sites which track to a distance of about 6×10^6 km. By contrast, the 63-m antenna can track to planetary encounter (about 80 mil-

Table 1. Mean residuals for encounter solutions at 10-minute sample interval. Doppler error weight = 1.67 mm/sec. Total RMS residual/error weight = 0.422 (no range). Total RMS residual/error weight = 0.425 (with range).

Station	No. of points	Doppler residual, no range (mm/sec)		Doppler residual with range (mm/sec)	
		RMS	Mean	RMS	Mean
Cebreros	354	0.691	−0.131	0.686	−0.063
Tidbinbilla	18	.502	.021	.504	−.029
Woomera	344	.653	.042	.568	−.127
Johannesburg	31			.424	.008
Goldstone, 12	177	.558	.025	.737	−.040
Goldstone, 14	181	.932	.209	.966	.258
Total	1105	.704	.010	.709	−.020

Table 2. Mean residuals for cruise solution (A.U.) known at 10-minute sample interval. Doppler error weight = 0.791 mm/sec. Range error weight = 162 m. Total RMS Doppler residual/error weight = 0.599. Total RMS range residual/error weight = 0.316.

Station	No. of Doppler points	Doppler residual (mm/sec)		No. of range points	Range residual (m)	
		RMS	Mean		RMS	Mean
Cebreros	410	0.567	0.235	6	72.6	20.2
Tidbinbilla	971	.424	.066	298	51.5	−5.3
Woomera	553	.408	.137	74	50.1	42.8
Robledo	1074	.377	.155	285	52.6	10.1
Goldstone, 12	457	.671	.093	65	41.2	−3.3
Goldstone, 14	249	.555	.121	140	2027	450
Total Doppler	3714	.474	.128			
Total range less Goldstone, 14				728	51.2	6.0

lion km and beyond), and the round-off error is proportionately larger. Of course, solutions by the weighted-least-squares method depend on the relative weighting of the range and Doppler data. The total RMS residual divided by the assumed error weight is about the same for both types of data (Table 2), which indicates that the relative weighting is reasonably correct.

In the future, the computation of the range data will be accomplished in extended precision, and the error weight will be reduced to the level of the measurement error. Also, the data for cruise and encounter will be combined in a single solution to obtain important information on the astronomical unit and certain elements of Earth's orbit. At present the best that can be said is that the radar value of the astronomical unit (149,597,890 km) is consistent with the Mariner V data, and that the cruise solutions accomplished to date (those which include corrections to Earth's orbital elements) indicate that significant corrections on the order of a 0.1 second of arc are needed to fit the data. One correction in particular requires that the instantaneous axis of revolution of Earth about Sun be moved by at least ½ second of arc toward Earth's axis of rotation to produce residuals as small as those indicated by Table 2.

J. D. ANDERSON
G. E. PEASE
L. EFRON
Systems Analysis Research Section, Jet Propulsion Laboratory, California Institute of Technology, Pasadena
R. C. TAUSWORTHE
Communications Systems Research Section, Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California

11 December 1967

Aluminum-26 and Beryllium-10 in Greenland Ice

Abstract. *Activities of beryllium-10 and aluminum-26 dissolved in 200-year-old Greenland ice were found to be 18.4 (+ 8.4, − 4.8) × 10^{-6} and 3.2 ± 0.9 × 10^{-7} disintegration per minute per liter, respectively. From these values and the precipitation rate (30 milliliters of water per square centimeter per year), the production rates of these isotopes are calculated to be 3.6 (+ 1.6, − 0.9) × 10^{-2} and 1.7 ± 0.5 × 10^{-4} atom per second • square centimeter. These rates agree with the rates calculated for the production of these isotopes by cosmic rays in the atmosphere. Probably all the Al^{26} in the ice is accounted for by such atmospheric production; however, an upper limit for the influx of cosmic dust bearing aluminum-26 is calculated at 3.2 × 10^5 tons per year for Earth. Only upper limits could be found for Al^{26} and Be^{10} in the undissolved particulate matter in the ice; their addition to the activities in the dissolved material leaves our conclusions unchanged.*

The nature and flux of extraterrestrial material accreted by Earth are highly uncertain. It has often been stated, but never proved, that the bulk of this material is in the form of micron-size particles from the interplanetary dust cloud. If extraterrestrial material is accreted uniformly over Earth's surface, sediments that accumulate at the lowest rate should contain the largest extraterrestrial component. It is clear, however (Table 1), that even the most slowly accumulating sediments are predominantly terrestrial. The problem of measuring and characterizing the extraterrestrial component is therefore not trivial.

If sufficient amounts of a sediment sufficiently rich in cosmic dust are available, isotope anomalies and radioisotopes produced by cosmic rays may be found in it (*1*) as they are in meteorites. Dust from Greenland ice and fractions of deep-sea sediments have yielded inert gases showing anomalous isotope ratios (*2*), and claims have been made for measurable amounts of Al^{26} produced by cosmic rays in deep-sea sediments (*3, 4*). However Al^{26} was not detected in the dust from Greenland ice that yielded the anomalous inert gases (*5*). This discrepancy is important. Anomalies in rare-gas isotopes may be explained by the presence of diverse types of extraterrestrial material (micron-size dust grains, cometary or asteroidal material entering the atmosphere as large chunks, or even organic material), but Al^{26}, if present, sets severe limits on the type of extraterrestrial material that can be hypothesized (*6*); it also provides a measure of the flux of extraterrestrial material reaching Earth, which the anomalies in rare-gas isotopes can do only by elaborate theoretical models (*7*).

Some Al^{26} is produced by cosmic rays in Earth's atmosphere, but the activities found in sediments by Lal and Venkatavaradan (*3*) and Wasson (*4*) are greater than would be expected from this source.

The search for Al^{26} in Greenland ice was incomplete in that only the undissolved particulate material [chiefly clay minerals and an unidentified organic component (*8*)] was collected and examined. There are many reasons for expecting that the bulk of the extraterrestrial material in the water from the ice is dissolved: extraterrestrial material may reach the ice as finely divided and readily soluble dust or smoke, the water is very pure and slightly acidic [total dissolved material, less than 1 part per million (ppm), *p*H 4.5], and the chemistry of the dissolved material in the ice indicates the presence of extraterrestrial material (*8, 9*). In any case, radioisotopes produced by cosmic rays in Earth's atmosphere may be expected to be dissolved in the ice, and a measurement of these would be of interest. We now report the measurement of Al^{26} and Be^{10} dissolved in Greenland ice; we also describe an attempt to measure Be^{10} and further attempts to find Al^{26} in the particulates. We find these isotopes only in the amounts expected from their production by cosmic rays in the atmosphere.

The facilities and methods for collecting particulate material from 100- to 300-year-old ice at Camp Century, Greenland, have been described (*5*). In our work, columns containing cation-exchange or mixed cation- and anion-exchange resins were inserted into the waterline. Later the resins were removed from the columns and homogenized, and the cation resins were leached with 4*M* HCl; the solutions so obtained have been subjected to extensive chemical study that will be reported elsewhere (*8*).

For the search for Al^{26} and Be^{10}, a solution obtained by leaching resins through which 1.2 × 10^6 liters of ice water had passed was used, and carriers for Be, Ti, Mn, and Co were added. The elements were then separated by standard methods of analytical chemistry. Final purification of the aluminum, chiefly to remove thorium and its decay products, was accomplished by solvent extraction with cupferron (*4, 10*). Beryllium was purified by repeatedly precipitating it as the