Taking 50 percent as an upper limit to the possible values of the porosity, the highest value of the grain dielectric constant is 5.9, which is inconsistent with the values found for chondritic meteorites (*15*). Because of the difficulty of measuring the dielectric constant in conducting materials, Fensler *et al.* (*16*) give values for only two chondrites—Leedy and Plainview; they did, however, measure enough other ultrabasic rocks to give assurance that high values of the dielectric constant (7.2 or more) are associated with such rocks.

The value of 5.9 is reached by stretching the data in each respect (bulk dielectric constant, conversion formula, and porosity) in favor of the hypothesis of very basic material. Using the more-plausible values of 2.7 for the bulk dielectric constant (*12*) and 40 percent for the porosity, and the Krotikov formula (*11*),

$$(\sqrt{\epsilon} - 1)/\rho = (\sqrt{\epsilon_0} - 1)/\rho_0 \qquad (3)$$

where ρ is the bulk specific gravity, and ρ_0 is that of the solid material, we find 4.3 plausible for the grain dielectric constant. This finding would indicate either an acid rock (granite, rhyolite, or tektite) or a vesicular basaltic rock.

The discrepancy between the maximum value of 5.9 for the grain dielectric constant, and the much higher values for chondrites, could be removed if we could assume that the chondritic material is highly vesicular, having 25 to 40 percent of the volume of the average grain occupied by voids (in addition, of course, to the intergranular voids amounting to 50 percent of the volume). Unfortunately it is well-established that chondritic meteorites are not vesicular in this way; some of them are porous, but the porosity is intergranular—not intragranular.

Shock might conceivably produce porosity in chondritic meteorites. If this were so, and if the lunar surface were the source of chondritic meteorites, then we would expect the porosity to be greatest in pieces that had been most strongly shocked. Pieces expelled from Moon would be more strongly shocked and therefore more porous than those that were simply knocked from one place to another on Moon. But in fact we find that chondritic meteorites reaching Earth are never porous; thus it follows that chondritic material on Moon should not be porous. Measurements now available of dielectric constant do not cover carbonaceous chondrites.

Although these results are based on soil experiments at only two lunar sites, the optical, radar, radiometric, and thermal data indicate that the lunar surface is much more homogeneous than Earth's; these experimental data are probably typical of the maria at least. We conclude that the surfaces of the lunar maria are probably not composed of material similar to ordinary chondritic meteorites.

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Galaxies as Gravitational Lenses

Abstract. *The probability that a galaxy gathers light from another remote galaxy, and deflects and focuses it toward an observer on Earth, is calculated according to various cosmologic models. I pose the question of whether an object called a quasar is a single, intrinsically luminous entity or the result of accidental alignment, along the line of sight, of two normal galaxies, the more distant of which has its light amplified by the gravitational-lens effect of the nearer galaxy. If galaxies are distributed at random in the universe, the former alternative is true. But, if we assume that most galaxies exist in pairs, we can find about 30 galaxies occurring exactly one behind the other in such a way as to enable amplification of the order of 50. This model explains also the variations in intensity in quasars, but fails to explain others of their observed properties.*

When electromagnetic waves pass near a massive object, within distance r of its center, they are deflected toward the mass by an angle (θ) predicted by the general theory of relativity:

$$\theta = 4\,GM/rc^2$$

where G is the gravitational constant, M is the mass of the object, and c is the velocity of light in vacuum.

Einstein (*1*) pointed out that the mass can act as a lens, deflecting the light coming from a distant star S and focusing it. This lens has peculiar focusing properties: light coming from infinity and grazing the limb of the mass is focused closer to the mass than is light passing at greater distance from the center of the mass (Fig. 1).

For certain values of M and d, the light from star S passes through an annulus of radius r and is deflected and reaches the observer. The minimum distance d_m at which this happens satisfies the equation

$$4\,GM/ac^2 = \theta = a/d_m;\ \text{or}\ d_m = a^2c^2/4\,GM$$

where a is the radius of the mass D. Thus the only objects in the sky that can act as gravitational lenses are those situated at distances greater than a^2c^2/-$4\,GM$. Using the known values for the radii and masses of stars and galaxies, one can predict which of them (Table 1) is a candidate for action as a gravitational lens.

One concludes from Table 1 that any object like Sun can demonstrate a gravitational-lens effect provided it is sufficiently distant and provided there is a star behind it. The probability of seeing the effect within our galaxy (*4*) is very slight. The possibility of galaxies acting as gravitational lenses has been discussed (*5*). One can see that all the normal galaxies (represented by M87 in Table 1) do not qualify as gravitational lenses. If the masses and radii assigned to quasistellar sources are correct, these sources are the only galaxies capable of acting as gravitational lenses.

Before calculating the probability of finding a galaxy behind a quasistellar source, let us write down the amplification factor A by which the flux from a distant source, as recorded by us, is multiplied if a mass is introduced between us and the source; the amplification is:

$$A = \sqrt{(4\,GM/dc^2\mu)}/\alpha$$

where α is the angular spacing between the two objects, and $\mu = 1 + (d/d_1)$. If we restrict ourselves to instances in which the second galaxy is in the "shadow" of the first, so that $\alpha \leq a/d$,

$$A \geqslant \sqrt{4\,GMd/c^2a^2\mu} = \sqrt{d/d_m}$$

which equation leads to amplification by at least 100 for our hypothetical galaxy and by 10^5 for 3C273 (if $a = 10^{17}$ cm).

This result shows that the amplification increases with distance from Earth to the deflecting object, a fact that was noted by Einstein as "most curious" (*1*). Thus, if a galaxy like M87 had a smaller radius and happened to be at distance $d > d_m$, it could become a gravitational lens.

Amplification of the apparent luminosity can become very high if one of the constituents of the galaxy (one of its stars, for example) stands exactly on the line between the observer and the center of the deflecting galaxy; the amplification then becomes

$$A \cong 2\sqrt{4\,GM/dc^2\mu}/\phi_o \qquad (4)$$

where ϕ_o is the angular diameter of the object. For an idea of the order of magnitude involved; $A = 3 \times 10^{11}$ if the object has the diameter of Sun and is situated behind a galaxy having 10^{11} times the mass of Sun at a distance 2×10^{27} cm. This amazingly high amplification means that if an object like Sun was at that distance (without amplification, its magnitude is

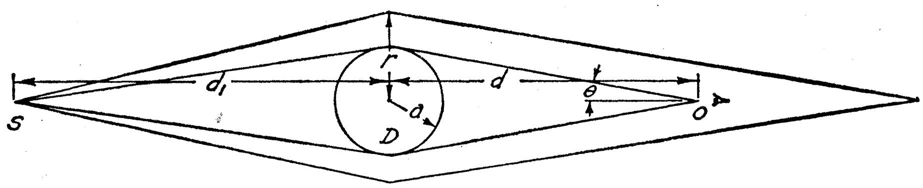

Fig. 1. The gravitational-lens focusing mechanism shown diagrammatically.

Table 1. Characteristics of some typical heavenly objects, bearing on their qualifications as gravitational lenses; those qualifying have d greater than d_m, d being the true distance of the object from Earth. A hypothetical galaxy is used to avoid reference to quasars.

Object	M (g)	a (cm)	d_m (cm)	d (cm)
Sun	1.95×10^{33}	6.9×10^{10}	8×10^{15}	1.5×10^{13}
M87	2×10^{45}	6×10^{22}	10^{28}	4.1×10^{25}
3C273 (*2*)	2×10^{44}	10^{17}*	1.7×10^{17}	1.4×10^{27}
3C273 (*3*)	2×10^{44}	3×10^{21}*	3×10^{26}	1.4×10^{27}
3C48 (*2*)	2×10^{44}	10^{17}*	1.7×10^{17}	3.3×10^{27}
Hypothetical	2×10^{44}	2×10^{19}	6.7×10^{23}	10^{27}

* Knowledge of the radius of the galaxy must not derive from the period of variation on luminosity.

+44) it would show as a 16th-magnitude star [compared to magnitude 16 for 3C48, 17.8 for 3C196, and 13 for 3C273 (*6*)]. If there is a transverse motion of the first galaxy, with a relative velocity between the observer and this galaxy of 10^3 km/sec, the line between the observer and the center of this first galaxy scans across this sun in 5 minutes, causing a scintillation having the same period. If a larger object is encountered, a longer period is recorded. I must emphasize that, if the obscured galaxy is similar to our own, the probability of an encounter with a star is small and will happen only once in 100 years for 10^3 km/sec, or once in 10 years for 10^4 km/sec, or for $d_1 = 10d$.

If there is evidence that a gravitational-lens effect prevails, the variation period can be used as a tool for investigation of the dimensions and specific luminosity of the individual constituents of the hidden galaxy. [As the upper limit to the diameter of a galaxy can be determined from the period of variation (for example, *7*), one should, in view of the preceding argument, avoid reliance on such limits if amplification is believed to occur.]

The probability of a galaxy being a lens depends on the number of galaxies to be found behind it, and this number depends on the cosmologic model. Using Sandage's tables 1 and 3 (*8*), I calculated the number of galaxies in a truncated cone having an opening angle a/d. The height of the cone, or the depth in space penetrated, was chosen as ten times the distance d: equivalently, the distance over which the apparent luminosity decreases by a factor of 100 (or 5 units of magnitude). The reason for choice of a factor of 100 is that the amplification, by our hypothetical galaxy, can be at least as great as 100-fold to compensate such loss in luminosity; one may still detect every galaxy in this cone. (In fact 60 percent of the galaxies in this cone have amplification between 1 and 2; 0.2 percent, amplification greater than 40.)

My results show that if the first galaxy is at a distance of 10^{27} cm (or the red shift $\Delta\lambda/\lambda = Z = 0.15$), the number ($N$) of galaxies in such a cone, having an opening angle of 1 deg, is $10^{3.86}$ for a deceleration parameter q_o of -1, $10^{3.8}$ for 2.5, $10^{3.75}$ for 5, and $10^{3.4}$ for 8.5. The number of galaxies does not, therefore, depend sharply on the model, but on the distance to the first galaxy, and gives $N = 10^{4.5}$ for $q_o = -1$ if $Z = 0.38$, and 10^5 for $q_o = -1$ if $Z = 0.75$ (for $Z > 2$ and $q_o > 7$, N starts to decrease, for most of the galaxies are already between us and the first galaxy and not behind it). The number of galaxies in a cone having an opening angle of $\theta = a/d = 2 \times 10^{19}/10^{27} = 2 \times 10^{-8}$ radians (for the hypothetical galaxy) is $10^{-12} \times 10^{3.8} = 10^{-8.2}$ ($q_o = 2.5$, $Z = 0.15$), or 10^{-7} ($q_o = 1$, $Z = 0.75$). There are 240 galaxies per square degree between $Z = 0.15$ and $Z = 0.3$, or $240 \times 41 \times 10^3 \cong 10^7$ over the entire sky, so that there is an 8-percent probability of finding one gravitational lens, in the whole sky, having a magnitude corresponding to the range $0.15 \leq Z \leq 0.3$. If one integrates to see how many gravitational lenses can be found, and if one uses a telescope powerful enough to penetrate until $Z = 2$, the answer is 40 galaxies for $q_o = 1$; 65, for -1; and 3, for 2.5. These numbers were found by use of the arbitrary assumption that all galaxies at distance are so compact that their radii are no greater than 2×10^{19} cm. Clearly this assumption maximizes these numbers.

If one wishes to confine himself to instances in which the amplification is so great that the total flux received is of the order of 50 times greater than the flux received from one normal galaxy, the deflecting galaxy should be 50 times smaller in radius than our model galaxy. Therefore the probability of finding a galaxy behind it is reduced by a factor of 2500, and the total number of galaxies that are 50 times more luminous than a normal galaxy over the entire sky becomes 2×10^{-2}!

One is tempted to explain the apparent large amount of energy emitted by quasars, and their fluctuations in luminosity, as results of accidental alignment, along the line of sight, of two normal galaxies in such a way that the light from the distant one is amplified by the gravitational-lens effect of the nearer. But the probability of such an alignment has just been calculated to be so small that such event cannot be found. One's conclusion is that quasars are not demonstrations of gravitational lenses unless additional assumptions are made: for example, if the density of galaxies increases much more rapidly with increasing distance, the probability of alignment increases. But this is an arbitrary assumption lacking theoretical or observational backing; such an assumption was made by Barnothy (*9*) who claimed that 3000 Seyfert galaxies can be found behind each other and act as gravitational lenses.

Another assumption that I would like to contemplate is the following: Let us assume that most of the galaxies in the universe come in doublets. Let us assume the distance between each two components to range between 10^{21} and 10^{22} cm (3 to 30 kiloparsecs) (the separation between the optical doublet of Cygnus A is of that order of magnitude), and the radius of each galaxy to range from 10^{17} to 10^{18} cm. When the line connecting the two galaxies is normal to the line to Earth, nothing significant happens, but, if one galaxy is behind the other, amplification may occur. (For $d = 10^{28}$, the angular separation of the doublets is no greater than 0.2 second.) The amplification in this instance is (for $d \gg d_1$)

$$A = \sqrt{\frac{4\,GMd}{a^2c^2\,[1 + (d/d_1)]}} \cong \sqrt{\frac{4\,GMd_1}{a^2c^2}}$$

$$= 25 \text{ for } a = 10^{18},\ d_1 = 10^{22}$$

$$= 79 \text{ for } a = 10^{17},\ d_1 = 10^{21}$$

The probability of finding one component behind the other is $2\ (a^2/d_1^2) = 2 \times 10^{-8}$. Therefore, of 1.6×10^9 galaxies over the entire sky up to a distance of $Z = 2$ (for $q_o = -1$ or 1, 2×10^9 for $q_o = \frac{1}{2}$), 32 are aligned—a number that is comparable to number of quasars found (*10*). This model can explain the great luminosity emitted by quasars, and the variation recorded (if the relative velocity between the two components in 10^3 km/sec, variations having short periods can occur, as I have just explained). But it fails to explain other characteristics of quasars, such as the excess of short wavelengths (blue) in the visible spectrum of quasi-stellar sources, the two sets of red shifts found for some of them, or the greater flux of radio emission from quasi-stellar sources than from normal galaxies. In order to obtain a number comparable to the number of observed quasars, I was forced to make assumptions that are unsupported by observation.

Another interesting idea follows: Every two galaxies in the universe define a line; there are approximately $(10^9)^{10^9}$ such lines. The density of photons on each line is higher than outside the lines because of the gravitational-lens effect. We have just calculated that the probability of finding a third galaxy on this line is very small; but, if there exist in the universe thin clouds of highly ionized material happening to be on this line, these can behave like mirrors and reflect the light toward us. Thus a cloud that is undetected under normal conditions becomes visible and reflects the properties of the distant two galaxies (like the red shift and the fluctuations). Here again are assumptions that are unsupported by observations, because we must assume that these clouds are as large as galaxies and that their densities are much greater than those of galaxies. Thus we conclude that quasars are unlikely to be amplified normal galaxies; that instead they are real entities having large masses, small radii, and intrinsically high luminosities.

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10. This is the order of magnitude of quasars for which red shifts have been found. The absolute number of quasars, according to sample survey, is of the order of 10^4. I should emphasize that when one allows the distance between the two galaxies d_1 to be 10^{20}, the number of gravitational lenses increases to 3200.
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Undersea Penetration by Ambient Light, and Visibility

Abstract. *Undersea observations from various submersibles reveal penetration by ambient light to depths as great as 700 meters. The range of horizontal viewing under ambient light in offshore tropic and subtropic areas varies from 5 to 6 meters (estimated) at 300-meter depth to more than 60 meters at 183-meter depth. Such observations indicate that ambient light may be usable for undersea tasks to greater depths than was anticipated.*

Before Beebe's dives by bathysphere in the 1930's (*1*) information was sparse concerning ranges of visibility at water depths much beyond 30 m. In Beebe's era this information was neither deemed necessary nor even so envisioned. Recent years, however, have witnessed a surge of interest in undersea operations, and concomitant growth of technology has made possible the employment of deep-diving manned submersibles.

Apart from Beebe's in situ observations, the major emphases in under-